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Expositions on the Variation of Torsional-Distortional Stresses and Deformations in Box Girder Bridges

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Abstract:- In this work a single cell and a double cell mono symmetric box girder study profile with the same cross sectional (enclosed) area and same material thickness were used to obtain the effect of sectorial properties on the distortional bending stresses and deformation of mono-symmetric box girders. The principles of Vlasov's theory of thin walled plates were used to obtain the torsional-distortional equilibrium equation for the analysis of the study profiles. The distortional bending stresses for the profiles were obtained by numerical differentiation of the warping function to obtain the distortion diagrasm which formed the base system for evaluation of the distortional bending moments. By solving the fourth order differential equations for torsional-distortional equilibrium the deformations of the study profiles were obtained. The warping functions, the distortion diagrams, the distortional bending moments and the torsional and distortional deformations of the study profiles were critically examined and conclusions were made.

Keywords:- box girder, torsional-distortional deformations, distortional bending stresses, sectorial quantities, strain modes.,

I. INTRODUCTION

The analysis of structures generally, requires determination of loads, shears, moments and deflections, all of which have a definite and expected results depending on the complexity of the structure. The procedures for such analysis follow the general rules of kinematics and basic laws for strength of material.

In the case of torsional and distortional analysis of structures, most engineers resort to ready made software programs mainly because a better understanding of the subject matter appears to be lacking. This work is undertaken to verify some of the salient trends akin to distortional stresses and deformations, particularly in mono symmetric box girder structures.

In his book entitled "The theory of thin-walled beams", Vlasov [1] stated as follows: "The deformation of the beam is not analyzed on the basis of the usual hypothesis of plane sections. In its stead the author uses the more general and natural hypothesis of an inflexible section contour and absence of shear stresses in the middle surface, which constitute the basis for the new law of distribution of longitudinal stresses in the cross section. This "new law" which the author calls the law of sectorial areas and which includes the law of plane section as a particular case, permits the computation of stresses in the most general cases of flexural-torsional equilibrium of beams".

This law of sectorial areas and the failure of Benoulis' hypothesis in thin walled structures, form the fundamental principles in the flexural-torsional-distortional analysis of thin-walled box girders using Vlasov's theory [1].

Because of the importance of torsional and distortional stresses in thin-walled box structures, it is necessary to study how these stresses are distributed on cross sections forming the box girder and also to ascertain the effect of sectorial quantities on the distribution of the stresses. Quantities which involve the warping coordinate ω are called sectorial quantities.

When twisting occurs as in the case of torsionaly loaded box girder structures, effects such as warping stresses are found to add to those arising from bending stresses. The tendency is that the torsional-distortional performance of the box girder section is drastically reduced thus, requiring the use of diaphragms and intermediate stiffeners to beef it up.

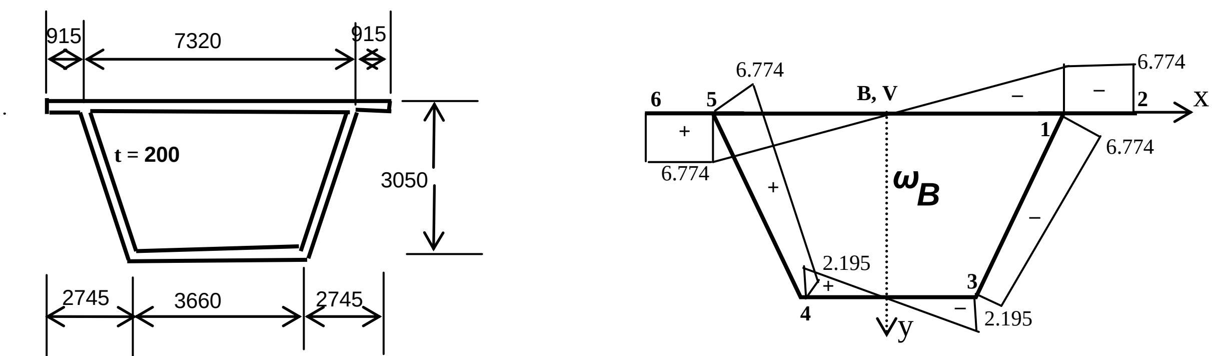
In this work, attempt is made to obtain and explain some of the usual (expected) and un-usual (un-expected) trends peculiar to distortional stresses and torsional- distortional deformations as a result of the law of sectorial areas and the failure of Benoulis' hypothesis in the theory of thin-walled structures propounded by Vlasov and modified by Verbanov [2].

