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Networked Robots Path Planning Using Simultaneous Localization and Mapping Algorithm

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Abstract:- Multi-robot map merging is an essential task for cooperative robots navigation. Each robot is building its own local map with different reference. To merge these maps to global one, the transformation between the local maps must be computed so that we have one reference. Data association and inter robot observations are two alignment methods which are used to compute the transformation parameters. Data association uses correspondences between each pair of maps and inter robot observations uses robot-robot observations when robots observed each other. This paper evaluates these two alignment methods; and presents an improved method for map alignment. Robots will be able to choose the suitable one of them depending on the degree of overlap between partial maps to merge these maps accurately. Navigation path for a single robot is used for testing the implemented map.

Keywords:- SLAM algorithm, EKF, Particle filters, FastSLAM, Multi-robot SLAM, Map alignment techniques.

I. INTRODUCTION

Building truly autonomous robots is a highly sought-after goal in the field of mobile robotics. Autonomous robots can be used for various purposes in our lives such as building maps for search and rescue operations and executing a lot of dangerous tasks instead of humans. To autonomously perform such tasks, a mobile robot must be able to localize and plan its path. If a robot has a model of its environment, it becomes able to plan its path easily to execute its task.

For a robot to build a map of its surrounding area, it must have accurate position information within the area, and to obtain accurate position information within the area, the robot needs to have an accurate map of the area. This circular problem is what is called Simultaneous localization and Mapping (SLAM) problem [1]. There have been several techniques published for solving this problem [2] [3] [4].The problem of SLAM can be solved by a single robot, but this task will be performed more efficiently if there is a team of robots which construct cooperatively a map of the environment. Therefore, the challenge is to find strategies to efficiently mix the information collected by sensors of different robots for different states of their environment. The partial maps are estimated using FastSLAM 2.0 algorithm [5] which is implemented by Tim Bailey for single robot [6], but this paper focuses on multi robot system, i.e. a team of robots.

The rest of the paper is organized as follows: Sections II presents solutions of multirobot SLAM problem. Section III focuses on the alignment methods which are used to compute the coordinate transformation between different reference systems. Section IV explains the map merging process for creating global map. Section V presents an evaluation of the alignment methods. Section VI explains our methodology and simulation results. Section VII discusses the choosing method of the shortest path for arriving to the target through building map. Finally section VIII concludes our work.

II. COOPERATIVE SLAM

The complexity of SLAM increases when robots cooperate to construct a single map of the area they explore. In the realistic case, the robots do not know the initial positions of the others and this adds extra challenges to the problem. Multi-robot SLAM problem can be solved in two different ways; one of them is to use only one map [7] which has to be updated by every team member. But in this way, the initial relative position of the robots should be known, which it is something that may not be possible in practice. The other way, every robot makes its own partial map and, at some point of the exploration, they merge their maps into a global one. One of the major advantages of this way is that can be performed even if the relative positions of the robots are unknown. Therefore in our work, we focuss on the other way, i.e., robots build local maps independently. In order to merge maps created by different robots, the transformation between their coordinate frames needs to be determined. It will permit to transform the other robot reference frame and its landmarks into the reference frame of the leader robot, which will result in a global frame for the whole team.

The coordinate transformation can be calculated in two ways. The first is to search for landmark matches in the two maps. The most probable transformation is the one that produces the maximum number of landmark correspondences (data association). The second way, robot-to-robot measurements can be used for computing the unknown coordinate transformation. When two robots meet and measure their relative distance and bearing, this information can be used to compute the transformation required for merging the two maps (inter-robot observations). Due to noise in these measurements, the estimated transformation may be inaccurate, which in effect will reduce the quality of the merged map. Therefore this paper focuses on study these two alignment methods under different conditions such as number of overlaps between partial maps and effect of noise to improve the quality of the merged map under any conditions of state of the environment.

