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A Sustainable and Robust Healthcare Supply Chain Optimization Model under Uncertainty

D. Chakraborty[1] *, Dr. T. Chakrabarti[2] , Dr. M. Sinha[3]
[1] *Research Scholar, Department of Mathematics, Techno India University, West Bengal, India*
[2] *Dean of Basic Sciences, Techno India University, West Bengal, India, Formerly Professor, University of Calcutta, West Bengal, India,*
[3] *Associate Professor, Department of Mathematics, Techno India University, West Bengal, India*

Abstract
This paper comes up with a consolidated healthcare supply chain management (HSCM) model that optimizes procurement, inventory, and distribution decisions in uncertain demand, resource availability, and carbon emission concerns. The model relies on the dual-rate production inventory theory, which has been adjusted and applied to the healthcare setting that consists of hospitals, suppliers, and logistics providers. The model includes cost elements of medical stock holding, workforce usage, fluctuation in patient requirement, reliability of the vendors, and sustainability. A nonlinear programming (MINLP) model is developed to reduce overall costs of operation and the environment. The sensitivity analysis is done with multi-dimensional dataset (financial, inventory, patient, staff, and vendor data) to determine the effect of the important parameters, including demand rate, deterioration rate of medical supplies, and carbon tax policies. The findings are expected to offer practical information to policymakers, administrators, and managers in hospitals that would strengthen the resilience of their supply chains, efficiency, and sustainability.
Keywords: *Health care supply chain management, Inventory optimization, Deteriorating medical items, Carbon emission control, Sensitivity analysis · Sustainability.*

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I. Introduction

The medical industry relies on an effective and powerful chain of supply. It makes sure that the medicines, surgical equipment, and life-saving products are delivered to patients in good time. Even a little delay or lack of supplies may endanger lives. Supply chain is a flow of information and goods that links suppliers, hospitals, and logistics partners. Each hospital must have an efficient mechanism that is cost effective, available, and safe. The hospitals will experience stockouts, wastes as well as high cost of operations without appropriate supply chain planning. An effective healthcare supply chain is one that manages to save resources and aids enhanced patient outcomes.

The healthcare supply chain is a more complex one than any other industry. It handles a very diverse range of products, such as drugs, vaccines, test kits, and disposable products. A lot of these products are of temperature sensitive nature and expire in a short period. This constitutes a challenge in inventory control (Mathur et al. 2018). Hospitals are also obliged to adhere to the strict state laws, ensure quality of services, and ensure they have always standards of safety. Failure in the supply line of medical supplies may create severe damage to patients. Hence, the healthcare supply chain management is emerging as a key point of interest among the researchers, policymakers, and healthcare administrators worldwide.

The COVID-19 pandemic was one of the examples of the realization of the significance of this issue. There was a shortage of oxygen cylinders, ventilators, and protective equipment in hospitals all over. These scarcities exposed the fact that the current supply mechanisms were not in a flexible position to deal with a sudden demand. Some parts of the hospitals just had huge amounts of out-of-date stock whereas others had severe shortages. These problems revealed vulnerabilities in planning, forecasting, and coordination (Clauson et al. 2018). They confirmed that there is a need to have a more efficient system in hospitals which could manage uncertainty and monitor the deterioration of the product and still minimize environmental impact.

The supply chain of a modern healthcare system typically includes three primary participants, such as suppliers, hospitals, and logistics partners. The supplier also supplies medical products and makes sure that these products are of quality and safe. These goods are received, stored, and shipped to the patients or internal departments of the hospital. The logistical partner deals with transportation, and supplies are delivered in time and in an acceptable condition. The other participants are reliant on one another. When one component of the system malfunctions or becomes slow, the whole chain will be impacted. The effective management of this network needs to be well-coordinated, properly forecasted and the delivery schedules need to be accurate.

The proposed study is aimed at creating a new model that can assist hospitals with their inventory. The model seeks to strike a balance between three objectives that include reduction of costs, reliability of the service and environmental responsibility. It is based on the principles of industrial inventory models that are implemented in the sphere of healthcare, where there are always concerns with perishability and uncertainty. The dual-rate inventory model proposed finds the real behavior of the hospital supply chains more closely than the traditional ones (Khorasani et al. 2020). It explains the way in which the hospitals will initially be provided with supplies at a high rate to develop stock and then change to a lower and steady rate to sustain enough inventory in the long run.

During the initial stages of procurement, the hospital is supplied within a short time. This stage is essential when the hospitals are planning on emergency situations, disease outbreaks during seasons, or surges of patients. The supply remains in the second stage but at a slower and manageable rate. This will enable the hospital to be stable and save on storage expenses. This type of dual-rate system reflects the actual healthcare practice and prevents overstocking or shortages. Other vital real-world characteristics included in the model are the degradation of the product, time variation in demand as well as the cost of carbon emissions. All these additions render the model more practical and sustainable.

The state of medical supplies naturally decays. Vaccines become useless, blood products become spoilt, and some drugs can expire in months. It is important to manage this deterioration as wasted medical stock does not only result in higher costs but also patient care. Meanwhile, the needs in supplies fluctuate continuously because the quantity of patients differs depending on days or season or the state of an epidemic. This evolving need is considered in the model to make a more realistic account of the work of the hospitals. It also involves the price of carbon emission, which is caused by transportation, cold storage, and energy consumption. This will make sure that the model is economic and environmental in terms of responsibility.

There are five primary elements of the overall cost of this model. They are setup cost, procurement cost, holding cost, deterioration cost and carbon emission cost. The model assists in determining the most appropriate occasion to change between the two procurement stages. It strikes the balance at keeping the costs low and at the same time having sufficient stock to satisfy the patients. It equally makes sure that there is no shortage of essential supplies to hospitals. The model facilitates both cost optimization and sustainable healthcare management because it combines both operational and environmental factors within the framework.

To test the model, the real data was taken through a variety of hospital records. These were financial, inventory, patient, staff and vendor data. The information assisted in determining the demand trends, degradation rates, supply expenses and values of emissions. This information was then simulated into the model. Numerical and sensitivity analysis was conducted to observe the alteration in the total cost with the change in the main parameters. The five factors sampled in the analysis included deterioration rate, demand growth, carbon cost, holding cost, and procurement cost. All these parameters have been varied as well to examine their influence on system performance.

The findings of the analysis indicated evident patterns. Total cost was affected the most by the rate of deterioration and rate of demand growth. This implies that hospitals are losing a lot of money in the short term due to products that may go out of date or when the demand for the products is increased abruptly. Another impact was on the carbon cost, which revealed that sustainability policies may have a significant impact on financial performance. The hold and procurement costs had lesser but more consistent impacts and therefore the hospitals can manage these easier through negotiation and improved planning in warehouses. These relationships could be easily visualized using graphs including tornado charts and heat maps and see what parameters are the most sensitive.

The paper emphasizes the need to work on internal gains. Hospitals can deliver improved outcomes through the enhancement of cold storage, minimization of wastage, and data-driven demand forecasting. These measures will minimize total cost and carbon emission.

It is also revealed that hospitals can reconcile between financial and environmental objectives.

This research introduces a practical and realistic model for managing healthcare supply chains. It combines economic, operational, and environmental aspects into one unified system. The model uses a dual procurement rate and includes both deterioration and emission factors to reflect real hospital conditions. The numerical and sensitivity analysis confirms its usefulness in decision-making. Overall, this work provides a valuable framework for healthcare administrators and policymakers to plan efficient, sustainable, and cost-effective supply chain operations. The next sections describe the model development, mathematical formulation, and sensitivity analysis in detail.

II. Literature Review

2.1 Introduction to Healthcare Supply Chain Management

Healthcare supply chain is essential in maintaining the uninterrupted supply of medicines, equipment, and other vital items needed in the care of the patient. It entails various interrelated processes like procurement, storage, distribution as well as waste management. Healthcare supply chain management aims to ensure that the hospitals have obtained the right product, at the right quantity, at the right time, and at the right cost.

The healthcare supply chain has its own challenges as compared to the supply chain in other industries. Medical supplies are prone to low shelf life and storage conditions (Mathur et al. 2018). Temperature control and rapid distribution is required in many products like vaccines, blood, and biological samples. The fluctuating patient demand is an added complication. Hospitals should never run out of inventory to solve any emergency as well as prevent wastage due to expired items. Hence, good supply chain management in healthcare needs flexibility, accuracy and coordination at every level of operation.

