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Maximum Flow in A Network With Rough Capacity

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Abstract:- The maximum flow problem is one of the combinatorial optimization problems. The objective of the problem is to find maximum amount of flow from the source to the sink in a network. Due to lack of information in some networks, different types of uncertainty in arc capacities arise. To deal with such situations some scholars consider the arc capacities as random variable or fuzzy variable and solve the problem using probability theory and fuzzy theory. In absence of sufficient data for fitting of appropriate probability distribution for arc capacities or unobservable conditions for measuring arc capacities, neither probability theory nor fuzzy set theory shall be applicable. In this situation, subjective estimation of arc capacities is done by the experts who are inclined to estimate the values in form of certain range as per their belief degree. These estimates can be best characterized by rough variables. In this paper, we have studied maximum flow problem using rough variables as arc capacities and augmenting path algorithm is used to find the sure maximum flow and possible maximum flow. Further, two compromise solutions are proposed using uncertainty theory.
Keywords: Maximum flow, Augmented path, Uncertainty theory, Rough variable, Uncertain variable

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I. INTRODUCTION

The Maximum Flow Problem (MFP) is one of the network optimization problems with wide applications. The problems like electric power transmission, traffic control, communication networks, computer networks, water supply network etc are modelled into MFP problems. In conventional MFP, it is assumed that the decision maker is certain about the capacities of the arcs, flows between different nodes through the arc joining the nodes. But in real life situations, there always exist uncertainty in arc capacities, demand and cost etc.

When the arc capacities of the network are uncertain, some authors have considered the arc capacities as random variables or fuzzy variables. Frank and Hakimi [1] assume that in a communication network, each arc has a random capacity and attempted to find the probability of a flow between vertices. Frank and Frisch [2] determined the Maximum Flow Problem where each arc capacity is a continuous random variable. Kim and Roush [3] are the first to develop the optimal flow taking arc capacities as fuzzy number but Chanas and Kolodziejezyk [4, 5, 6] introduced the main works on the field of fuzzy maximum flow. Chanas and Kolodziejczyk [4] have presented an algorithm for a graph with crisp structure and fuzzy capacities (the arc has a membership function associated in their flow). Chanas and Kolodziejczyk [5] have taken the flow as real number and the capacities have upper bounds and lower bounds with a satisfaction function. Further, Chanas and Kolodziejczyk [6] have studied the integer flows in a network with fuzzy capacity constraint. Again, Chanas et al. [7] developed an optimal flow on imprecise structure called fuzzy graph. Liu and Kao [8] have investigated the network flow when the arc lengths are fuzzy number, generalised fuzzy number etc.

In reality, there exists indeterminacy about the parameters like arc capacities, costs and demands in a network flow. That indeterminacy can be described by random variables if samples are available. But, when sample is not available or the values are unobservable, then the techniques based on probability theory or fuzzy set theory is not appropriate to deal with the problems. In this case, experts are invited for their subjective assessment of the parameters. To deal with the subjective assessment, uncertainty theory as developed by Liu [9, 10] can be successfully applied. Han et al. [11] have applied 99-method to find the maximum flow with uncertain capacities. Ding [12] has used Zigzag uncertain variable as arc capacities to solve maximum flow problem with uncertainty.

In this paper, we have tried to solve the maximum flow problem taking rough variable as arc capacities. Two crisp equivalents as α-pessimistic and α-optimistic value of arc capacities are evaluated. Applying the augmented path algorithm, possible maximum flow and surely maximum flow are calculated. Further, two compromise solution processes are also proposed using uncertainty distribution. For illustration a numerical example is taken.

This paper is organised as follows: In Section II, some basic concepts used in this paper are introduced. In Section III, a maximum flow problem is formulated. In Section IV, two compromise solutions are proposed. In Section V, a numerical example is taken. Finally in Section VI, conclusion of the work is given.

II. PRELIMINARIES

A. Uncertainty Theory

Liu [9][10] has developed uncertainty theory which is considered as a new approach to deal with indeterminacy factors when there is a lack of observed data. In this section, some basic concepts of uncertainty theory has been reviewed which shall be used to establish a compromise solution of maximum flow problem under uncertainty.