III. MAP ALIGNMENT TECHNIQUES

This section presents the alignment methods which compute the transformation between local maps. This transformation consists of three alignment parameters: translation in x and y (dx and dy) and rotation (Θ) which can be expressed as a transformation matrix T:

$$T = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) & d_x \\ \sin(\Theta) & \cos(\Theta) & d_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} T_\theta & & d_x \\ & & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Given two maps m and m’, T transforms the reference system of m’ into the reference system of m.

A. Map Alignment Using Data Association

The data association method depends on establishing a list of correspondent landmarks among the maps. Although the landmarks are 3D (x, y, z), the alignment is performed in 2D, because the robots move in a 2D plane (x, y). Nevertheless, the third component of the landmarks (z) is also compared when establishing correspondences. One of the best methods for performing data association is RANSAC (Random Sample Consensus) according to [8].

This technique is performed as follows [8] [9]:

a) In the first step, a list of possible correspondences is obtained. The matching between landmarks of both maps is done based on the Euclidean distance between their associated signatures. This distance should be the minimum and below a threshold th_0 = 1m. As a result of this first step, we obtain a list of matches consisting of the landmarks of one of the maps and their correspondences in the other map, i.e., m and m′.

b) In the second step, two pairs of correspondences ([(x_i, y_i, z_i), (x'_i, y'_i, z'_i)] and [(x_j, y_j, z_j), (x'_j, y'_j, z'_j)]) are selected from the previous list. These pairs should satisfy the following geometric constraint [9]:

$$(A^2 + B^2) - (C^2 + D^2)| < th_1 \tag{2}$$

Where $A = (x'_i - x'_j)$, $B = (y'_i - y'_j)$, $C = (x_i - x_j)$ and $D = (y_i - y_j)$, the threshold th_1 = 0.8 m. The two pairs of correspondences are used to compute the alignment parameters (d_x, d_y, Θ) with the following equations:

$$d_x = x_i - x'_i\cos\Theta - y'_i\sin\Theta \tag{3}$$
$$d_y = y_i - y'_i\cos\Theta + x'_i\sin\Theta \tag{4}$$
$$d^2 = d^2_x + d^2_y \tag{5}$$
$$\Theta = \arctan\frac{BC-AD}{AC+BD} \tag{6}$$

c) The third step is to look for possible correspondences that support the transformation (d, Θ) which is calculated in the previous step, i.e. The calculated transformation is applied to each point of the set m’ to transform the landmarks of the second map to the same reference system as the first map. Then, for each landmark of the transformed map, we find the closest landmark of the first map in terms of the Euclidean distance between their positions. The pairing is done if this distance between them would be less than Threshold th_2=1m. These matches are called supports. Finally, the second and third steps are repeated for every possible two pairs of correspondences which satisfy geometric constraint (2). The final solution will be transformation with the highest number of supports.

B. Map Alignment Using Inter-Robot Observations

This method, presented in [10] [11], depends on meeting robots, i.e., allowing the robots to meet at some point where they can observe each other. When they meet, they inform each other and share knowledge to merge their own maps as shown in figure (1). We will consider the situation from the point of view of only one robot.

The message from the other robot includes:

– $\mathbf{M_{other}}$: the map estimation of the other robot. Each entity $m_{other,i}$ contains the position $p_{other,i}$ and a 2x2 covariance matrix $\Sigma_{other,i}$ of the ith landmark.

– $\mathbf{p_{other}}$: the pose of the other robot. Note that p_{other} is the other robots pose and $p_{other,i}$ is the position of the ith landmark of other robot.

– $\mathbf{z_{other,self}} = \{ \rho_{other,self}, \theta_{other,self} \}$ is the observation of the other robot to self robot.