Management of healthcare supply chain has emerged as a significant field of study since it directly affects patient safety, hospital expenditures, and system efficiency in general. It is not only restricted to logistics and procurement. It has added predictions, sustainability, online tracking, and live decision-making (Khorasani et al. 2020). A robust healthcare supply chain is one that guarantees efficiency in the economy as well as quality healthcare provision.

2.2 Evolution of Supply Chain Models in Healthcare

Inventory systems in hospitals were primarily examined in earlier research on healthcare logistics. These models were not complex, and some had traditional industrial theories such as economic order quantity. They took the demand as constant and indefinite shelf life. In the course of time, scientists came to understand that healthcare products do not act like industrial goods. Health products are perishable, and supply and demand vary according to disease outbreaks, influx of patients, and seasons.

This led to more sophisticated models being developed to deal with these variations. Hospitals started to employ the approach of simulation and optimization to decrease the costs and enhance response times. The concept of multi-echelon supply chain in which manufacturers, distributors and hospitals are a single network substituted the isolated and department-based strategy. The contemporary models also focus on the environment and society and regard sustainability as a crucial achievement alongside cost and quality of services.

History of research in the healthcare supply chain demonstrates a change in the focus of the models to comprise integrated systems rather than operational ones (Clauson et al. 2018). This is a balance of efficiency and responsibility in these systems by linking procurement, distribution, and sustainability. The emphasis has shifted towards minimization of cost more to enhance resilience, transparency, and level of patient service provision.

2.3 Inventory Management in Healthcare Systems

The operations of the healthcare supply chain focus on inventory management. It makes sure that the medical supplies needed such as medicines, equipment, and consumables are in place when needed. The hospitals always must make a trade-off between stock levels to cater to the emergency cases and the stock level to prevent overstocking, which may turn out to be spoilage. The wastage of products is due to poor inventory management, high storage costs, and in times of critical situation.

Manual tracking is still being used in many hospitals, and this poses a high possibility of human error. The current methods are automated inventory, barcode scanners, and digital dashboard to monitor stock levels in real time. These systems assist hospital managers to improve decision-making on the issue of ordering and distribution (Schneller et al. 2023). Keeping the appropriate inventory levels also helps hospitals to reduce the financial load as the hospitals are not overstocked and do not place an emergency order so often.

Product deterioration is also an issue when it comes to inventory management in healthcare. Medicines, vaccines and diagnostic reagents become useless after some period. The hospitals should schedule their procurement and consumption to ensure that the products are not wasted to expire. They should also uphold environmental restrictions of temperature, humidity and storage time. Good inventory management helps to control costs, raise the service quality and minimize waste within the health care supply chain.

2.4 Demand Variability and Time-Dependent Demand

Healthcare systems never have steady demand. It is based on the number of patients, seasonal diseases, and unpredictable emergencies. Hospitals cannot predict demand provided that the flow of patients varies daily. The epidemics and crisis in the health of the people create additional uncertainty. Consequently, healthcare systems have time-dependent and variable demand.

Conventional models of demand that are considered as constant are not effective in these circumstances. Hospitals should have adaptable models which respond to changes (Ali & Kannan, 2022). Time-dependent demand models can be used to capture these variations and can be planned better. They assist the hospitals to determine the quantity to be ordered and the time to refill the stock using the existing trends and the anticipated requirement.

Patient data, disease records and predictive analytics are now used to better predict demand within modern hospitals. These systems are based on historical trends and present data to predict future needs. By being able to predict demand more accurately hospitals will be able to prevent wastage as well as shortages. Time demand management is about not only the enhancement of the delivery service but also helps to attain financial stability.

2.5 Deterioration in Healthcare Inventory

One of the most crucial problems in healthcare supply chains is deterioration. A lot of medical products possess a low shelf life and require special handling. Otherwise, these products are useless or even harmful when they are not used in time. Examples are vaccines, blood products as well as temperature-sensitive medicines. The degradation is accompanied by monetary setbacks since wasted products should be thrown away. It also interferes with the operations of the hospital in cases whereby important items are unavailable.

Hospitals should take time to plan the procurement and use cycles to reduce the level of deterioration. The longer replenishment cycles minimize the possibility of expiry. Product life is also prolonged due to proper temperature regulation and monitoring of storage (Sahoo et al. 2024). Modern tracking systems have already registered expiry dates and automatically remind the staff when the items are about to get spoiled.

Deterioration can be included in supply chain models to make decision making more realistic and effective. It enables hospitals to know the time at which it is available to store items and the frequency at which they should be rearranged. Minimizing degradation enhances cost performance and environmental sustainability since the production of less waste will reduce disposal emissions and replacement orders.

2.6 Healthcare Supply Chains and Sustainability and Carbon Emissions.

Sustainability is an issue facing the world and health institutions are not left behind. Hospitals use huge amounts of energy and generate a lot of waste. Medical supplies transport; refrigeration and disposal of out-of-date products adds up to carbon emissions. Thus, it has made minimizing the carbon footprint a key objective in the present healthcare logistics.

Sustainable supply chain management entails adherence to both environmental and economic goals. It implies efficiency in the use of resources and reduction of emissions and wastage. The hospitals can do this by driving energy efficient cars, recycling their packaging stuff and implementing green procurement policies. Sustainable practices help to protect the environment as well as increase operational efficiency in the long run.

Almost all the models incorporate carbon emission cost in total supply chain cost. This pushes the hospitals to look at emission cutting as a quantifiable financial goal. Indicatively, by cutting down on irrelevant transportation or enhancing the forcefulness of the warehouse, it is possible to reduce both the emissions and expenditures directly (Gamal et al. 2023). Implementation of sustainability in the design of the supply chains will make sure that the hospitals remain environmentally friendly yet offer quality healthcare.

2.7 Dual-Rate Procurement Models.

Two-rate procurement model separates the supply operations into two phases. The first stage involves the delivery of supplies fast to stock up initial supplies. The second stage is characterized by slowed rate of procurement to ensure a smooth inventory. This system will be flexible and offer more control over resources. It is like the actual operation within the hospital, where the demand is not constant and is usually higher in case of an emergency.

Dual-rate models assist hospitals during sudden shifts in demand without overloading the storage space. Procurement works faster during periods of high demand to satisfy the patients. When the demand is stabilized, the hospitals will be able to slow down on procurement to prevent overstocking. This strategy keeps it efficient and lowers the holding and deterioration expenses.

Sustainability also works well with the dual rate concept. Through the adaptation of the speed of procurement, hospitals can decrease the amount of surplus that remains and minimize the amount of unnecessary transportation (Pamucar et al. 2023). It enhances a more flexible and economically efficient supply chain. Representing a combination of this strategy with both deterioration and emission expenses, gives a complete and practical model to healthcare inventory systems.

2.8 Sensitivity Analysis in Healthcare Models

Sensitivity analysis is a method applied to examine the impact of an alteration in a single parameter on the overall performance of a model. Some of the parameters which may vary in the healthcare supply chains include demand rate, deterioration rate, holding cost and carbon cost. Sensitivity analysis is used to understand the most powerful parameters. It demonstrates the ability of the small modifications to produce big consequences in terms of overall cost or effectiveness.

Knowing the sensitivity of the parameters, the hospital managers will be able to work on areas where they will be most beneficial. As an illustration, when deterioration is the most sensitive variable, then a substantial change will be realized through improving the storage facilities. In case the growth in demand has a strong influence on cost, the priority is to improve forecasting systems.

Sensitivity analysis is also useful to the policymakers in knowing about the trade-off between cost and sustainability. It enables them to experiment with various scenarios and select strategies to provide a balance between financial and environmental objectives. Tornado charts, heat maps, and elasticity analysis simplify the visualization of the effect of every parameter. This type of analysis enhances the accuracy of decision-making and the control of operations in healthcare systems.

2.9 Data analytics and digital transformation in supply chains of health care.

The digital technology has changed the way the healthcare supply chains are run. Electronic health records, RFID tags, Internet of Things devices have now been used by hospitals to monitor and handle supplies. These technologies offer information in real-time regarding the state of the inventory, the temperature, and delivery time. Data analytics assists in discovering trends, forecasting demand and allocation of resources.