II. WARPING FUNCTION AND TORSIONAL RIGITY

Warping can be described as a distortion of (usually) thin-walled plate or its assemblage as a result of a twisting load or bimoment. Warping function is a sectorial quantity obtained by plotting the out of plane displacement of the cross section (of say a box girder) when the girder is twisted about its sectorial axis, one radian per unit length without bending in either the x or y directions and without longitudinal extension [3]. Warping function therefore represents a scaled value of the warping or out of plane distortion of the cross section.

Figures 1 and 2 show the single cell and double cell mono-symmetric study profiles with their warping functions. Even though the configurations and the net cross sectional areas of the study profiles are different, their overall dimensions (gross areas) and plate thicknesses are the same. Their warping functions also came out to be exactly the same as shown in Figs.1(b) and 2(b). This necessitated a probe into the understanding of the physical meaning of warping function. Warping function can therefore be seen to be related to the gross (enclosed) cross sectional area of the profile and not the net area. It is indeed an area whose magnitude depends on the location of the shear center **B** and the sectorial origin **V** in the profile from which the area integral started.

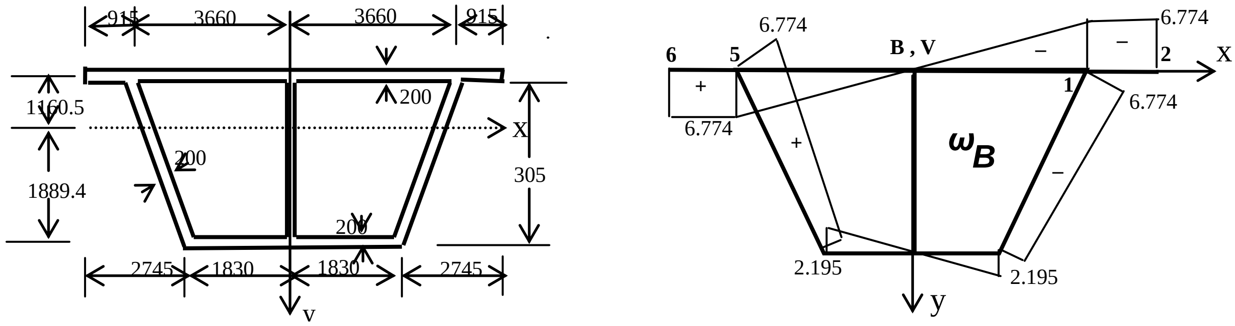
The single cell and double cell study profiles have the same over all enclosed cross sectional area, common line of symmetry, the same shear center, and for the same sectorial origin their warping functions are the same. Therefore, as long as the gross cross sectional area of the profiles does not vary and their shear centres (the poles) remain the same, the warping function remains the same irrespective of the configurations of the profiles.