When the robot receives the message from the other robot, it calculates the transformation matrix between the coordinate frames as following:

$$d_x = x_1 + \rho \cos(\varphi_1 + \theta_{self,other}) - (x_2\cos\Theta - y_2\sin\Theta) \quad (7)$$

$$d_y = y_1 + \rho \sin(\varphi_1 + \theta_{self,other}) - (x_2\sin\Theta + y_2\cos\Theta) \quad (8)$$

$$\Theta = \varphi_1 + \theta - \varphi_2 \quad (9)$$

Where x_1, y_1 are P_{self}, x_2, y_2 are P_{other}, $\varphi 1$ and φ_2 are the final orientations of robot one and two, respectively, $\rho = (\rho_{self,other} + \rho_{other,self})/2$, and θ is the relative orientation between the robots: $\theta = \pi + \theta_{self,other} - \theta_{other,self}$.

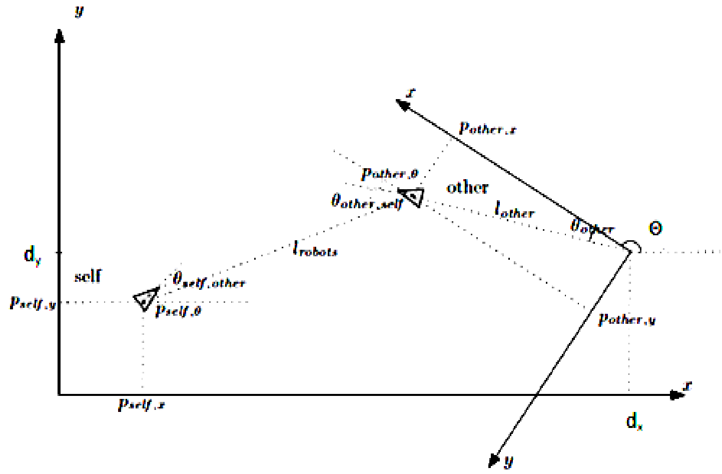

Fig.1: The configuration and parameters when robots observe each other

IV. MAP MERGING

After calculating the coordinate transformation by one of the two methods: inter-robot observations or data association, with the transformation matrix eq (1), each entity in the incoming map is transformed with the following equations [10] [12].

$$\begin{bmatrix} p_{other,x,i'} \\ p_{other,y,i'} \\ 1 \end{bmatrix} = T \begin{bmatrix} p_{other,x,i} \\ p_{other,y,i} \\ 1 \end{bmatrix} \quad (10)$$

$$\Sigma_{other,i'} = T_\theta^T \Sigma_{other,i} T_\theta \quad (11)$$

T_Θ is rotational transformation matrix.

After transforming, the Euclidean distance is used to find duplicate landmarks when merging the incoming map of the other robot with another map. If a landmark is a new landmark, it is simply added to the global map. If the landmark is also known by itself, the other robot's estimation is considered as evidence and the resulting state is calculated as:

$$\Sigma_{merged} = \Sigma_{self} - \Sigma_{self}\left[\Sigma_{self} + \Sigma_{other'}\right]^{-1}\Sigma_{self} \quad (12)$$

$$p_{merged} = p_{self} + \Sigma_{self}\left[\Sigma_{self} + \Sigma_{other'}\right]^{-1}\left(p_{other'} - p_{self}\right) \quad (13)$$

V. EVALUATION OF THE ALIGNMENT METHODS

In this section, the results of three simulation experiments are presented to provide a comparison between the two basic strategies (data association and inter-robot observations) used to combine partial maps in multi-robot systems.

A. Experiment 1

After performing FastSLAM 2.0, there are M maps of every robot. However, to perform the alignment process, only one map is needed for each robot. In [8], the map with the maximum weight is chosen. In [13], the combination of the M maps that each robot has is described. It should be noted that the M particles $\mu_i^{[L]}$, L = 1, 2...M associated with each landmark i, i = 1, ...,N_j, in which j is the index of the robot, are independent, and can be combined as follows:

$$\mu_i = w_1\mu_i^{[1]} + w_2\mu_i^{[2]} + \ldots + w_M\mu_i^{[M]}. \tag{14}$$

Where μ_i is the coordinate of landmark i, w_L is weight of particle L. The covariance matrix for every landmark in the joint map is given by:

$$\Sigma_i = \sum_{p=1}^{M} w_p^2 \Sigma_i^{[p]}. \tag{15}$$

This experiment was performed using both the map with the greatest weight and the estimated map using equations (14), (15). The results are shown in table I.