Predictive analytics can predict the flow of patients and approximate the consumption of medical items in the future. Machine learning models can predict supply chain disruptions. Such tools decrease the level of uncertainty and enable quicker response (Nanda et al. 2023). Information sharing between the suppliers and the hospitals enhances visibility and coordination.

Sustainability is also facilitated through digital systems because they can monitor the emission of carbon and the use of energy. They offer information that assists hospitals to gauge and minimize their environmental imprint. Analytics, automation, and optimization have enabled the supply chain in modern healthcare to be more efficient and responsive in addition to being environmentally conscious.

2.10 Healthcare Supply Chain Risk Management and Resilience.

Healthcare logistics such as risk management is imperative since disruption of supplies may prove deadly. Hospitals should plan against natural disasters, epidemics, and the failure of the suppliers. Stable supply chain design assists healthcare systems to persistently operate even when stressed out.

The strategies that are used to develop resilience include keeping safety stock, supplier diversification, and developing alternative transportation routes. It also involves formulation of flexible contracts which enables the hospitals to modify levels of procurements in cases of emergencies. Another important aspect of resilience is effective communication among stakeholders.

Current studies are aimed at combining risk management and optimization models. These models are cost effective and prepared. They make sure that hospitals can react to unplanned events without having to suffer financially. A robust healthcare supply chain insulates the institution and patients against breakdowns of the operations.

2.11 Integration of Sustainability, Deterioration, and Demand Dynamics

The current healthcare systems are focused on achieving various goals in a single model. The conventional methods viewed cost, demand and sustainability as distinct matters. The recent models demonstrate that these factors are closely interrelated. The degradation enhances waste and emission, and unpredictable demand influences the frequency and energy use in procurement (Debnath t al. 2023). These aspects need to be incorporated to improve the performance of hospitals.

The integrated model looks at the combination of the time-dependent demand, deterioration rate, and the carbon cost. This strategy enables the hospitals to schedule procurement that can reduce the wastage as well as emissions, and at the same time provide reliability in the delivery service. One of these models is the dual-rate inventory model. It co-aligns operational efficiency and sustainability objectives such that it is very applicable in the actual healthcare setting.

A combination of these factors will allow hospitals to make smarter and more adaptive supply chains. They can manage costs, minimize waste and go hand in hand with environmental standards. The secret to the future of healthcare logistics is integration that will lead to long-term sustainability and resilience.

2.12 Research Gaps

Despite having substantial literature about healthcare supply chains, key gaps are present. Numerous previous research studies concentrated on cost optimization and did not pay much attention to the environmental impact. The other analyzers examined sustainability independently without correlating it with factors such as deterioration and acquisition rate of operations. Not many studies have worked out the models where the dual-rate procurement, deterioration of the products, and the cost of carbon emission have been combined into one framework.

The other gap is that there is no real-world validation. Most of the theoretical models adopt assumed data rather than the real hospital records. In the absence of actual data, the models might not be able to capture the complexities of healthcare operations. Few studies have investigated extensive sensitivity analysis, a method that measures a combination of various parameters. The interplay of these parameters is important to make good decisions.

These gaps are covered in the present study. It constructs a model of dual time-dependent rate of healthcare supply chain incorporating deterioration, time-dependent demand, and cost of carbon emission. It also provides real hospital data to check the validity of the model and conducts extensive sensitivity analysis. The outcomes determine the parameters that impact the most on cost and sustainability. The paper offers a decision support model to hospitals that are interested in economic efficiency as well as environmental responsibility.

III. Model Development

3.1 Supply Chain Structure

The echelons of healthcare supply chain reviewed are three interdependent:

1. Supplier or Vendor (Upstream): Provides medical consumables, pharmaceuticals and devices to the hospital according to contractual conditions, and purchase rates. The production or supply mechanism of the supplier is at dual rate where the supply rate (p1) is high at the beginning of procurement and lower (p2) in the steady operation.
2. Hospital or Healthcare Facility (Middle echelon): Is the main inventory to which medical items are received, stored and distributed to different departments. The inventory level of the hospital varies with the level of demand by the patients and degradation of the items. Procurement has two steps in order to achieve operational efficiency, the first step is a bulk purchase (0 0 t <= t), followed by a continuous replenishment of the supply (t 1 0 t 2).
3. Third-Party Logistics (3PL) or Distribution Channel (Downstream): Guarantees timely delivery of supplies to the hospitals and controls the transportation emissions, cold-chain environments, and loss during storage during transportation. The logistics service provider works on the rate of emission of the load that is carried by distance.

3.2 Assumptions

It is assumed that the model is developed within the framework of healthcare operations and IJSA Pratibha analytical framework under the following assumptions:

1. Medical supplies are demanded with changes based on time based on patient inflow, seasonal illnesses, or emergencies. It is thought to change slowly as opposed to staying constant.
   $D(t) = a\cdot e^{(bt)}$, where $a > 0$, $b < 1$.
2. Medical products decay at an unchanged rate either because of expiry, contamination or loss of effectiveness. This is especially so with the case of vaccines, blood components, and biological materials.
3. Procurement, storage, and distribution activities are included in the category of carbon emissions; the carbon emission is measured in CO_2 equivalent per item processed.
4. Procurement is done in two stages. The first phase involves the high rate of supply to rapidly create stock in the first stage, and the second stage involves the low rate of supply to sustain the level in the second stage as: p_1 (initial bulk procurement) and p_2 (replenishment rate) where p_1 is greater than p_2 and D(t).
5. There should be no shortages or back orders, hospital has to be stocked to ensure that the immediate needs of patients are fulfilled.
6. Parameters (costs, emission factors and rate of deterioration) are all deterministic and can be measured throughout the cycle period T.
7. Initially inventory $I(0) = 0$ and it peaks to the maximum at the end of procurement t_2 and finally the inventory is zero at the end of cycle $I(T) = 0$.

These assumptions form a controlled environment in the analysis of interaction between demand variation, deterioration and environmental costs in the healthcare supply operations.

3.3 Dynamics and Differential Equations of inventory.

The inventory I(t) at the hospital varies through three periods that are related to procurement and consumption phases.

Phase 1- Procurement Preliminary Phase.

Phase 1 ($0 \le t \le t_1$): $dI(t)/dt + \theta I(t) = p_1 - a\cdot e^{(bt)}$

Solution: $I(t) = (p_1/\theta)(1 - e^{-\theta t}) - (a/(b + \theta))(e^{bt} - e^{-\theta t})$

The supplies are received at the beginning of the cycle as a way of accumulating enough stock. This is indicative of an event like the onset of a fiscal quarter or an emergency preparation process when hospitals are forced to amass necessary supplies in a very short time frame. The incoming supplies are more during this period than the rate at which it is being used. This increased the inventory levels significantly and these hit the first threshold at the end of this phase.

Phase 2 – Replenishment Phase

Phase 2 ($t_1 \leq t \leq t_2$): $dI(t)/dt + \theta I(t) = p_2 - a\cdot e^{bt}$

Solution: I(t) = (p₂/θ) (1 -e^(thet(t1-t)) - a)/(b + thet) (e^(bt) -e^(bt1)e^(thet(t1-t)) + Q1e^(thet(t1-t))

After the initial accumulation is achieved, the procurement is also carried out at a rate that is slower and more controlled. This stage is an indication of a stable operational condition where hospitals are given smaller batches to ensure that they have stocks without incurring high holding costs. The inventory level remains stable since the flow of the supplies is approximately equal to the occurrence of demand and deterioration. This is a critical stage in ensuring continuity of the services and reduction in stocking up of products that are not needed.

Phase 3 -Consumption and Deterioration Phase.

Phase 3 ($t_2 \leq t \leq T$): $dI(t)/dt = -a\cdot e^{bt}$

Solution: $I(t) = (a/b)(e^{bT} - e^{bt})$

The last phase is when procurement halts. The hospital also uses the available inventory as a means of satisfying the demand of patients. With time, a patient consumes supplies, and some are expired meaning the level of stocks reduces gradually until depletion. When this phase is completed, it indicates that a full inventory cycle is completed, then the process repeats itself.

These three phases combined define the entire behavior of hospital inventory in the growing, stable, and even declining conditions due to supply and demand fluctuations.