(a) Single cell mono-symmetric study profile

(b) Warping function of single cell mono-symmetric study

Fig.1 Single cell mono-symmetric study profile and its warping function



(a) Double cell mono-symmetric study profile

(b) Warping function of double cell study profile

Fig.2 Double cell mono-symmetric study profile and its warping function

Torsional Rigidity

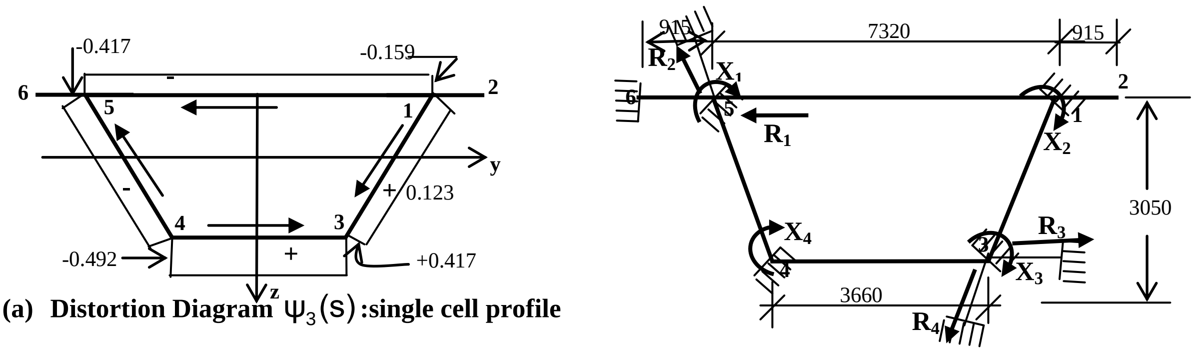
From strength of materials study it is known that the rigidity of a hollow shaft, for example, is given by GJ, where G is shear modulus which depends on the material property, and J is the polar moment of inertia of the cross section profile given by $J = \sum dA(R^2-r^2)$. A is the gross area of the hollow shaft with external radius R, and internal radius r.

In the case of the single cell and double cell study profiles, the gross areas are the same, the material thicknesses are the same, hence their torsional rigidities are the same.

III. DISTORTION DIAGRAM

Closely related to the warping function parameter is the issue of obtaining the distortion diagram for the study profiles. As stated in [4] and [5], the distortion diagram is obtained by numerical differentiation of the warping function diagram. In the study of distortional deformations, distortion diagrams form the base system for evaluation of the distortional bending moments

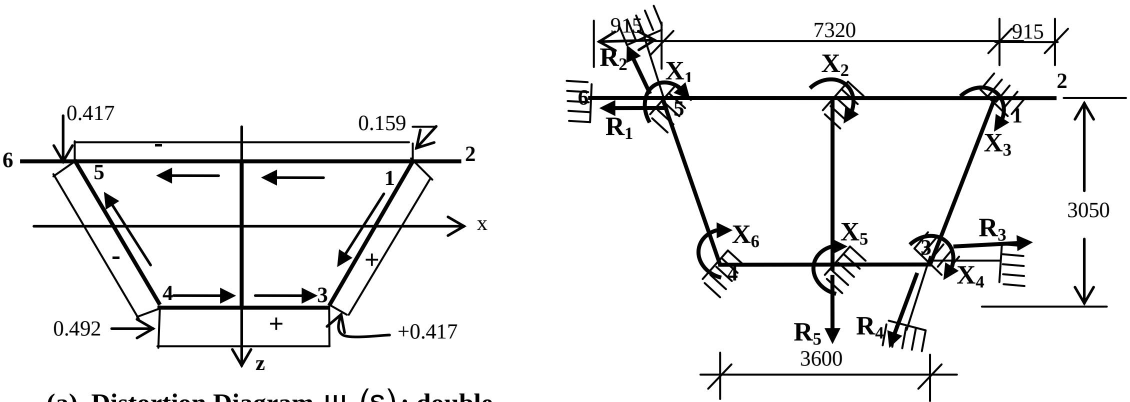
The distortion diagrams for the single cell and double cell study profiles are shown in Figs 3(a) and 4(a) respectively. Here, again, the distortion diagrams, being the numerical integration of warping functions, are the same.



(a) Distortion Diagram $\psi_3(s)$:single cell profile

(b) Base System single cell profile

Fig. 3 Distortion diagram and base system for evaluation of distortional bending moment: Single cell study profile



(a) Distortion Diagram $\psi_3(s)$: double cell profile

(b) Base system: double cell profile

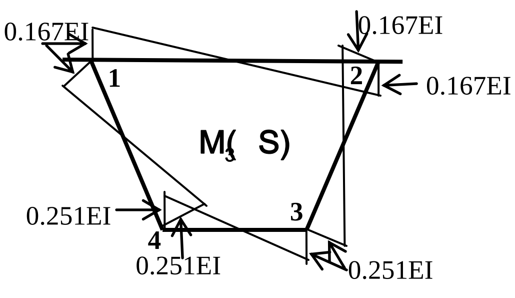
Fig. 4 Distortion diagram and base system for evaluation of distortional bending moment: Double cell study profile