Uncertainty measure

Let L be a δ - algebra on a nonempty set Γ. A set function M : L $\rightarrow$[0,1] is called an uncertain measure if it satisfies the following axioms

Axiom 1: (Normality axiom) M (Γ) = 1 for the universal set Γ

Axiom 2: (Duality axiom) $M(\Lambda)+M(\Lambda^C)=1$ for every event Λ

Axiom 3: (sub-additive axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, ...$ we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M(\Lambda_i)$$

The triplet (Γ, L, M) is called an uncertain space.

Axiom 4: (Product measure) Let (Γ_k, L_k, M_k) be uncertainty spaces for k = 1, 2, ...The product uncertain measure M is an uncertain measure satisfying

$$M\{\prod_{k=1}^{\infty}\Lambda_k\} = \wedge_{k=1}^{\infty} M_k(\Lambda_k)$$

where, Λ_k an arbitrary chosen events for L_k for k = 1, 2, . . . respectively.

Uncertain variable

An uncertain variable ξ is essentially a measurable function from an uncertainty space to the set of real numbers. Let ξ be an uncertain variable. Then the uncertainty distribution of ξ is defined as $\phi(x) = M\{\zeta \le x\}$ for any real number x.

Definition 1: An uncertain variable ξ is called linear if it has linear uncertainty distribution L(a, b) such that

$$\phi(x) = \begin{cases} 0 & if \quad x \le a \\ \dfrac{x-a}{b-a} & if \quad a \le x \le b \\ 1 & if \quad x > b \end{cases} \tag{1}$$

where, a and b are real numbers with a < b.

Definition 2: An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution Z(a, b, c) such that

$$\phi(x) = \begin{cases} 0 & if \quad x \le a \\ \dfrac{x-a}{2(b-a)} & if \quad a \le x \le b \\ \dfrac{x+c-2b}{2(c-b)} & if \quad x \ge b \\ 1 & if \quad x \ge c \end{cases} \tag{2}$$

Definition 3: An uncertain distribution ϕ is said to be regular if its inverse function $\phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$.

The linear uncertainty distribution L(a, b) is regular and its inverse uncertainty distribution is

$$\phi^{-1}(\alpha) = (1-\alpha)a + \alpha b \tag{3}$$

The zigzag uncertainty distribution Z(a,b,c) is also regular and its inverse uncertainty distribution is

$\varphi^{-1}(\alpha) = (1 - 2\alpha)a + 2\alpha b$ if $\alpha < 0.5$

$= (2 - 2\alpha)\, b + (2\alpha - 1)\, c \qquad \text{if} \quad \alpha \geq 0.5 \qquad (4)$

B. Rough variable

The concept of rough variable is introduced by Liu [10] as uncertain variable. The following definitions are based on Liu [10]. Definition 4: Let Λ be a non empty set, A be σ - algebra of subsets of Λ, Δ be an element in A, and π be a non negative, real- valued, additive set function on A. The quadruple $(\Lambda, \Delta, A, \pi)$ is called a rough space.

Definition 5: A rough variable ξ on the rough space $(\Lambda, \Delta, A, \pi)$ is a measurable function from Λ to the set of real numbers $\Re$ such that for every Borel set B of $\Re$, we have $\{\lambda \in \Lambda \mid \xi(\lambda) \in \mathrm{B}\} \in A$.

Then the lower and upper approximation of the rough variable ξ are defined as follows

$\bar{\zeta} = \{\zeta(\lambda) \mid \lambda \in \Lambda\}$ (Upper approximation)

$\underline{\zeta} = \{\zeta(\lambda) \mid \lambda \in \Delta\}$ (Lower approximation)

Definition 6: ([a , b], [c , d]) with c ≤ a < b ≤ d is a rough variable, where ξ(λ) = λ from the rough space to the set of real numbers and $\Lambda = \{\lambda \mid c \leq \lambda \leq d\}$ *and* $\Delta = \{\lambda \mid a \leq \lambda \leq b\}$, A is the Borel algebra on Λ, π is the Lebesgue measure.