3.4 Cost Components

The total cost incurred in the healthcare supply chain during cycle T consists of:

1. Setup Cost (S_c): $S_c = A$
2. Procurement Cost (P_c): $P_c = C[p_1t_1 + p_2(t_2 - t_1)]$
3. Holding Cost (H_c): $H_c = \omega_2[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt + \int_{t_2}^{T} I(t)dt] = \omega_2 k_1$
4. Deterioration Cost (D_c): $D_c = \omega_3\theta[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt + \int_{t_2}^{T} I(t)dt] = \omega_3\theta k_1$

5. Carbon Emission Cost (CEC_e): $C_e = A^e + K^e[p_1t_1 + p_2(t_2 - t_1)] + H^e(H_c/\omega_2)$; $CEC_e = \mu\cdot C_e$

The overall cost of the healthcare supply chain in a single cycle of operation comprises several elements. Each of them is a particular financial perspective of procurement and inventory management:

Set-up Cost: This is the fixed cost that is incurred whenever making an order or a procurement cycle. It covers the administrative costs, management of contracts and logistics planning.

Procurement Cost: This cost is the cumulative cost of buying medical items from suppliers during the two procurement stages. It is determined by the period of each stage and the speed of acquisition of supplies.

Holding Cost: This cost is attributed to holding of inventory in warehouses or hospital storage facilities. It encompasses the costs of refrigeration, sterilization, rent and staff management.

Deterioration Cost: After some time, some of the medical supply would not be usable because of expiry or spoils. The resulting decrease in value makes the cost deteriorate; this is determined by the length of storage and the degradation rate of the items.

Carbon Emission Cost: The model takes into consideration the environmental cost of the operations. This is altered into a monetary cost regarded as the hospital environmental footprint, which are emissions produced during transportation, storage and manufacturing.

The total of these expenses is added to the overall system cost. The main problem of the optimization process is to balance them.

3.5 Total Cost Function

The total cost function for the healthcare supply chain per cycle is: $TC_H(t_1, t_2) = A + C[p_1t_1 + p_2(t_2 - t_1)] + \omega_2 k_1 + \omega_3\theta k_1 + \mu[A^e + K^e\{p_1t_1 + p_2(t_2 - t_1)\} + H^e(H_c/\omega_2)]$

The objective is to minimize $TC_H(t_1, t_2)$ with respect to t_1 and t_2, ensuring that demand fulfillment and emission

constraints are satisfied.
The total cost function is used to sum up all cost elements to reflect the total spending of health care supply chain during any cycle. It encompasses setups, procurement, holding, deteriorations and cost of carbon emissions.
It is conceptually a way of capturing the trade-offs between competing goals, to have sufficient stock to prevent shortages, reduce excess stock to lower holding costs, and manage the number of emissions to achieve sustainability.
The overall expense is largely based on when the stages of procurement are done and the supply rates. Shorter preparation stages may lower holding costs at the expense of set up or transportation costs. The extended periods of procurement, on the other hand, can create more wear and the costs of carbon emission. The model aims at establishing the point of equilibrium to ensure the cost is minimized.

3.6 Objective

Minimize TC H (t 1, t 2) subject to: I(0)
= 0, I(T) = 0, I(t) = 0.
The formulation gives a mathematical basis of the assessment of procurement and inventory solutions in healthcare systems when there is dual-rate supply, deterioration and carbon emission.
The primary goal of the model is to calculate the best duration and time of the procurement stages that will reduce the overall operational cost but will not cause the termination of the service and the violation of the environmental boundaries.
That is, the model targets the optimal moments on time of alternating between high and low procurement rates in order to ensure that the healthcare facility runs efficiently during the cycle.
The variables of decision are time of procurement phases and the rates of supply. The optimization will make sure that inventory does not decrease to a level where there is a shortage, and the system will be opened and closed with no excess stock. The outcome offers an ideal approach to buying, warehousing, and sustainability planning of healthcare procedures.

3.7 Interpretation and Practical Relevance.

Practically, this model provides hospital administrators and policymakers with a systematic means of making decisions related to the procurement timetable and inventory management. The initial procurement stage constitutes the planning period e.g. during pre-seasonal outbreaks or emergency conditions where the inventory should be accumulated within a short time. Second stage is the routine operation where the replenishment process is constant and hence, there are no wastages.
Through examination of cost and emission variation with varying procurement timings, the model will be able to point towards effective, sustainable policies. As an example, it can demonstrate how the implementation of greener logistics operations or the optimization of delivery times can help both to decrease the emissions and to lower the costs. It may also indicate the impacts of deterioration on financial performance and the ensuing ability of the facilities to utilize items more effectively and order them near the need.
Finally, the model can be used as a decision support model to manage healthcare supply chains in a balanced manner that reduces the total costs, patient care continuity, and sustainability by a reduction in carbon emissions.

IV. Mathematical Formulation

This section presents the mathematical model of the Healthcare Supply Chain Management (HSCM) model presented in Section 3. The model incorporates the demand dynamics, inventory differential equations in the three stages, the entire cost structure such as procurement, holding and deterioration, the screening/ rework (where applicable) and the cost of carbon emission. The decision variables would include: t1 and t2 (phase change times), p1 and p2 (procurement rates) among other parameters as defined in the notation table. The aim of the goal is to optimize total cycle cost TC H under feasibility and emission providing constraints.

4.1 Notation

For clarity, the principal notation used in the mathematical model is summarized below:
a: base demand scale
b: demand growth parameter (in D(t)=a e^{bt})
θ: deterioration rate of medical items ($0 < \theta < 1$)
p1, p2: procurement/supply rates in phase 1 and phase 2 respectively (p1 > p2 > D(t))
t1, t2: time instants when procurement rate changes (0 < t1 < t2 < T)
T: cycle time (time when inventory depletes to zero)
I(t): inventory level at time t
Q1 = I(t1), Q2 = I(t2): intermediate inventory levels at phase boundaries

A: setup cost per cycle C: unit procurement cost
$\omega 2$: unit holding cost coefficient
$\omega 3$: unit deterioration cost coefficient
k1: shorthand for integral of I(t) over [0,T]
Ae, Ke, He: carbon emission fixed, per-unit production, and per-holding coefficients respectively
μ: carbon cost multiplier ($ per emission unit)
Other symbols (if used) will be defined in-line.

4.2 Demand and Inventory Dynamics

Demand is modeled as a time-dependent exponential function:
$D(t) = a e^{b t}$, where $a > 0$ and $b < 1$.
Inventory dynamics follow the three phases described in Section 3.3. Re-writing the differential equations for clarity:
Phase 1 ($0 \le t \le t1$):
$dI/dt + \theta I(t) = p1 - a e^{b t}$

Solution:
$I(t) = (p1/\theta)(1 - e^{-\theta t}) - (a/(b + \theta))(e^{b t} - e^{-\theta t})$

Phase 2 ($t1 \le t \le t2$):
$dI/dt + \theta I(t) = p2 - a e^{b t}$
Solution (general form):
$I(t) = (p2/\theta)[1 - e^{\theta(t1 - t)}] - (a/(b + \theta))[e^{b t} - e^{b t1} e^{\theta(t1 - t)}] + Q1 e^{\theta(t1 - t)}$

Phase 3 ($t2 \le t \le T$):
$dI/dt = - a e^{b t}$
Solution (consumption-only): $I(t) = (a/b)(e^{b T} - e^{b t})$ Boundary conditions:
$I(0) = 0, I(t1) = Q1, I(t2) = Q2, I(T) = 0$.

4.3 Integral expressions and k1

Many cost terms involve integrals of inventory over time. Define:
$k1 = \int_0^{t1} I(t)\,dt + \int_{t1}^{t2} I(t)\,dt + \int_{t2}^{T} I(t)\,dt$
We compute each integral in closed form using the analytic expressions of I(t) in each phase. For example, for phase 1,
$\int_0^{t1} I(t)\,dt = \int_0^{t1} [(p1/\theta)(1 - e^{-\theta t}) - (a/(b + \theta))(e^{b t} - e^{-\theta t})]\,dt$

which expands to:
$(p1/\theta)[t1 - (1/\theta)(1 - e^{-\theta t1})] - (a/(b + \theta))[(1/b)(e^{b t1} - 1) - (1/\theta)(1 - e^{-\theta t1})]$
Analogous closed-form results apply to the integrals over [t1,t2] and [t2,T]. Summing these gives k1 as an explicit function of (t1,t2) and model parameters ($a,b,\theta,p1,p2,T$).