IV. DISTORTIONAL BENDING STRESSES

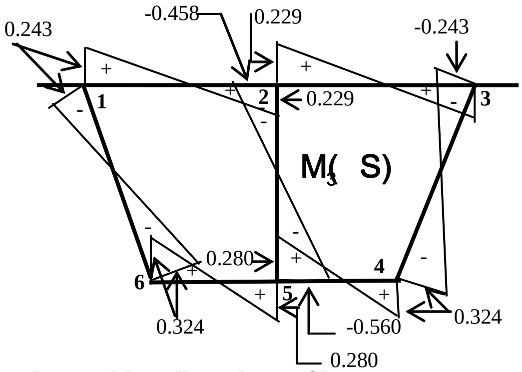
The basis for computation of distortional bending moment is the distortion diagram obtained by numerical differentiation of the warping function diagram. The base systems (based on the distortion diagrams) for evaluation of distortional bending moment diagrams for the single cell and double cell study profiles are shown in Figs.3(b) and 4(b) respectively. In the base systems, R_1, R_2, R_3 etc, represent the magnitude and direction of the forces required to give unit translation to the joints in the direction of the forces (determined by the arrows in the distortion diagrams). X_1, X_2, X_3, etc, represent the forces required to give unit rotation to each of the joints, applied separately.

Following the procedure available in literatures [4] and [5], the distortional bending moment diagrams for the single cell and double cell study profiles were evaluated and presented as Figs 5(a) and 5(b) respectively.

As indicated in the subsequent section of this report, the pure torsional and distortional components of the applied torsional load are the same for both single cell and double cell study profiles.



(a) Single cell study profile



(b) Double cell study profile

Fig. 5 Bending moment diagrams due to distortion of mono-symmetric box girder study profiles

Observations

A critical examination of the distortional bending moment diagrams for the study profiles, particularly at the four free edges of the box frames, show that the numerical values of these bending moments are higher in the double cell box girder even though the torsional rigidity and distortional components of the applied torsional load have the same magnitudes respectively in both profiles. Consequently, it is expected that where higher distortional bending stresses occur, higher distortional deformations are inevitable. In other words, it is expected that double cell frame with higher bending stresses will experience higher distortional deformation while single cell frame with lower bending stresses will experience lower distortional deformation. These presumptions are further investigated in the subsequent torsional-distortional analysis of the study profiles.

V. TORSIONAL-DISTORTIONAL EQUILIBRIUM EQUATIONS

From literature [], [7], [8], the general expression for torsional-distortional equilibrium of all mono-symmetric box girder sections in given by:

$$ka_{33}U_3'' - b_{33}U_3 - c_{33}V_3' - c_{34}V_4' = 0 \quad (1)$$

$$c_{33}U_3' - ks_{33}V_3 + r_{33}V_3'' + r_{34}V_4'' = -\frac{q_3}{G} \quad (2)$$

$$c_{43}U_3' + r_{43}V_3'' + r_{44}V_4'' = -\frac{q_4}{G} \quad (3)$$

where, U_3 and U_4 are unknown functions governing the displacements in the longitudinal and transverse directions respectively,

q_3 and q_4 are distortional and rotational components of the applied torsional load respectively,

k is shape factor, G is shear modulus,

$a_{33,}$ b_{33} and c_{33} are Vlasov's coefficients [1] due to the interaction of strain mode 3 with itself,

c_{34}, c_{43}, r_{34} and r_{43} are Vlasov's coefficients due to interaction of strain mode 3 with strain mode four.

S_{33} is distortional bending moment coefficient due to strain mode 3 and is given by:

$$s_{hk} = s_{kh} = \frac{1}{E}\int_s \frac{M_3(s)M_3(s)}{EI_s}, \quad (4)$$

r_{44} is Vlasov's coefficient due to interaction of rotational strain mode 4 with itself.

$M_3(s)$ is the distortional bending moment of the relevant cross section.

