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Equations of motion are five in nature not three.

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ABSTRACT
We have derived several equations of motion from velocity-time graph. This paper shows that only five different equations of motion are derived from velocity time graph without needing to know the normal and frictional forces acting at the point of contact. We also discuss all the conditions to be an equation of motion. After deriving these five equations of motion, we examine the importance and the educational benefits of these equations of motion.

Keywords: - Five kinematical quantities, velocity-time graph representation, Conditions to be an equation of motion, importance and educational benefits.

I. INTRODUCTION

In mathematical physics, equations of motion are equations that describe the behaviour of a physical system in terms of its motion as a function of time. More specifically, the equations of motion describe the behaviour of a physical system as a set of mathematical functions in terms of dynamic variables: normally spatial coordinates and time are used, but others are also possible, such as momentum components and time.

There are two main descriptions of motion: dynamics and kinematics. Dynamics is general, since momenta, forces and energy of the particles are taken into account. In this instance, sometimes the term refers to the differential equations that the system satisfies (e.g., Newton's second law or Euler–Lagrange equations), and sometimes to the solutions to those equations. However, kinematics is simpler as it concerns only spatial and time-related variables. In circumstances of constant acceleration, these simpler equations of motion are usually referred to as the "SUVAT" equations, arising from the definitions of kinematic quantities: displacement (S), initial velocity (u), final velocity (v), acceleration (a), and time (t).

Equations of motion had discovered by Galileo Galilee but he did not manage to prove it practically that his equations was right. Because during his time there was no instrument exists for measuring time and distance accurately, also at that time experimentation was not permitted in Italy. Later on Sir Isaac Newton proved the three equations of motion practically as well as graphically. Now are often called Newton's three equations of motion. Up to now, we have read three equations of motion i.e.

1. $a = (v – u) \div t$
2. $S = ut + ½ at^2$
3. $S = (v^2 – u^2) \div 2a$

Where
u = initial velocity (m/s)
v = final velocity (m/s)
a = acceleration (m/s²)
t = time (s)
S = displacement (m)

The first equation is mainly used to calculate acceleration produced by the body however we also calculate initial velocity, final velocity and time taken when other three variables are given. If acceleration is constant, this implies that there is a uniform rate of change of velocity. The longer acceleration is occurring, the greater the change in velocity is. When acceleration is constant, the rate of change of velocity is directly proportionate to time. If there is no acceleration present, final velocity is equal to initial velocity. The second equation of motion is used to calculate the displacement of an object that is undergoing uniform acceleration when final velocity is not given and the third equation of motion is used to calculate the displacement of an object that is undergoing uniform acceleration when time is not given. All these three equations of motion are used to describe various components of a moving object.

II. EQUATIONS OF MOTION BY GRAPHICAL METHOD

When an object moves along a straight line with uniform acceleration, it is possible to relate its velocity, acceleration during motion and the distance covered by it in a certain time interval by a set of equations known as the equations of motion. There are five such equations. These are:

1. $v = u + at$
2. $S = ut + ½ at^2$
3. $2aS = v^2 – u^2$
4. $S = vt – ½ at^2$
5. $S = ½ (u + v) t$

where u is the initial velocity of the object which moves with uniform acceleration a for time t, v is the final velocity, and S is the distance travelled by the object in time t. Equation-1 describes the velocity-time relation and Equation-2 represents the position-time relation. Equation-3, which represents the relation between the position and the velocity, can be obtained from Equations-1 and Equation-5 by eliminating t. Equation-4 represents position-time relation on the other hand Equation-5 describes the displacement-time and displacement-velocity relation. These three equations can be derived by graphical method.

Consider the linear motion of a body with initial velocity u. The body accelerates uniformly and in time t, it acquires the final velocity v. The velocity-time graph is a straight line AB as shown in figure. It is evident from the graph that:

Initial velocity (at t = 0) = OA = CD = u
Final velocity (at t) = OE = BD = v

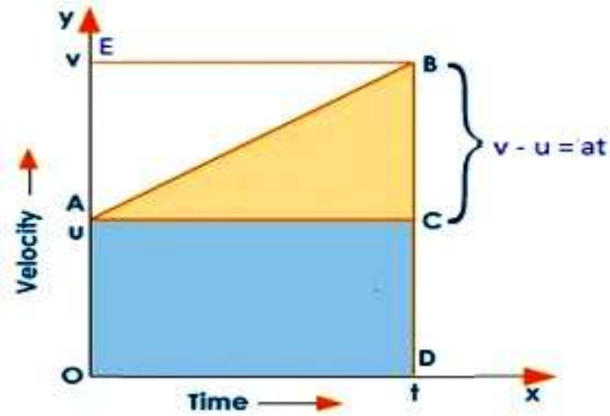

Figure-1. A velocity–time graph for an object undergoing uniform acceleration

Acceleration of the body (a) = Slope of the line AB

$$a = \frac{BC}{AC} = \frac{BD - CD}{OD}$$

$$\underline{\mathbf{a = (v - u) \div t}} \qquad \text{------- (1)}$$

Equation (1), represent First equation of motion and is called displacement independent equation. This equation is often useful in kinematics problems where you do not know the displacement of the body but still have to work with the initial velocity, final velocity, acceleration, and time.

In Fig-1, the distance travelled by the object is obtained by the area enclosed within OABD under the velocity-time graph AB.

Thus, Distance travelled by a body in time 't' is equal to area of the trapezium OABD.

$$S = \text{area of rectangle OACD} \times \text{area of triangle ABC}$$

$$S = (OD \times OA) + ½ (AC \times BC)$$

$$S = (t \times u) + ½ [t \times at]$$

$$\underline{\mathbf{S = ut + ½ at^2}} \qquad \text{--------- (2)}$$

Equation (2) represents Second equation of motion. It is derived directly from velocity-time graph using basic definitions of velocities, time and acceleration. What is important to notice is that the quantity of final velocity is not present in this equation. We say, therefore, that the equation is final velocity independent equation of motion. This equation is often useful in kinematics problems where you do not know the final velocity of the body but still have to work with the initial velocity, acceleration, displacement and time.

From the velocity-time graph shown in Fig-1, the distance 'S' travelled by the object in time t, moving under uniform acceleration 'a' is given by the area enclosed within the trapezium OABC under the graph.

Distance travelled by a body in time 't' is equal to area of the trapezium OABD.

$$S = ½ (\text{sum of} \parallel \text{sides}) \times (\perp \text{distance between parallel sides})$$

$$S = ½ (OA + BD) \times OD$$

$$S = ½ (u + v) \times t$$

From first equation of motion "t = (v – u) ÷ a", we get

$$S = ½ \times (u + v) \times (v - u) \div a$$

$$S = ½ \times (v^2 - u^2) \div a$$

$$\underline{\mathbf{S = (v^2 - u^2) \div 2a}} \textbf{ or } \underline{\mathbf{2aS = (v^2 - u^2)}} \qquad \text{----- (3)}$$

Equation (3) represents Third equation of motion. It is derived directly from velocity-time graph using basic definitions of velocities, time and acceleration. What is important to notice is that the quantity of time is not present in this equation. We say, therefore, that the equation is time independent equation of motion. This equation is often useful in kinematics problems where you do not know the time taken by the body but still have to work with the initial velocity, final velocity, acceleration, and displacement.

In 2001, we had derived fourth equation of motion incidentally and later on in April 2008, we had taken copyright of fourth equation of motion from the Govt. of India under registration number L-30407/2008. In Fig-1, the distance travelled by the object is obtained by the area enclosed within OABD under the velocity-time graph AB.

Distance travelled by a body in time 't' is equal to area of the trapezium OABD.

$$S = \text{area of rectangle OEBD} - \text{area of triangle ABE}$$

$$S = (OD \times OE) - ½ (AE \times BE)$$

$$S = (t \times v) - ½ (at \times t)$$

$$S = (vt) - ½ (at^2)$$

$$\underline{\mathbf{S = vt - ½ at^2}} \qquad \text{----------- (4)}$$

Equation (4) represents Fourth equation of motion. It is derived directly from velocity-time graph using basic definitions of velocities, time and acceleration. What is important to notice is that the quantity of initial velocity is not present in this equation. We say, therefore, that the equation is initial velocity independent equation of motion. This equation is often useful in kinematics problems where you do not know the initial velocity of the body but still have to work with the final velocity, displacement, acceleration, and time.