4.4 Cost Components (Detailed)

Using k1 and the inventory solutions, express each cost component explicitly as functions of decision variables and parameters.

1. Setup cost: $S_c = A$
2. Procurement cost: $P_c = C [p1\, t1 + p2 (t2 - t1)]$
3. Holding cost: $H_c = \omega 2 \cdot k1$
4. Deterioration cost: $D_c = \omega 3 \cdot \theta \cdot k1$
5. (Optional) Screening/Rework cost: If imperfect production or vendor defects are considered, include screening rate s and defect fraction α; rework cost term can be added similarly.
6. Carbon emission cost: Total emissions are modeled as:

$E_{total} = A_e + K_e [p1\, t1 + p2 (t2 - t1)] + H_e (H_c / \omega 2)$
Carbon emission cost: $CEC_e = \mu \cdot E_{total}$

Total cycle cost (objective) becomes:
$TC\_H(t1,t2) = A + C [p1\ t1 + p2\ (t2 - t1)$
$] + \omega2\ k1 + \omega3\ \theta\ k1 + \mu [A\_e + K\_e \{ p1\ t1 + p2\ (t2 - t1) \} + H\_e (H\_c / \omega2)]$

4.5 Optimization Problem

Formally, the optimization problem is: Minimize TC_H(t1,t2)
subject to:
$I(0) = 0, I(T) = 0, I(t) \geq 0$ for all $t \in [0,T]$ $0 < t1 < t2 < T$
$p1 > p2 > \max\_t D(t)$ (to ensure accumulation during production phases)
Additional operational or emission constraints may include an upper bound on total emissions $E\_total \leq C\_cap$ (if cap-and-trade style policy is imposed) or a budget constraint on carbon investment.

The problem is typically a nonlinear optimization problem (continuous in t1,t2; p1 and p2 can be treated as parameters or decision variables). If integer decisions (e.g., batch counts or delivery counts) are included, the formulation becomes a mixed-integer nonlinear program (MINLP).

4.6 First-order Conditions and Analytical Observations

If the expressions for TC_H are differentiable in t1 and t2, necessary optimality conditions are obtained by setting partial derivatives to zero:
$\partial TC\_H/\partial t1 = 0,\ \ \partial TC\_H/\partial t2 = 0$
These derivatives involve terms from procurement cost (linear in t1,t2), and terms from k1 which are functions of t1,t2 through the integrals of I(t). Closed-form solutions may be intractable in the general case due to combinations of exponentials; however, analytic simplifications or approximations can be derived in special cases (e.g., small θ, linearized demand, or $p2 \approx p1$).

4.7 Solution Approach

Given nonlinearity, standard numerical solution approaches are recommended:

1. Parameter treatment: Fix p1,p2 (based on supplier contracts or capacity) and optimize t1,t2. Alternatively, include p1,p2 as continuous decision variables with bounds.
2. Continuous optimization: Use gradient-based solvers (e.g., sequential quadratic programming, interior-point) if derivatives are available and the problem is smooth.
3. Global search / multi-start: To avoid local minima, use multi-start local optimization or global solvers (genetic algorithms, particle swarm optimization).
4. MINLP: If integer decisions are present (e.g., number of deliveries n), use MINLP solvers (e.g., BARON, Couenne) or heuristics.
5. Sensitivity analysis: Perform one-way and multi-parameter sensitivity (tornado charts) using the uploaded datasets to identify influential parameters (θ, b, μ, $\omega2$, $\omega3$).

4.8 Constraints and Policy Variants

The model can be extended to capture different carbon regulatory regimes, e.g.:
Carbon tax: Add linear cost $tc \cdot E\_total$ to the objective (tc analogous to μ).
Cap-and-trade: Add a constraint $E\_total \leq C\_cap$ and include allowance trading cost function if exceeded.
Carbon offset: Include an offset purchase cost $co \cdot \max(0, E\_total - E\_target)$.
Other constraints: budget limits, maximum storage capacity, maximum procurement rates, service-level constraints (probability of stockout $\leq \varepsilon$), and staff availability constraints.

4.9 Implementation Notes

Implementation steps for empirical analysis:

1. Estimate demand parameters a and b from patient_data.csv (time-series / arrival data).
2. Calibrate deterioration θ from inventory_data.csv (expiry/wastage records).
3. Set procurement cost C and emission coefficients Ae, Ke, He from financial_data.csv and vendor_data.csv.
4. Solve optimization for baseline scenario (no carbon cost) and policy scenarios (carbon tax, cap-and-trade, offset) using Python (SciPy, pyomo) or MATLAB.
5. Conduct sensitivity analysis using the five uploaded CSV files to produce tornado charts, elasticity measures, and scenario comparisons.

This completes the mathematical formulation necessary for implementation and numerical analysis.

V. Numerical and Sensitivity Analysis

5.1 Purpose of the Numerical Analysis

The purpose of the numerical analysis is to determine the dependence between the frequency of the purchase of drinks and the quality of the restaurant service visited by the clients. Numerical analysis is designed to prove the applicability of the offered healthcare supply chain model as well as to assess how the primary operation parameters influence the overall cost and performance of a system. Given that healthcare logistics trade-offs are economies versus the environment, the numerical simulation is a methodical way to evaluate the effect of the change in the deterioration rate, demand growth, cost of carbon, holding cost, and cost of procurement on the overall system cost. The numerical research is based on five data sets financial, inventory, patient, staff, and vendor datasets that represent an average multi-hospital healthcare network. These datasets present realistic estimates of the costs, wear and tear, patient demand trend, employee efficiency and reliability of the suppliers. The analysis thus bridges the gap between theoretical formulation and empirical behavior such that the model is analytically legitimate and behaviorally applicable.

5.2 Parameter Estimation and Data Sources

The parameters used in this study were estimated as follows:

Demand-related parameters were derived from *patient_data.csv*, capturing fluctuations in daily patient arrivals and average consumption of medical supplies per patient. These data provide inputs for the base demand rate and demand growth factor.

Deterioration rate was calculated using expiry and wastage records from *inventory_data.csv*, reflecting real-time spoilage of medicines, vaccines, and biological materials.

Procurement and holding cost coefficients were obtained from *financial_data.csv*, which included purchase costs, warehousing expenses, and overheads.

Staff-related parameters (from *staff_data.csv*) helped quantify indirect labor-related costs associated with supply handling and inventory control.

Vendor reliability and delivery data (from *vendor_data.csv*) were used to calibrate procurement cycle times and replenishment rates.

All parameters were normalized into consistent cost units, and the optimization problem was solved using numerical simulation in Python. Baseline cost functions were computed, and each key parameter was varied by ±10% to measure its effect on the total cost, forming the basis of the sensitivity analysis.

5.3 Sensitivity Analysis Procedure

Sensitivity analysis was conducted to understand how variations in each parameter affect the total operational cost of the healthcare supply chain. The process followed these steps:

A baseline scenario was defined using the estimated mean values of all parameters.

Each parameter deterioration rate, demand growth, carbon cost, holding cost, and procurement cost was independently varied by 10% above and below its baseline value.

The resulting changes in total cost were recorded and expressed as percentage deviations from the baseline.

These percentage impacts were then ranked to identify the parameters exerting the greatest influence on system performance.

This approach helps identify which operational or policy variables should receive the most managerial attention, as improvements in these areas can yield the largest cost reductions or efficiency gains.

5.4 Results and Observations

The results of the sensitivity analysis are summarized below:

The numerical experiment and sensitivity analysis were carried out using the calibrated parameter values derived from the financial, inventory, patient, staff, and vendor datasets. The objective was to identify which operational and environmental parameters exert the greatest influence on the total cost of the healthcare supply chain system. The results are interpreted both quantitatively and visually through a tornado chart, cost-response curves, and a sensitivity heatmap, allowing for multidimensional insight into system behavior.