Definition 7: Let $(\Lambda, \Delta, A, \pi)$ be a rough space. Then the upper and lower trust of event A is defined by

$$\mathrm{T}\underline{\mathrm{r}}(\mathrm{A}) = \frac{\pi\{\mathrm{A}\}}{\pi\{\Lambda\}} \quad \text{and} \quad \mathrm{T}\overline{\mathrm{r}}(\mathrm{A}) = \frac{\pi\{\mathrm{A} \cap \Delta\}}{\pi\{\Lambda\}}$$

The trust of the event A is defined as

$$\mathrm{Tr}(\mathrm{A}) = \frac{1}{2}\left(\mathrm{T}\underline{\mathrm{r}}(\mathrm{A}) + \mathrm{T}\overline{\mathrm{r}}(\mathrm{A})\right)$$

Definition 8: Let ξ_1 , ξ_2 be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$. Then their sum and product are defined as

$$(\xi_1 + \xi_2)(\lambda) = \xi_1(\lambda) + \xi_2(\lambda)$$
$$(\xi_1 . \xi_2)(\lambda) = \xi_1(\lambda) . \xi_2(\lambda)$$

If ξ = ([a , b] , [c , d]) and η = ([p , q] , [r , s]) be two rough variables, then

ξ + η = ([a+p , b+q] , [c+r , d+s]

kξ = ([ka , kb] , [kc , kd], if k ≥ 0

= ([kb , ka] , [kd , kc], if k < 0

Definition 9: Let ξ be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$ and $\alpha \in (0,1]$ then

$\xi_{\sup}(\alpha) = \sup\{\mathrm{r} \mid \mathrm{Tr}\{\xi \geq r\} \geq \alpha\}$ and called α -optimistic value of ξ.

$\xi_{\sup}(\alpha) = \inf\{\mathrm{r} \mid \mathrm{Tr}\{\xi \geq r\} \geq \alpha\}$ is called α -pessimistic value of ξ.

In our work, we have considered the rough variable ζ as ζ = ([a, b] , [c, d]) where c ≤ a < b ≤ d.

The α-optimistic value of ξ = ([a, b] , [c, d]) is

$$\xi_{\sup}(\alpha) = \begin{cases} (1 - 2\alpha)\, d + 2\alpha\, c & \text{if } \alpha \leq \dfrac{d - b}{2(d - c)} \\ 2(1 - \alpha)\, d + (2\alpha - 1)\, c & \text{if } \alpha \geq \dfrac{2d - a - c}{2(d - c)} \\ \dfrac{d(b - a) + b(d - c) + 2\alpha(b - a)(d - c)}{(b - a) + (d - c)} & \text{otherwise} \end{cases} \qquad (5)$$

The α-pessimistic value of ξ = ([a, b] , [c, d]) is

$$\xi_{\inf}(\alpha) = \begin{cases} (1-2\alpha)c + 2\alpha d & if\ \alpha \le \dfrac{a-c}{2(d-c)} \\ 2(1-\alpha)c + (2\alpha-1)d & if\ \alpha \ge \dfrac{b+d-2c}{2(d-c)} \\ \dfrac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)} & otherwise \end{cases} \tag{6}$$

Further, $\xi_{\inf}(\alpha) \ge \xi_{\inf}(\alpha)$ if $\alpha > 0.5$ and $\xi_{\inf}(\alpha) \le \xi_{\inf}(\alpha)$ if $\alpha \le 0.5$

Definition 10: The trust distribution $\phi : [-\infty, \infty] \to [0, 1]$ of a rough variable ξ is defined by

$$\Phi(x) = \mathrm{Tr}\{\lambda \in \Lambda \mid \xi(\lambda) \le x\} \tag{7}$$

If $\xi = ([a,b],[c,d])$ be a rough variable such that $c \le a < b \le d$, then the trust distribution $\phi(x) = \mathrm{Tr}\{\xi \le x\}$ is

$$\phi(x) = \begin{cases} 0 & \text{if } x \le c \\ \dfrac{x-c}{2(d-c)} & \text{if } c \le x \le a \\ \dfrac{[(b-a)+(d-c)]x + 2ac - ad - bc}{2(b-a)(d-c)} & \text{if } a \le x \le b \\ \dfrac{x+d-2c}{2(d-c)} & \text{if } b \le x \le d \\ 1 & \text{if } x \ge d \end{cases} \tag{8}$$

III. PROBLEM DESCRIPTION AND MODEL FORMULATION

The objective of this paper is to solve maximum flow problem with uncertain arc capacities where the arc capacities are taken as rough variables.