Table I: Error of the Coordinate Transformation

Inter_robot observations	**maximum weight**	[4.7347m 5.6185m 0.1096radian]
	combination maps	[4.015m 4.9955m 0.1105 radian]
Data Association	**maximum weight**	[0.3309m 0.4887m 0 radian]
	combination maps	[0.3938m 0.3386m 0 radian]

From the results, two cases [maximum weight & combination maps], are a plausible option for the map alignment process

B. Experiment 2

In this experiment, two types of maps were used. The first is a map with small degree of overlap and the second is map with large degree of overlap. The results from this experiment are shown in table II.

Table II: Error of the Coordinate Transformation

Map Alignment		**Error of coordinate Transformation**		
		dx	**dy**	**theta**
Data Association	Very little overlap	100%	100%	100%
	Large overlap	7.303%	0.76%	0%
Inter_robot observations	Very little overlap	0.953%	4.492%	0.39%
	Large overlap	7.58%	3.3403%	1.84%

From the results, it can be concluded that robustness of the estimation procedure using associations strongly depends on the degree of overlapping between partial maps as shown in table II. And when using inter-robot observations, this fact is negligible, and this strategy is preferred in the case of disjoint partial maps.

To ensure that the performance of data association method is best with increasing number of overlapping of landmarks between partial maps and performance of inter_robot observations method is little change, we showed this by increasing number of overlap landmarks as shown in table III.

C. Experiment 3

This experiment is designed to evaluate the effect of the observation noise on the two alignment methods. We changed the observation noise from 0.01m to 10m and we kept the bearing error constant. For each noise value, we ran the experiment 10 times. The result obtained of this experiment is shown in figure 2.The result shows that the inter-robot observations method gives larger error as the observation noise increases and gives good performance when the observation noise is moderate. Therefore the data association method is more suitable than the inter-robot observation method when the observation noise is large.

Table III: Compare Two Alignment Methods with Different Number of Overlap Landmarks

	Error	**Overlap=22**	**Overlap=33**
Data association	**dx**	0.5635	0.0202
	dy	0.1698	0.0497
	landmark_x_{rms}	0.36515	0.0500
	landmark_y_{rms}	0.15465	0.0662
Inter_robot observations	**dx**	8.0398	8.2956
	dy	0.97155	0.8102
	landmark_x_{rms}	2.83225	2.81375
	landmark_y_{rms}	1.74885	1.50365

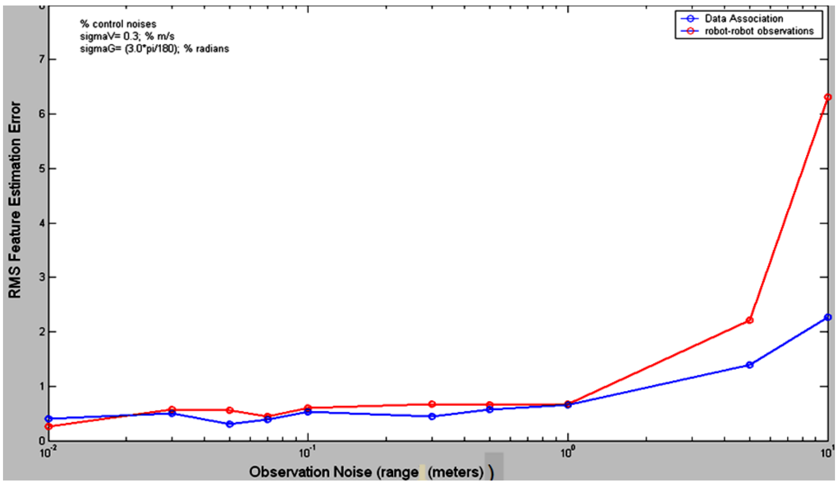

Fig.2: A comparison of the accuracy of data association and inter-robot observations on simulated data with varying observation noise.