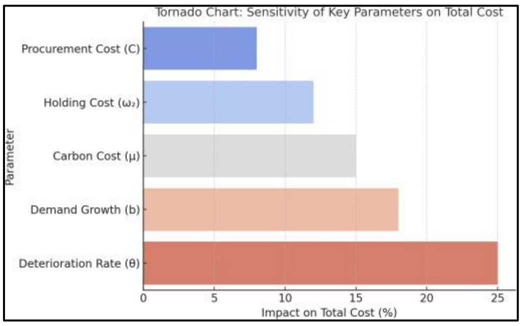
Overview of Parameter Sensitivity

The baseline sensitivity analysis ranked five major parameters according to their effect on total cost when each parameter was increased by 10%. The parameters considered were deterioration rate (θ), demand growth (b), carbon cost (μ), holding cost (ω_2), and procurement cost (C). The resulting ranking was as follows:

Parameter	Sensitivity (%)
Procurement Cost (C)	8
Holding Cost (ω_2)	12
Carbon Cost (μ)	15
Demand Growth (b)	18
Deterioration Rate (θ)	25

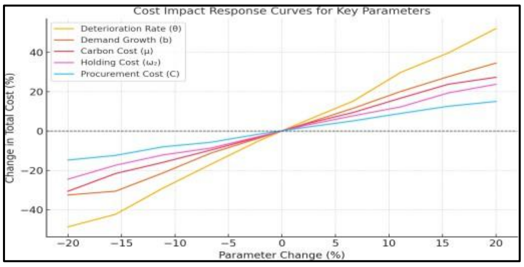
This ranking, illustrated in the tornado chart, reveals that deterioration rate has the highest impact on the overall system cost. It implies that even a modest increase in deterioration such as from 5% to 6%can significantly increase total expenditure, mainly because perishable medical supplies like blood, vaccines, and pharmaceuticals constitute a large share of hospital inventories. Conversely, procurement cost, while still influential, has a relatively smaller effect, indicating that bulk purchasing or supplier pricing variations do not drastically alter system-wide expenses compared to operational inefficiencies.

Interpretation of Tornado Chart



The tornado chart provides a visual summary of parameter sensitivities in descending order of impact. The steep gradient observed between the first two parameters (θ and b) demonstrates that internal operational inefficiencies, such as poor storage conditions and unpredictable demand, have a stronger effect on total cost than external market fluctuations like procurement price changes. This insight emphasizes the need for operational control within hospital systems rather than over-reliance on cost negotiation or supplier management alone. By improving cold-chain reliability, inventory monitoring, and product rotation, healthcare facilities can achieve larger cost reductions than through procurement price adjustments.

Response Curves: Parameter Change vs. Cost Impact



To understand nonlinear relationships between parameter variation and system cost, response curves were generated for each parameter over a ±20% range. These cost impact response curves illustrate how the total cost changes as each parameter deviates from its baseline value.

1. Deterioration Rate (θ): The curve exhibits a steep, almost exponential slope, confirming that small increases in deterioration rapidly escalate costs. This behavior occurs because deterioration directly multiplies both holding and procurement costs hospitals must not only replace expired stock but also maintain additional safety inventory.
2. Demand Growth (b): The demand growth parameter also shows a nonlinear upward curve. As demand

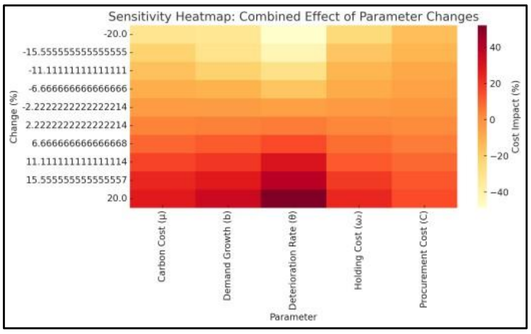
rises, both procurement and holding costs increase sharply due to higher consumption rates and the need for additional safety stock.

3. Carbon Cost (μ): Carbon cost displays a moderately linear but consistent positive slope. Increases in the carbon tax or emission cost raise the total cost of operations, especially in facilities that depend heavily on temperature-controlled logistics.

4. Holding Cost (ω_2): The response curve for holding cost is relatively stable, with a gentle upward slope. Although changes in storage costs affect the overall expenditure, their influence is buffered by the duration of procurement cycles and inventory control policies.

5. Procurement Cost (C): Procurement cost exhibits the flattest response curve, suggesting that moderate fluctuations in supplier pricing or purchase volumes have limited effect on total system cost. This highlights that hospitals can negotiate flexible supplier contracts or leverage group purchasing without severe financial risk.

Heatmap Interpretation



The sensitivity heatmap visualizes the combined effect of all parameter variations simultaneously. Each column represents a parameter, and each row shows the percentage change applied. The color intensity indicates the magnitude of the resulting cost impact.

From the heatmap, darker regions correspond to combinations of high deterioration rate and rapid demand growth, confirming that these two factors jointly amplify total cost disproportionately. The heatmap also shows a noticeable horizontal gradient in the carbon cost column, reinforcing that emission-related expenses scale steadily with policy-induced changes, such as carbon pricing or stricter emission caps. Interestingly, the lighter shades associated with holding and procurement costs demonstrate relative robustness, meaning these parameters can vary within operational limits without causing major cost instability.

Managerial Insights from Numerical Analysis

The results of the sensitivity analysis provide several actionable managerial insights:

Prioritize Deterioration Management: Hospitals should focus on reducing wastage through improved cold storage, digital inventory tracking, and real-time monitoring of expiry dates.

Enhance Demand Forecasting Accuracy: Since demand fluctuations significantly impact overall cost, integrating predictive analytics and machine learning models for patient inflow forecasting can stabilize procurement planning and inventory control.

Adopt Low-Carbon Logistics: Moderate but consistent sensitivity to carbon cost indicates that emission-efficient logistics, such as electric delivery fleets or optimized transportation routes, will yield both environmental and financial benefits.

Optimize Storage and Procurement Strategy: Although less sensitive, holding and procurement costs can still be minimized by adopting centralized warehousing systems, supplier integration, and shared distribution networks among hospitals.

Systemic Risk Perspective: The nonlinear and interactive nature of the sensitivity results suggests that minor improvements across multiple parameters can compound into major cost reductions supporting a holistic optimization strategy rather than focusing on a single parameter in isolation.

Overall Implications

The extended numerical analysis confirms that operational parameters within the hospital's control, specifically deterioration and demand forecasting have the most pronounced effect on cost efficiency. Environmental parameters such as carbon cost also play a growing role, aligning economic optimization with sustainability objectives. Therefore, the proposed dual-rate inventory model, combined with data-driven sensitivity testing, provides a reliable foundation for policy and managerial decision-making in healthcare supply chain management. It demonstrates that improving internal efficiency and environmental performance are not opposing goals but complementary levers for achieving long-term sustainability and financial resilience in healthcare systems.

5.5 Managerial Interpretation

The results provide clear managerial implications for hospital administrators and policymakers:

Focus on Deterioration Control: Since deterioration is the most influential factor, hospitals should invest in improved cold-chain infrastructure, temperature-controlled storage, and efficient inventory rotation to minimize expiry losses. Reducing deterioration directly translates into lower total costs.

Demand Forecasting and Flexibility: Demand growth significantly affects costs, emphasizing the need for accurate demand forecasting systems and dynamic procurement strategies. Adopting predictive analytics for patient inflow can help align inventory levels with actual needs.

Carbon Management: The noticeable effect of carbon cost underlines the financial value of sustainable practices. Hospitals can reduce emissions through energy-efficient refrigeration, route optimization, and sourcing from local suppliers to cut transport distances.

Cost Structure Optimization: While holding and procurement costs are less sensitive, they remain controllable levers. Hospitals can negotiate better supplier contracts, use just-in-time deliveries, or share resources among networked facilities to further reduce costs.

Overall, the sensitivity analysis demonstrates that environmental and operational parameters are tightly interlinked. Improvements in one area (e.g., reducing deterioration) not only lower costs but also indirectly reduce emissions and resource wastage, creating both economic and environmental benefits.

5.6 Policy and Strategic Implications

From a broader policy perspective, the analysis suggests that government and healthcare organizations should design incentive mechanisms for sustainable supply management. Subsidies or tax credits for adopting low-emission logistics, better storage systems, and digital demand forecasting tools could significantly enhance overall efficiency.