Consider a directed flow network N = { V,A,c,s,t}

Where V = { 1,2,3,…..} is the set of nodes.

A = { (i, j)/ i , j ϵ V} is the set of arcs.

C is the non-negative real valued capacities function dependant on A.

s is the source node.

t is the sink node.

Let u = $\{u_{ij} \mid i,j \in A\}$ be the set of arc capacities.

Let f be the total flow from the source node to the sink node t .

Let x_{ij} be the flow along the arc (i, j).

Let Γ_k and $\acute{\Gamma}_k$ be the set of nodes preceding and following the node k respectively. A flow is feasible if it satisfy the following conservation conditions.

$$\begin{cases} \sum_{j \in \tilde{\tau}_{\bar{j}}} x_{sj} - \sum_{j \in \tau_j} x_{js} = f \\ \sum_{j \in \tilde{\tau}_{\bar{j}}} x_{ij} - \sum_{j \in \tau_j} x_{ji} = 0 \\ \sum_{j \in \tilde{\tau}_{jt}} x_{tj} - \sum_{j \in \tau_t} x_{jt} = -f \\ 0 \le x_{ij} \le u_{ij} \ (\text{capacity constraint}) \end{cases} \tag{9}$$

In deterministic maximum flow problem, the arc capacities are crisp values. But in many real world problem, due to uncertainty the arc capacity cannot be taken as crisp. Enough data may not be available to deal the uncertainty through probability approach. Even if data is available, they may be useless due to change in conditions or environment. In this situation the capacity data can be obtained from the decision maker's subject estimation. In case of subjective estimation, it is desirable to give a range instead of a particular value. In this paper, we use rough variable to determine the capacity of the arcs under the following assumptions.

i) The network is directed.
ii) The network does not contain parallel arcs.
iii) All capacities are non-negative.

Let $\xi = \{ \xi_{ij} \mid (i, j) \in A\}$, where ξ_{ij} is a rough variable attached to the arc (i, j).
Let $\xi_{sup}(\alpha)$ and $\xi_{inf}(\alpha)$ be the α-optimistic and α-pessimistic value of ξ respectively where $0 < \alpha < 1$ be the predetermined confidence level provided by the decision maker.

Sure maximum flow model

$$\begin{cases} \text{Max f} \\ subject\,to \quad \sum_{j\in\bar{\tau}_j} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i = s \\ 0 & \forall \quad i \in V - \{s,t\} \\ -f & for \quad i = t \end{cases} \\ \mathrm{x_{ij}} \le \xi_{\sup}(\alpha), \quad \text{(i, j)} \in \mathrm{A} \\ \mathrm{x_{ij}} \ \ge\ 0\ , \qquad \text{(i, j)} \in \mathrm{A}\ . \end{cases} \tag{10}$$

Possible maximum flow model

$$\begin{cases} \text{Max f} \\ subject\,to \quad \sum_{j\in\bar{\tau}_j} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i = s \\ 0 & \forall \quad i \in V - \{s,t\} \\ -f & for \quad i = t \end{cases} \\ \mathrm{x_{ij}} \le \xi_{\inf}(\alpha), \quad \text{(i, j)} \in \mathrm{A} \\ \mathrm{x_{ij}} \ \ge\ 0\ , \qquad \text{(i, j)} \in \mathrm{A}\ . \end{cases} \tag{11}$$

The above two models are converted to two deterministic models which can be solved by augmented path algorithm giving two solutions one as possible maximum flow and the other surely maximum flow.

Algorithm:
We can design the following optimal solution algorithm for obtaining maximum flow.
Step 1: Set a predetermined confidence level 'α' and calculate α-pessimistic value of rough variable and α-optimistic value of rough variable.
Step 2: Construct the corresponding deterministic network N = (V,A,C,s,t) and set the capacity of each arc u_{ij} equal to ξ_{inf} , ξ_{sup} .
Step 3: Apply the Augmented path algorithm to find the Maximum Flow in each case.
Hence a possible maximum flow and sure maximum flow are obtained.