VI. PROPOSED METHOD

Based on the results of the three previous experiments, this paper proposes an improved method for map alignment for avoiding the effect of noise and the number of overlap landmarks on accurate merging partial maps. Leader robot has the two alignment methods (data association, inter-robot observations) and after receiving information from follower robots; it begins to check the degree of overlapping between their partial maps using landmark's signature (its width) then decides which of the two methods is currently more appropriate for calculating coordinate transformation between the robot reference frames. The selection between the two methods is illustrated in figure 3. The threshold used in this algorithm, is determined from experiment 2. From the results of this experiment, we found that if the number of overlap landmarks between the two partial maps is greater than a certain threshold (thr =10), data association method gives good results than inter robot-robot observations.

Method of map alignment = check (map1, map2)

If the number of overlap landmarks >= threshold **then**

Warning ('Map alignment using data association');

[Coordinate transformation]= RANSAC (map1, map2);

else

Warning ('Map alignment using inter robot observations');

[Coordinate transformation]=robot_robot (map1, map2);

end

Fig.3: Selecting one of the two alignment methods algorithm

To verify our method, we will perform two experiments. The first experiment will be performed using two robots and the second experiment by four robots.

A. *Experiment 1*

This experiment is designed to evaluate the proposed method in this work using two robots. The map used in this experiment has gradual crowding (the number of landmarks increases gradually in the map), thus the two robots travel from empty to crowded area as shown in figure 4. At the first point of rendezvous, robot1 receives a message from the other robot and checks the number of overlaps between the two partial maps then uses the selection algorithm in figure 3 to select the appropriate method of alignment as follows:

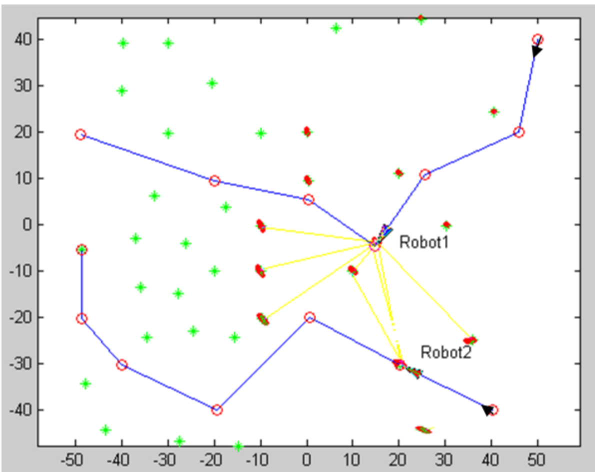

Fig.4: The two robots at the first point of rendezvous

no_overlap = 2 (small degree of overlaps), warning: "Map alignment using inter robot observations" and the transformation matrix:

$$T= \begin{bmatrix} 1.0000 & 0.0013 & 39.0889 \\ -0.0013 & 1.0000 & -39.4249 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

The first part of the global map after merging the two partial maps using equations (10, 11, 12, and 13) is shown in figure 5. After merging the maps partially, the two robots continue in their paths to reach to the second point of rendezvous. Then robot1 receives data from the other robot and check degree of overlap. The result is as follows:

no_overlap = 13 (large degree of overlaps), warning: "Map alignment using data association", and the transformation matrix:

$$T=\begin{bmatrix} 1.0000 & -0.0000 & 41.1119 \\ 0.0000 & 1.0000 & -39.2071 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

The second part of the global map after merging the two partial maps is shown in figure 5. Once the two robots reach the end of the path, we get the final estimated of global map for the simulated environment as shown in figure 6.