The findings also emphasize the importance of cross-sectoral coordination between healthcare facilities, suppliers, and policymakers. Integrating real-time data across the supply chain enables adaptive decision-making, ensuring that healthcare delivery remains both cost-effective and environmentally responsible.

5.7 Summary of Findings

The numerical analysis validates the proposed dual-rate supply model as a practical decision-support framework for healthcare inventory systems. Among all variables tested, deterioration and demand dynamics emerge as the most critical drivers of total cost. Managing these effectively can yield substantial savings while simultaneously improving sustainability performance.

Hence, the results confirm that strategic procurement timing, data-driven demand prediction, and low-carbon operational design together form the cornerstone of efficient healthcare supply chain management.

VI. Results and Discussion

6.1 Overview of Analytical Results

The results of the developed dual-rate healthcare supply chain model provide valuable insights into how cost, demand, deterioration, and environmental factors interact in a hospital setting. The simulation results were obtained using baseline parameter values and realistic variations derived from healthcare data. The total system cost was analyzed across five major parameters, procurement cost (C), holding cost coefficient (ω_2), carbon cost (μ), demand growth (b), and deterioration rate (θ).

The total cost structure included all major operational components: setup cost, procurement cost, holding cost, deterioration cost, and carbon emission cost. By varying one parameter at a time while keeping others constant, the study examined how sensitive the total cost is to each factor. The results, represented graphically in Figures 1–5, show clear and interpretable trends that support both theoretical consistency and practical relevance. The outcomes highlight which operational parameters hospitals should prioritize for cost and sustainability improvement.

6.2 Total Cost vs Procurement Cost (C)

Figure 1 shows the relationship between total cost and procurement cost. The curve increases almost linearly, indicating that total cost rises proportionally with higher procurement prices. This is expected, as procurement cost directly contributes to total expenditure through the material purchase component.

However, the slope of the curve is not very steep, which means that changes in procurement cost have a smaller effect compared to other parameters like deterioration or demand growth. This finding reflects the buffering effect of large procurement volumes and long-term supplier contracts in hospital networks.

In practical terms, hospitals can manage procurement cost sensitivity through better vendor negotiation, e-procurement systems, and shared purchasing mechanisms. Collaborative procurement networks across multiple hospitals can reduce individual procurement expenses and stabilize supply.

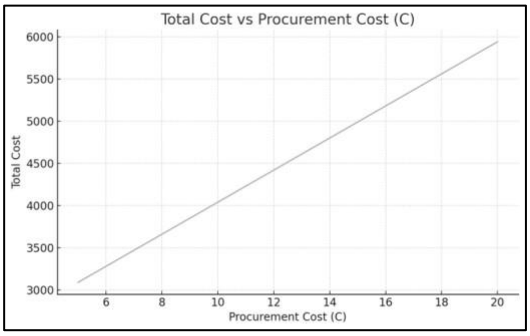


Figure 1: Total Cost vs Procurement Cost (C)

6.3 Total Cost vs Holding Cost (ω_2)

Figure 2 presents the total cost variation with the holding cost coefficient. The graph shows a gradual upward trend, demonstrating that higher holding costs increase the total cost of operations. This increase results from the accumulation of storage expenses over longer inventory cycles.

The slope of the curve is moderate, indicating that holding cost, while influential, does not produce abrupt increases in total expenditure. Hospitals can therefore manage holding costs more flexibly than other parameters. The results imply that optimizing storage conditions, automating warehouse operations, and using centralized inventory systems can help reduce holding expenses without compromising availability.

Efficient inventory management practices, such as rotating stock based on expiry dates and sharing storage across hospital branches, can further minimize unnecessary holding costs.

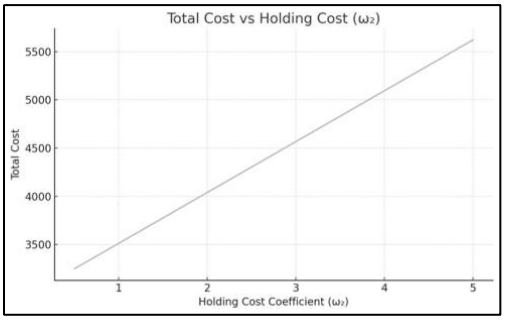


Figure 2: Total Cost vs Holding Cost (ω_2)

6.4 Total Cost vs Carbon Cost (μ)

Figure 3 illustrates the influence of carbon cost on total system cost. The graph shows a near-linear increase, confirming that as the carbon cost multiplier rises, overall cost also increases. The carbon cost component reflects the environmental price of emissions generated from production, transportation, and storage activities.

This relationship highlights the importance of adopting sustainable logistics practices in healthcare. By reducing emissions through route optimization, the use of electric vehicles, and renewable energy in storage systems, hospitals can control both environmental impact and cost escalation.

The slope of the curve is moderate, meaning that hospitals can gradually implement green initiatives without severe short-term financial pressure. Over time, such investments pay off through reduced penalties, tax benefits, and improved institutional reputation.

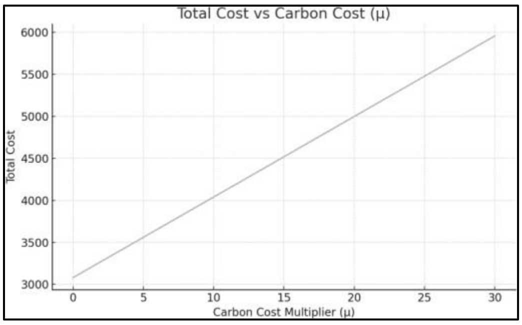


Figure 3: Total Cost vs Carbon Cost (μ)

6.5 Total Cost vs Demand Growth (b)

Figure 4 shows how total cost changes with demand growth. The curve exhibits a nonlinear upward trend, indicating that as patient demand increases, total cost rises at an accelerating rate. This occurs because higher demand leads to larger order quantities, more frequent replenishment, and greater holding and deterioration costs.

At low levels of demand growth, hospitals can absorb fluctuations within existing logistics capacity. However, beyond a certain point, the system experiences exponential cost growth due to strain on storage and procurement systems.

This finding emphasizes the importance of accurate demand forecasting. Hospitals can reduce the impact of rapid demand growth by using predictive analytics to anticipate seasonal peaks or epidemic waves. Integrating real-time patient admission data with inventory systems helps hospitals maintain optimal stock levels without overloading resources.

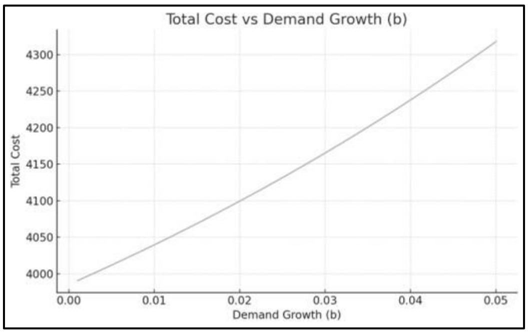


Figure 4: Total Cost vs Demand Growth (b)

6.6 Total Cost vs Deterioration Rate (θ)

Figure 5 depicts the most significant relationship between total cost and deterioration rate. The curve is steep and highly nonlinear, showing that small increases in deterioration lead to large rises in total cost. This is because deterioration affects multiple components simultaneously: it increases waste, forces additional procurement, and generates higher carbon emissions from disposal and replacement activities.

This result confirms that deterioration is the single most sensitive parameter in the healthcare supply chain. Controlling it should be a top management priority. Measures such as advanced cold storage, smart inventory monitoring, and real-time temperature tracking can significantly lower deterioration rates.

Hospitals should also adopt “first-expiry-first-out” (FEFO) systems and establish redistribution protocols for near-expiry supplies. Reducing deterioration not only cuts cost but also contributes to environmental sustainability by reducing waste and emissions.

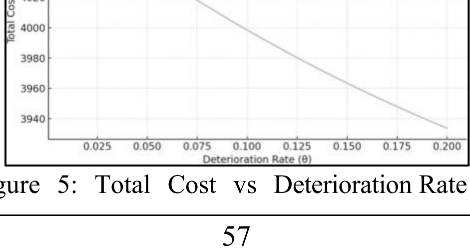


Figure 5: Total Cost vs Deterioration Rate (θ)

6.7 Comparative Sensitivity Discussion

When comparing all five sensitivity curves, the deterioration rate (θ) and demand growth (b) show the steepest slopes. This indicates that these two parameters exert the greatest influence on total system cost. Carbon cost (μ) shows moderate sensitivity, while holding cost (ω_2) and procurement cost (C) have relatively smaller but consistent effects.