Simplifying eqns (1), (2), and (3) by eliminating U_3 and its derivatives we obtain:

$$\begin{aligned} &\beta_1 V_4'' - \gamma_1 V_3 = K_1 \\ &V_3^{iv} + \alpha_2 V_4^{iv} - \beta_2 V_4'' = K_2 \end{aligned} \quad (5)$$

where,

$$\alpha_2 = \frac{r_{44}}{c_{43}}, \tag{7}$$

$$\beta_1 = r_{34}c_{43} - c_{33}r_{44}; \tag{8}$$

$$\beta_2 = \frac{b_{33}r_{44} - c_{34}c_{43}}{ka_{33}c_{43}}, \tag{9}$$

$$\gamma_1 = c_{43}ks_{33}; \tag{10}$$

$$K_1 = c_{33}\frac{q_4}{G} - c_{43}\frac{q_3}{G}; \tag{11}$$

$$K_2 = \left(\frac{b_{33}}{ka_{33}c_{43}}\right)\frac{q_4}{G} \tag{12}$$

Equations (1) to (12) apply to all mono-symmetric box girder sections irrespective of the number of cells or size of the box girder. They are relevant only for torsional and distortional analysis of such box girder sections. For all other analysis say, flexural-torsional-distortional analysis of such box girders, a new and different set of differential equations will be required.

IV. EVALUATION OF COEFFICIENTS OF THE EQUILIBRIUM EQUATIONS

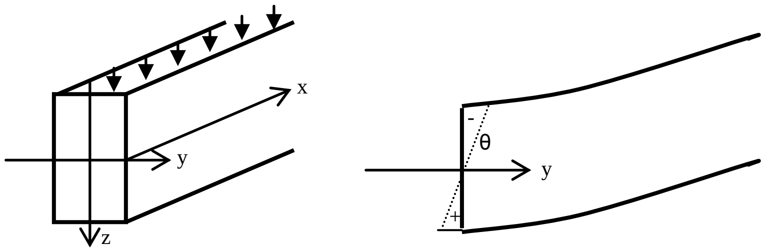
Strain Modes Diagrams

If we consider a simply supported box girder loaded as shown in Fig. 6(a) and assume the normal beam theory, the distortion of the cross section will be as shown in Fig. 6(b) where Θ_1 is the distortion angle (rotation of the vertical axis). The displacement φ_1 at any distance R from the centroid is given by $\varphi_1 = R\Theta_1$. For a unit rotation of the vertical (z) axis, Θ_1 = 1rad, then φ_1 = R at any point on the cross section. Thus φ_1 is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian. Incidentally strain mode one (φ_1) is as a result of vertical loads inducing pure bending in the box girder section and is inconsequential in the distortional analysis of the box girder section since it does not contribute to distortion.

A transverse load acting in a horizontal direction as shown in Fig. 7(a) will also induce bending in x-z plane and y axis is made to rotate through angle Θ_2. The out of plane displacement φ_2, can be obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian. Here again, strain mode two (φ_2) diagram is as a result of loads causing bending in the minor axis of the box girder which can be very negligible. More over, studies [5] have shown that strain mode two has no interaction with strain modes three and four. Hence strain mode two is also neglected in this study.

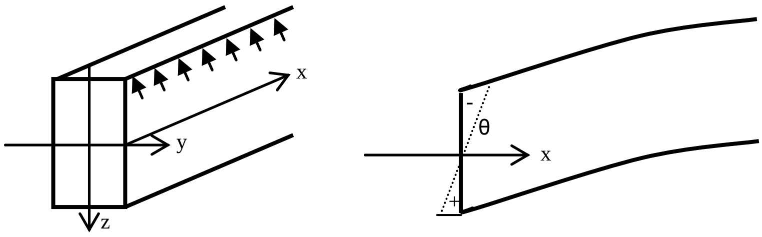
Strain mode three, (φ_3) is the warping function of the box girder section, Figs 1(a) and 2(a). It has a derivative $\varphi_3' = \psi_3(s)$, obtained by numerical differentiation of φ_3 diagram. Both φ_3 and $\varphi_3' = \psi_3(s)$ diagrams are useful in the computation of the coefficients of the differential equation.

Strain mode four, ψ_4 is the displacement of the box girder section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section.