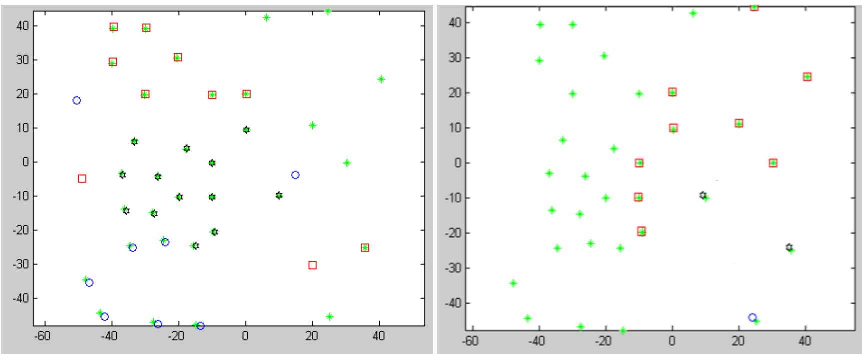

Fig.5: The first part of the global map after the first point of rendezvous (left) and the second part of the global map after the second point of rendezvous (rigth)**,** The squares are the landmarks only mapped by robot one, circles are the landmarks only mapped by robot two and hexagram are the combined estimates of landmarks considered correspondent once the transformation is applied.

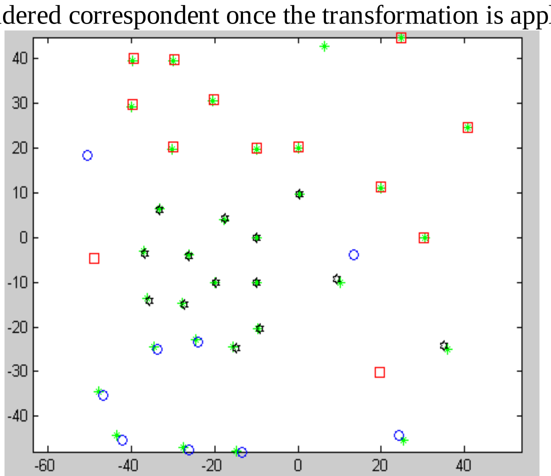

Fig.6: The final global map

B. Experiment 2

One of the reasons for using multirobot systems is reducing the task accomplishment time. Thus the multi robot case, in which the number of robots is higher than 2. In this experiment, we used four robots to explore an unknown environment to decrease time of mapping task as shown in figure 7.

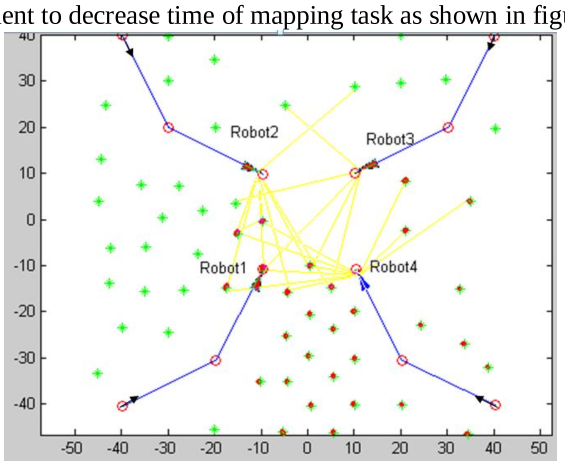

Fig.7: Simulated environment with four Robots

Robot1 is considered a leader robot and the others are follower robots. Each robot explores part of the environment and forms its local map by FastSLAM 2.0, then at the rendezvous point each follower robot sends its partial map, its position and its observation to the leader robot as explained in section III. When leader robot receives these messages, it begins to use the selection algorithm in figure 3 to decide which map alignment method is best to merge its partial map with all partial maps that are received, as shown in figure 8.