IV. COMPROMISE SOLUTIONS

In solving the rough model of maximum flow problem, we find two solutions as possible solution and sure solution. But in many cases, the decision maker shall prefer one set of solution rather being confused with two sets of solution. In this section, we propose two compromise solutions.

In the proposed compromise solutions, the arc capacities are reduced to uncertain variable and uncertain distributions are used.
When the arc capacities are uncertain variables, the model is transformed to the following form

$$\begin{cases} \text{Max f} \\ Subject\,to \quad \sum_{j\in\bar{\tau}_j} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i = s \\ 0 & \forall \quad i \in V - \{s.\ \ t\} \\ -f & for \quad i = t \end{cases} \\ \mathrm{M}\{\zeta_{\mathrm{ij}} \le x_{ij}\} \ \le \alpha\ , \quad \text{(i, j)} \in \mathrm{A} \\ \mathrm{x_{ij}} \ \ge\ 0\ , \qquad \text{(i, j)} \in \mathrm{A} \end{cases} \tag{12}$$

The capacity constraint is to be transformed to deterministic constraint in order to solve the problem. If the uncertain variable is regular with distribution function φ(.) , then $M\{\xi_{ij} \le x_{ij}\} \le \alpha$ can be transformed to
$x_{ij} \le \varphi_{ij}^{-1}(\alpha)$.

We proposes following theorem to find the compromise solutions

Theorem 1
If $\xi = ([a,b],[c,d])$ be a rough variable with $c \le a < b \le d$, then

$$i)\ \ \xi_{\sup}(\alpha) \le a, \quad if \quad \alpha \ge \frac{2d-a-c}{2(d-c)}$$

$$ii)\ \ \xi_{\inf}(\alpha) \ge b, \quad if \quad \alpha \ge \frac{b+d-2c}{2(d-c)}$$

$$iii)\ \ if \quad 0.5 \le \alpha \le \min\left\{\frac{2d-a-c}{2(d-c)}, \frac{b+d-2c}{2(d-c)}\right\}$$

$$then \quad a \le \xi_{\sup}(\alpha) \le b \quad and \quad a \le \xi_{\inf}(\alpha) \le b.$$

Proof :

$$(i)\ If \quad \alpha \ge \frac{2d-a-c}{2(d-c)}$$
$$then \quad \xi_{\sup}(\alpha) = 2(1-\alpha)d + (2\alpha-1)c$$
$$if \quad \xi_{\sup}(\alpha) > a, then \quad 2(1-\alpha)d + (2\alpha-1)c > a$$
$$\Rightarrow 2\alpha(c-d) > a - 2d + c$$
$$\Rightarrow \alpha < \frac{2d-a-c}{d-c}$$

which is a contradiction.
Hence ,$\xi_{sup} \le$ a .

$$(ii)\ If \quad \alpha \ge \frac{b+d-2c}{2(d-c)}$$
$$then \quad \xi_{\inf}(\alpha) = 2(1-\alpha)c + (2\alpha-1)d$$
$$if \quad \xi_{\inf}(\alpha) > b, \ then \quad 2(1-\alpha)c + (2\alpha-1)d > b$$
$$\Rightarrow 2\alpha(c-d) > b + d - 2c$$
$$\Rightarrow \alpha < \frac{b+d-2c}{2(d-c)}$$

which is a contradiction.
Hence ,$\xi_{inf} \ge$ b .
(iii) The third one follows from (i) and (ii) $\square$

Let the rough variable ξ_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}]) with $c_{ij} \le a_{ij} < b_{ij} \le d_{ij}$ be the arc capacity of the arc (i, j) ϵ A. For a predetermined confidence level α, let p_{ij} is the α-optimistic value of ξ_{ij} and q_{ij} is the α- pessimistic value of ξ_{ij}.