(a) Simply supported girder section **(b) Cross section distortion in x – y plane**

Fig. 6 Simply Supported Girder and Cross Section Distortion

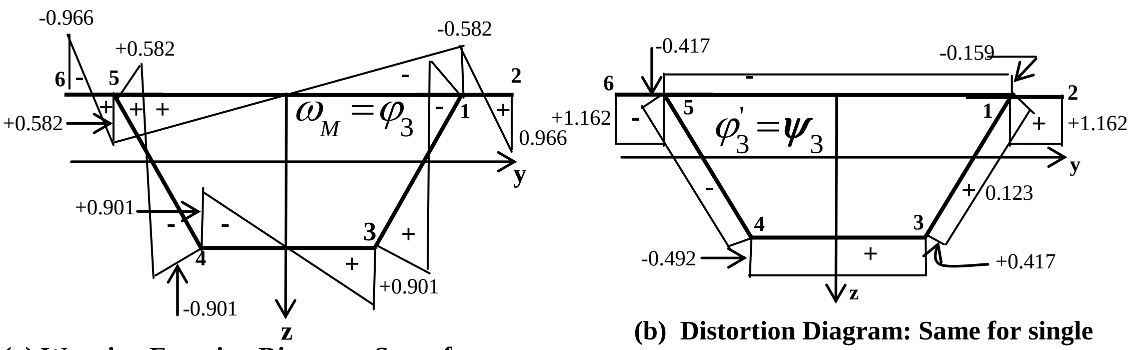


(a) Simply supported girder section **(b) Cross section distortion in x – z plane**

Fig. 7 Simply Supported Girder and Cross Section Distortion

The interaction of strain modes three and four constitutes the main consideration in torsional-distortional analysis of box girder structures. This interaction is inseparable as can be seen from eqns. (1), (2), (3) where V_3 and V_4 are inter- related.

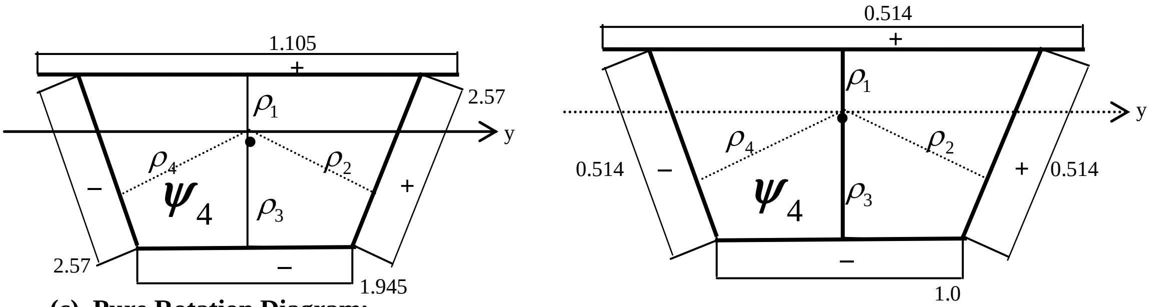
Strain mode three (φ_3) and strain mode four (ψ_4) diagrams for the single cell and double cell study profiles are shown in Figs. 8 and 9 respectively.



(a) Warping Function Diagram: Same for single & double cell

(b) Distortion Diagram: Same for single & double cell

Fig. 8: Warping function and distortion diagrams for single cell and double cell study profiles [5]



(c) Pure Rotation Diagram: Single cell

(b) Pure rotation Diagram: double cell

Fig. 9: Rotation diagrams for single cell and double cell study profiles [5]

Evaluation of Vlasov's Coefficients

The coefficients involved in the differential equations of equilibrium, eqns (1) to (3), are in this section computed for the single cell and double cell study profiles. In an earlier work, [8] the expressions for these coefficients are given as follows:

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA \qquad (a)$$

$$b_{ij} = b_{ji} = \int \varphi_i'(s)\varphi_j'(s)dA \qquad (b)$$

$$c_{kj} = c_{jk} = \int \varphi_k'(s)\psi_j(s)dA \qquad (c) \qquad\qquad (13)$$

$$c_{ih} = c_{hi} = \int \varphi_i'(s)\psi_k(s)dA \qquad (d)$$

$$r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; \qquad (e)$$

Using the strain modes diagrams (Figs 8 and 9) the relevant coefficients were computed with the aid of well known formulae for evaluating volume integrals. The values obtained are recorded in Table1.