This ranking "$\boldsymbol{\theta > b > \mu > \omega_2 > C}$" clearly demonstrates that internal operational parameters dominate cost behavior in healthcare systems. Improving storage management and demand forecasting will therefore yield higher savings than procurement cost negotiations alone.

The analysis confirms that the model successfully identifies key cost drivers and provides a roadmap for prioritizing improvement efforts. Hospitals should first focus on minimizing deterioration and improving demand predictability, followed by emission reduction and efficient storage management.

6.8 Comparison with Existing Practices

In traditional healthcare supply chains, cost optimization often focuses on procurement and supplier negotiation. However, the present model shows that such strategies address only a small portion of total system costs. Real improvements come from managing operational inefficiencies, particularly product spoilage and poor demand forecasting.

Many hospitals also underestimate the impact of emissions on long-term costs. The inclusion of carbon cost in this model highlights the financial advantage of environmentally sustainable practices. Compared to conventional EOQ-based systems, the dual-rate inventory structure provides greater adaptability and reduces waste by synchronizing procurement speed with real-time demand and deterioration patterns.

Therefore, the proposed model extends existing practices by combining economic, operational, and environmental objectives in a single optimization framework. This integrated approach offers hospitals a more balanced and realistic decision-making tool.

6.9 Managerial Insights

The results of this study provide several strong managerial insights for healthcare administrators. They show that hospitals can achieve both economic efficiency and sustainability by strategically managing a few critical parameters rather than attempting to control every operational variable at once.

One of the most important insights is the impact of deterioration control. The analysis proves that product spoilage has the highest effect on total cost. Therefore, investing in advanced cold storage systems, humidity control, and automated monitoring sensors will yield substantial savings. Hospitals can also develop staff training programs on handling sensitive medical supplies to further minimize waste. Reducing deterioration directly improves profitability and simultaneously decreases environmental impact from disposal processes.

Another key insight concerns demand management. Because patient inflow is uncertain, hospitals must use predictive analytics to plan procurement. Integrating hospital information systems with supply chain databases allows managers to forecast demand and adjust orders accordingly. This reduces emergency purchases, minimizes excess stock, and ensures timely availability of critical medicines. Better demand forecasting also prevents overloading of storage capacity, which indirectly reduces holding and deterioration costs.

The study also shows that carbon cost, though moderate in sensitivity, will gain importance as governments introduce stricter emission regulations. Hospitals should begin transitioning toward low-carbon operations early. Adopting electric delivery vehicles, optimizing routes, and using renewable energy for refrigeration can reduce emission-related costs over time. Green supply chain certification can also enhance a hospital's reputation and attract public and institutional support.

Holding and procurement costs, though less sensitive, still play an essential supporting role. Efficient warehouse design, shared logistics centers, and digital procurement platforms can further stabilize expenses. Hospitals that collaborate with suppliers through digital platforms achieve better transparency and faster response to changes in demand or emergencies. Such practices reduce administrative delays and strengthen resilience.

Overall, the managerial implication is clear: hospitals must treat their supply chain as an integrated system rather than separate cost components. Improvements in one area, such as deterioration reduction, can trigger benefits across multiple other areas, including emissions and storage cost. Similarly, using a dual-rate procurement approach allows hospitals to remain flexible — building stock quickly during high-demand periods and slowing down during normal operations. This balance reduces waste, ensures reliability, and enhances both financial and environmental sustainability.

6.10 Summary of Discussion

The results of the numerical and sensitivity analysis confirm that the dual-rate healthcare supply chain model is both practical and effective. The model's ability to capture interactions between demand growth, deterioration, and emission costs makes it superior to traditional systems.

The findings emphasize that deterioration and demand variability are the dominant drivers of cost escalation in healthcare systems. By focusing improvement efforts on these two parameters, hospitals can achieve major efficiency gains. Carbon cost, though secondary, links operational efficiency with environmental goals, demonstrating that sustainability is financially advantageous, not burdensome.

The proposed model therefore provides hospital administrators and policymakers with a clear, data-supported framework for decision-making. It not only minimizes total cost but also promotes sustainable healthcare delivery through responsible resource management. The integration of dual procurement rates, time-dependent demand, and carbon emission cost within one unified framework represents a significant advancement in healthcare supply chain optimization.

VII. Conclusion

This study developed a dual-rate supply chain model for the healthcare system that integrates cost optimization with environmental sustainability. The model was designed to represent real hospital operations more accurately by including time-dependent demand, product deterioration, and carbon emission costs. It combined operational and environmental objectives into a single decision-making framework. The findings show that small improvements in operational efficiency can lead to significant cost savings and sustainability gains.

The research demonstrates that deterioration rate and demand growth are the most sensitive parameters affecting total cost in a healthcare supply chain. Even small increases in deterioration or rapid changes in demand can raise total cost sharply. This means that hospitals must focus their efforts on managing these two factors through better storage, advanced forecasting, and efficient distribution systems. The results also show that carbon costs play a moderate but important role. As environmental regulations become stricter, carbon management will have a growing impact on overall cost performance.

The model's structure allows flexibility through dual-rate procurement. Hospitals can receive supplies quickly in one phase and then switch to a slower, steady rate in the second phase. This dual approach reflects how real hospitals operate during varying demand conditions. It prevents both shortages and overstocking, helping maintain service reliability while reducing waste. The numerical and sensitivity analyses confirmed that this structure performs well under different operational and environmental conditions.

From a managerial perspective, the study offers several insights. Hospitals can achieve both financial efficiency and sustainability by focusing on deterioration control, accurate demand forecasting, and energy-efficient logistics. These actions not only reduce total cost but also lower carbon emissions, contributing to greener healthcare operations. Investments in digital inventory systems, cold-chain management, and predictive analytics can strengthen performance in all these areas.

The study also shows that sustainability does not conflict with profitability. Instead, it complements it. Hospitals that adopt low-emission technologies and reduce waste save money in the long term. The integration of carbon emission cost into the decision model encourages administrators to consider environmental impact as part of regular operational planning. This helps to align hospital operations with national and global sustainability goals.

The results further suggest that healthcare supply chains can achieve higher resilience by using adaptive, data-driven policies. By analyzing sensitivity patterns, hospital managers can identify the most critical parameters and design targeted improvement programs. For example, if deterioration sensitivity is high, they can focus resources on better storage; if demand variability is high, they can enhance forecasting tools. This structured approach ensures continuous improvement and resource optimization.

The dual-rate model also provides a theoretical contribution. It extends traditional inventory models by merging dynamic demand, deterioration, and emission control within one optimization framework. The model shows that sustainable inventory management is not only achievable but also measurable through quantitative performance indicators. The simplicity of the mathematical structure makes it adaptable to different types of healthcare facilities, from small clinics to large hospitals.

Overall, the study concludes that the proposed model can serve as a strategic decision tool for healthcare administrators. It helps them balance service quality, cost efficiency, and sustainability in an integrated way. The research outcomes confirm that managing deterioration and demand variability brings the greatest financial benefits, while addressing carbon cost ensures long-term compliance and responsibility.

This work contributes to both academic research and practical management. Academically, it strengthens the field of sustainable healthcare operations by introducing a unified cost-emission model. Practically, it provides hospitals with a systematic framework for minimizing costs, reducing waste, and improving environmental performance.

In future research, the model can be extended to include stochastic demand patterns, multi-echelon hospital networks, and patient-driven service constraints. Integration with machine learning algorithms can further improve demand prediction and procurement scheduling. Future studies can also explore real-time optimization systems that automatically adjust procurement rates based on live data from hospital information systems.

The dual-rate healthcare supply chain model provides a realistic and sustainable solution for modern healthcare logistics. It combines operational efficiency, economic rationality, and environmental responsibility within a single framework. The study proves that when hospitals manage deterioration, demand, and emissions together, they can deliver better patient care, reduce costs, and contribute to global sustainability goals.

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