Hence, $p_{ij} \le q_{ij}$ for the choice of $\alpha \ge 0.5$
From (7) it is evident that the trust distribution of ξ_{ij} is linear in [c, a], [a, b] and [b, d] having same slope in [c, a] and [b, d] and different slope in [a, b].
By theorem 1, for the choice of α , the trust distribution of ξ_{ij} shall follow the following uncertainty distribution.
If both p_{ij} and q_{ij} lies in [a_{ij}, b_{ij}] then, the trust distribution of ξ_{ij} can be approximated by the linear distribution L[p_{ij}, q_{ij}]. If $a_{ij} < p_{ij} < b_{ij} < q_{ij}$, then the trust distribution of ξ_{ij} can be approximated by the zigzag uncertain distribution Z(p_{ij}, b_{ij}, q_{ij}). If $p_{ij} < a_{ij} < b_{ij} < q_{ij}$, then the trust distribution of ξ_{ij} can be approximated by the zigzag uncertain distribution Z(a_{ij}, b_{ij}, q_{ij}). If $p_{ij} < a_{ij} < q_{ij} < b_{ij}$, then the trust distribution of ξ_{ij} can be approximated by the zigzag uncertain distribution Z(a_{ij}, q_{ij}, b_{ij}).
Accordingly, four models are proposed which gives compromise solutions to the problem. In all models $\alpha \ge 0.5$.
Case 1: If $a_{ij} < p_{ij} < q_{ij} < b_{ij}$, then the model is transformed to the following deterministic form

$$\begin{cases} \text{Max f} \\ Subject\,to \quad \sum_{j\in\bar{\tau}_j} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i=s \\ 0 & \forall \quad i \in V-\{s,t\} \\ -f & for \quad i=t \end{cases} \\ x_{ij} \le (1-\alpha)p_{ij} + \alpha\, q_{ij}, \quad (i,j) \in A \\ x_{ij} \ge 0, \qquad (i,j) \in A \end{cases} \tag{13}$$

Then the model can be solved by augmented path algorithm to give a compromise solution.
Case 2: If $a_{ij} < p_{ij} < b_{ij} < q_{ij}$, then the model can be transformed to the following deterministic form

$$\begin{cases} \text{Max f} \\ \textit{Subject to} \quad \sum_{j\in\bar{\tau}_{\bar{j}}} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i = s \\ 0 & \forall \quad i \in V - \{s,t\} \\ -f & for \quad i = t \end{cases} \\ x_{ij} \le (2\text{-}2\alpha)b_{ij} + (2\alpha - 1)\, q_{ij}, \quad (i, j) \in A \\ x_{ij} \ge 0, \quad (i, j) \in A \end{cases} \tag{14}$$

Then the model can be solved by augmented path algorithm to give a compromise solution.

Case 3: If $p_{ij} < a_{ij} < b_{ij} < q_{ij}$, then the model can be transformed to the following deterministic form

$$\begin{cases} \text{Max f} \\ \textit{Subject to} \quad \sum_{j\in\bar{\tau}_{\bar{j}}} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i = s \\ 0 & \forall \quad i \in V - \{s,t\} \\ -f & for \quad i = t \end{cases} \\ x_{ij} \le (2\text{-}2\alpha)\, b_{ij} + (2\alpha - 1)\, q_{ij}, \quad (i, j) \in A \\ x_{ij} \ge 0, \quad (i, j) \in A \end{cases} \tag{15}$$

Then the model can be solved by augmented path algorithm to give a compromise solution.

Case 4: If $p_{ij} < a_{ij} < q_{ij} < b_{ij}$, then the model can be transformed to the following deterministic form form

$$\begin{cases} \text{Max f} \\ \textit{Subject to} \quad \sum_{j\in\bar{\tau}_{\bar{j}}} x_{sj} - \sum_{j\in\tau_j} x_{js} = \begin{cases} f & for \quad i = s \\ 0 & \forall \quad i \in V - \{s,t\} \\ -f & for \quad i = t \end{cases} \\ x_{ij} \le (2\text{-}2\alpha)\, q_{ij} + (2\alpha - 1)\, b_{ij}, \quad (i, j) \in A \\ x_{ij} \ge 0, \quad (i, j) \in A \end{cases} \tag{16}$$

Then the model can be solved by augmented path algorithm to give a compromise solution.

V. NUMERICAL EXAMPLE

Consider a power supply network whose transmission lines are numbered (i, j). Consider that the capacities of the line (i, j) are rough variables denoted as $([a_i, b_i][c_i, d_i])$ such that $c_i \le a_i < b_i \le d_i$.