In figure 8.a, the result is as follows:

no_overlap = 14, therefore the map alignment method which is used by the leader robot is data association and the transformation matrix is:

$$T = \begin{bmatrix} 1.0000 & -0.0000 & -39.7589 \\ 0.0000 & 1.0000 & 39.7943 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

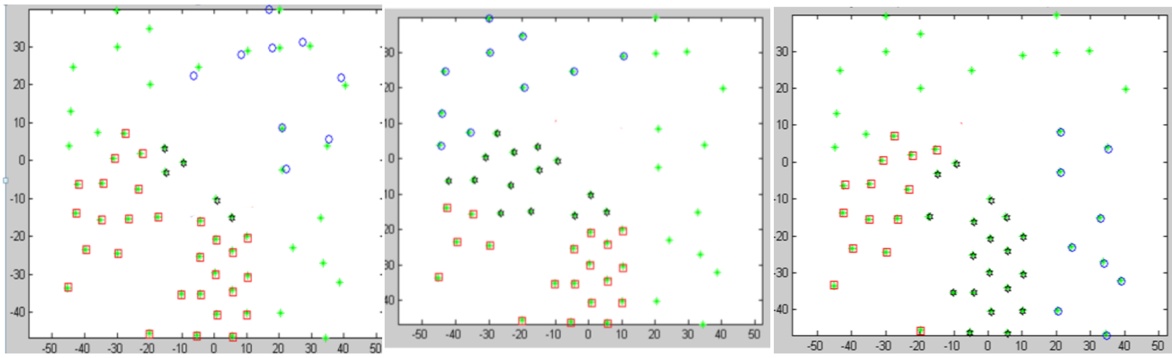

Fig.8: The global map between leader robot and robot2, (b) between leader robot and robot3 and (c) between leader robot and robot4

In figure 8.b, the result is as follows:

no_overlap = 5, therefore the map alignment method which is used by the leader robot is inter-robot observations and the transformation matrix is:

$$T = \begin{bmatrix} 0.9940 & -0.1097 & 36.8141 \\ 0.1097 & 0.9940 & 41.6536 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

In figure 8.c, the result is as follows:

no_overlap = 19, therefore the map alignment method which is used by the leader robot is data association and the transformation matrix is:

$$T= \begin{bmatrix} 1.0000 & -0.0000 & 40.7603 \\ 0.0000 & 1.0000 & -40.4778 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

After transforming all partial maps for leader robot reference, robots get the final estimate of global map as shown in figure 9 within less time than when one robot is used to explore this environment.

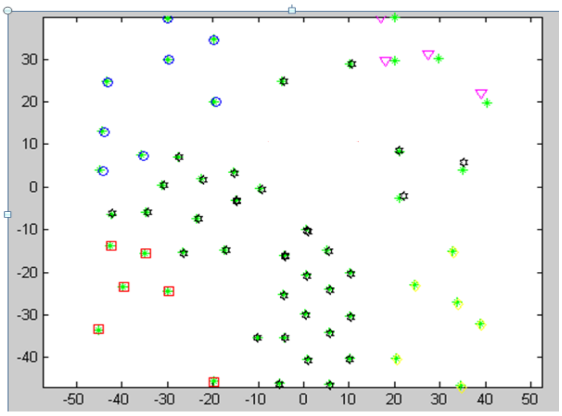

Fig.9: The final global map by four robots, square represents estimated landmarks by robot1, circle is by robot2, triangle is by robot3, diamond is by robot4 and hexagram is duplicate landmarks.

VII. PATH PLANNING

Robot navigational path planning has been cited as a vital research in the field of robotics. Path planning is pertaining to active navigation that guides a robot to locations within the built map to improve localization. It is a task of determining the optimal path by minimizing a cost function such as the distance travelled. A path is optimal if the sum of its transition costs is minimal across all possible paths leading from the initial position to goal position. Several approaches have been proposed over the last decades to deal with path planning problem such as A* algorithm [14], Bug algorithm [15], and Potential functions (PF) [16]. This research doesn't need to use any of these algorithms to solve this problem. As soon as a robot gets map of its surrounding area by the proposed method in this work, it can easily reach to its target by the most direct path as explained in Pseudo Code:

Pseudo code:

Input: Initial position (x_r, y_r, Φ_r), target position (x_t , y_t) and global map.