Table 1: Values of Vlasov's coefficients for Single cell and double cell study profiles

Coefficient Elements	Single cell	Double cell
a_{33}	0.75	0.750
b_{33}	1.407	1.533
c_{33}	1.407	1.533
r_{33}	1.407	1.533
$c_{34}=c_{43}$	1.265	1.295
$r_{34}=r_{43}$	1.265	1.295
r_{44}	14.616	14.485

VII. TORSIONAL-DISTORTIONAL ANALYSIS OF THE STUDY PROFILES

Load Evaluation

The pure torsional and distortional components of the applied torsional loads used in this work were those derived in earlier studies [4], [9]. These are distortional component q_3 and rotational component q_4, whose values are:

q_3 = 157.16kN and q_4 = 1446.50kN.

Single Cell Profile

Substituting the relevant coefficients in Table 1 into the parametric equations (7) to (12) and using the results in differential equations (5) and (6) we obtain:

$$2.371V_3^{iv} + 27.405V_4^{iv} - 18.963V_4'' = 2.120*10^{-6}$$
$$-18.964V_4^{iv} - 5.503*10^{-4}V_3 = 1.9163*10^{-4} \qquad (14)$$

Integrating by method of trigonometric series with accelerated convergence we have,

$$V_3(x) = 3.268*10^{-2}\,\text{Sin}\frac{\pi x}{50}$$
$$V_4(x) = 2.80*10^{-3}\,\text{Sin}\frac{\pi x}{50} \qquad (15)$$

Double Cell Profile

Substituting the appropriate coefficients in Table 1 into the parametric equations (7) to (12) and using the results in differential equations (5) and (6) we obtain:

$$2.428V_3^{iv} + 27.16V_4^{iv} - 20.528V_4'' = 2.87845*10^{-4}$$
$$-20.528V_4^{iv} + 1.632*10^{-3}V_3 = 2.613*10^{-4} \qquad (16)$$

Integrating by method of trigonometric series with accelerated convergence we have,

$$V_3(x) = 1.500*10^{-2}\,\text{Sin}\frac{\pi x}{50}$$
$$V_4(x) = 3.526*10^{-3}\,\text{Sin}\frac{\pi x}{50} \qquad (17)$$

VIII. DISCUSSION OF RESULTS

For a simply supported girder, the maximum torsional and distortional deformations occur at the mid span, i.e., at x = L/2, where the length of the girder L, was taken as 50m for the purpose of this study. Substituting this into eqns. (15) and eqns (17) we obtain:

V3 = 32.68mm, V4 = 2.80mm [for single cell profile]

V3 = 15.00mm, V4 = 3.53mm [for double cell profile]

These results show that:

- Distortional deformation V3 was higher in the case of single cell study profile.
- Torsional deformation V4 was higher in the case of double cell study profile

In other words, decrease in the distortional deformation of a mono-symmetric box girder profile was accompanied by increase in torsional deformation. Quantitatively, the decrease in the distortional deformation of double cell profile was 118% while the increase in its torsional deformation was 21%, compared with those of single cell profile.

Thus, the introduction of the vertical web member in the double cell study profile increased the distortional capacity of the box girder and reduced its torsional capacity.

These results are not in consonance with the expectations based on the distortional bending stresses of the study profiles, i.e. , it was expected that where higher distortional bending stresses occur, higher distortional deformations are inevitable. The results show that single cell frame with lower distortional bending stresses exhibit higher distortional deformation while double cell frame with higher distortional bending stresses show higher torsional deformation.

This deviation from the expected trend can be attributed to the law of sectorial quantities explicable within the theory of thin-walled structures.[1]

IX. CONCLUSIONS

1**.** Warping function was found to be related to the gross cross sectional area of the profiles and not the net area.

2. The sectorial areas or warping functions of the single cell and double cell mono-symmetric box girder profiles were found to be the same and will always remain unchanged as long as the gross area of the cross section and the shear center (the pole) do not vary. Similarly, the distortion diagrams for the single and double cell mono symmetric study profiles were the same because their warping functions were the same.

3. Even though the torsional rigidity and the pure torsional and distortional components of the applied torsional load were the same for the study profiles, their distortional bending stresses and deformations varied.

4. The expectation that the double cell profile with higher distortional stresses would undergo higher distortional deformation did not hold true. Rather, the single cell profile with lower distortional bending stresses exhibited higher distortional deformation.

5. Decrease in the distortional deformation of a mono symmetric box girder was accompanied by increase in torsional deformation.

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