Considering rough techniques, we obtained α-optimistic maximum flow and α-pessimistic maximum flow under different trust level (α= 0.8 and α=0.9)

The Fig-1 depicts the flow network and the table-1 depicts the rough arc capacities, α-optimistic value and α-pessimistic value of each arc under trust level α= 0.8 and α=0.9.

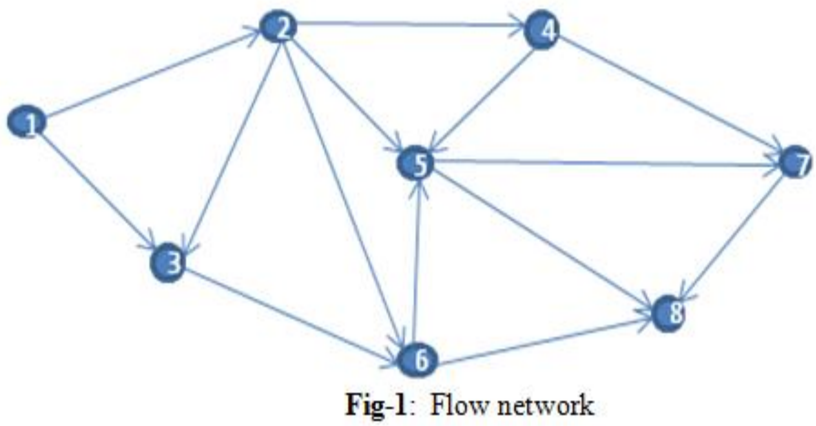

Fig-1: Flow network

Table-1: Rough arc capacities and related parameters

Arc	Arc Capacity	α = 0.9			α = 0.8		
		α-Optimistic value	α-Pessimistic value	Inverse Zigzag Distribution $\phi^{-1}(\alpha)$	α-Optimistic value	α-Pessimistic value	Inverse Linear Distribution $\phi^{-1}(\alpha)$

(1, 2)	([50,70], [40, 80])	48	72	71.6	56	64	62.4
(1, 3)	([70, 90], [60,100])	68	92	91.6	76	84	82.4
(2, 3)	([25, 45], [15, 55])	23	47	46.6	31	39	37.4
(2, 4)	([40, 60], [30, 70])	38	62	61.6	46	54	52.4
(2, 5)	([30, 50], [20, 60])	28	52	51.6	36	44	42.4
(2, 6)	([10,30], [00, 40])	8	32	31.6	16	24	22.4
(3, 6)	([80,100], [70,110])	78	102	101.6	86	94	92.4
(4, 5)	([20, 40], [10, 50])	18	42	41.6	26	34	32.4
(4, 7)	([55,75], [45, 85])	53	77	76.6	61	69	67.4
(5, 7)	([15, 35], [05, 45])	13	37	36.6	21	29	27.4
(5, 8)	([20, 40], [10, 50])	18	42	41.6	26	34	32.4
(6, 5)	([25, 45], [15, 55])	23	47	46.6	31	39	37.4
(6, 8)	([35, 55], [25, 65])	33	57	56.6	41	49	47.4
(7, 8)	([60, 80], [50, 90])	58	82	81.6	66	74	72.4

Using augmented path algorithm, the maximum flows under different conditions are depicted in the following table (Table 2).

Table 2: Result

$\alpha = 0.9$	Value	Model	$\alpha = 0.8$	Value	Model
Sure Maximum flow	99	10	Sure Maximum flow	128	10
Possible Maximum flow	164	11	Possible Maximum flow	148	11
Compromise Maximum flow	163.2	15	Compromise Maximum flow	147.2	13

VI. CONCLUSION

In absence of sufficient data, the decision maker shall take the values of the parameters by the subjective estimation of the domain experts. It is natural that the experts give their subjective estimation in a range of values which can be characterized by rough variables. In this work, the arc capacities of a network are taken as rough variables. In order to find the maximum flow, the rough variables are converted to two crisp values as α-optimistic and α-pessimistic values. Two solutions as possible flow and sure flow are obtained. Further, using uncertain theory, four compromise solutions are also proposed depending on the value of the confidence level α.

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