Output: Optimal trajectory of motion for robot from (x_r, y_r) to (x_t, y_t).

P_{robot} ←position of robot, P_{target}← position of target, Θ_s ←Steering_ angle, gmap←global map, rmax← max _range in front of the robot to check in its map, V← velocity of robot (m/s), dt← time interval between control signals.

Begin

Θ_s= compute_steering (P_{robot}, P_{target}); //calculate steering angle for robot to move toward target (direct path).

While (continue==1) // robot has not reached to its target.

{

Θ_s =check_steering (P_{robot}, gmap, Θ_s, rmax); // check, is current steering angle suitable for reach to target without any landmark hinder its direct path, If yes→return the same angle, if not→calculate new suitable steering angle.

$P_{robot\ (t)}$ = predict_true ($P_{robot(t-1)}$,Θ_s, V,dt);//determine current position of robot(localization)[17].

If ($P_{robot\ (t)}$ == P_{target})

continue =0; // reach to target;

} // end of while loop.

End

Θ_s =check_steering (P_{robot}, gmap, Θ_s, rmax)

For landmarks i=1to N

{

If landmarks found within semicircle around robot with radius rmax

{

Calculate bearing angle (Θ_b) between robot & landmark;

If ($-1^0 < \Theta_b < 1^0$**)**

sub_target=position this landmark; // landmark is front robot in its path to target so must change current steering angle to avoid this landmark;

If **(**$-\Theta^0 < \Theta_b < 0^0$**)**

$\Theta_{s=} \Theta_s + \Theta^0$; // turn left with angle=Θ^0 as shown figure10;

Else if **(**$0^0 < \Theta_b < \Theta^0$**)**

$\Theta_{s=} \Theta_s - \Theta^0$; // turn right with angle=Θ^0 as shown figure10;

If (P_{robot}== sub_target) // robot avoid landmark;

Θ_s= compute_steering (P_{robot}, P_{target}); // calculate again steering angle to move direct toward target;

}//end if

Else

Return the same steering angle;

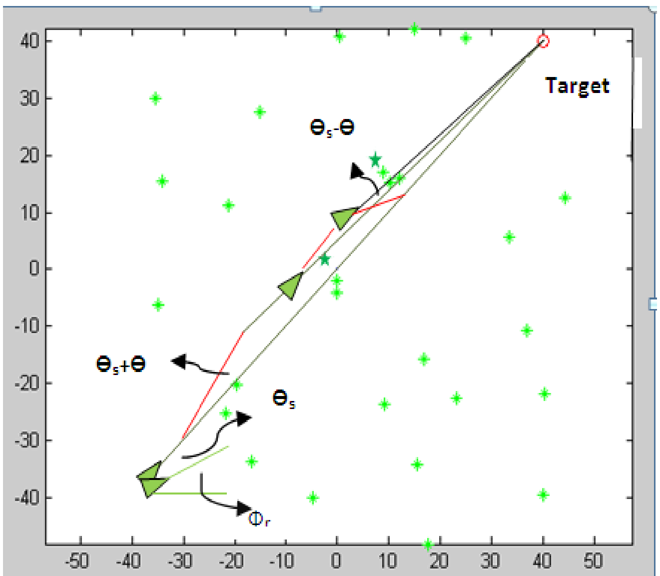

Fig.10: Pathplanning through global map

VIII. CONCLUSIONS

Although data association was the more accurate solution in terms of the alignment quality, this depends on the overla-pping degree between maps. Thus, the method does not work for disjoint maps. Moreover, the use of the relative distance measure between robots avoids this problem; these because the estimation of coordinate transformation depends on the pose of robot and the performed observation only, and not on correspondences between landmarks. Therefore our algorithm uses inter-robot observations method in case of small degree of overlap and uses data association method in case of large noise for robots can travel in any environment and merge their partial maps with highest quality. After building map by the proposed method in this work, robot can use this map to select the shortest path length to reach its target without hitting any obstacles in the world map.

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