When we were studying equations of motion deeply we noticed that the publishers were publishing one more equation at that time but it was not considered as equation of motion. From the velocity-time graph shown in Fig-1, the distance 'S' travelled by the object in time t, moving under uniform acceleration 'a' is given by the area enclosed within the trapezium OABC under the graph. That is,

Distance travelled by a body in time 't' is equal to area of the trapezium OABD.

$$S = ½ (\text{sum of} \parallel \text{sides}) \times (\perp \text{distance between parallel sides})$$

$$S = ½ (OA + BD) \times OD$$

$$\mathbf{S = ½ (u + v) \times t} \qquad \text{---------- (5)}$$

Distance covered = Average velocity × time

Equation (5) represents Fifth equation of motion. It is derived directly from velocity-time graph using basic definitions of velocities, time and acceleration. What is important to notice is that the quantity of acceleration is not present in this equation. We say, therefore, that the equation is acceleration independent equation of motion. This equation is often useful in kinematics problems where you do not know the acceleration of the body but still have to work with the velocities, time, and displacement.

III. CONDITIONS TO BE AN EQUATION OF MOTION.

For any equation to be considered as an EQUATION OF MOTION it must satisfy the following conditions: -

1. An equation must consist of kinematic variables i.e. time, displacement, final velocity, initial velocity & acceleration.

2. The value of all the quantities must be correct, so that we can get correct value of the unknown quantity. For e.g. if S = n (here n is any natural number) than in every equation in which we get the value of displacement (S) is equal to "n" is an equation of motion, i.e.

$$S = (v^2 - u^2) /2a = ut + ½ at^2 = ½ (u + v) t = vt - ½ at^2 = n$$

3. An equation must be in its shortest form.

For e.g. if "$\underline{a = (v^2 - u^2) \div 2vt - at^2}$" is an equation, then its shortest form is $\underline{a = (v - u)/t}$;

$$a = (v^2 - u^2) \div 2vt - at^2$$

$$a = [(v - u)(v + u)] \div [t(2v - at)]$$
$$a = [(v - u)(v + u)] \div [t(2v - v + u)]$$
$$a = [(v - u)(v + u)] \div [t(v + u)]$$
$$\underline{a = (v - u) \div t}$$

4. We use an equation as formulae, so we must remember that LHS variable must not be present in RHS. In case if LHS variable is also present in RHS that means derivation is yet incomplete. After removing LHS variable from RHS, we get an independent kinematical equation of motion.
For e.g., $\underline{u = 2u - v + at}$, Here initial velocity 'u' is in both sides, so we removing 'u' from RHS by putting $u = v - at$ and get an equation of motion, i.e. $\underline{u = v - at}$

$$\underline{u = 2u - v + at}$$
$$u = 2(v - at) - v + at$$
$$u = 2v - 2at - v + at$$
$$\underline{u = v - at}$$

5. The equation should not be the opposite view of its own.
For e.g. $\underline{u = v - at}$ is an equation of motion than $\underline{v = u + at}$ is not a new equation.

Now only 5 equations can satisfy above mention all conditions.

i. $a = (v - u) \div t$ [Displacement (S) independent equation]
ii. $S = ut + \frac{1}{2} at^2$ [Final velocity (v) independent equation]
iii. $S = (v^2 - u^2) \div 2a$ [Time (t) independent equation]
iv. $S = vt - \frac{1}{2} at^2$ [Initial Velocity (u) independent equation]
v. $S = \frac{1}{2} (u + v) t$ [Acceleration (a) independent equation]

IV. METHOD

Graphical analysis is an important tool for physicists to use to solve problems. Sometimes, however, we have enough information to allow us to solve problems algebraically. Algebraic methods tend to be quicker and more convenient than graphical analysis. If you were in the vehicle, you would simply use the vehicle's speedometer to determine the speed of the vehicle. Knowing the speed of your vehicle, you could easily determine how far it would travel in a given time interval using the equation **v = S/t.** As you can see, the best way to solve a problem is usually determined by the information that is available to you. To be able to solve problems related to motion with uniform acceleration, in which the velocity may change but the acceleration is constant, we need to use algebraic equations to solve the numerical that describe this type of motion. Equations of motion are very useful to locate the position and calculate the final velocity, initial velocity, acceleration and time taken by the object or body in uniform motion.

Table-1 shows the five key equations of accelerated motion. You should be able to solve any kinematic numerical regarding equations of motion by correctly choosing one of these five equations. They involve the variables for displacement, initial velocity, final velocity, acceleration, and time interval. In table-1, we see that in each equation one variable is missing. When solving uniform acceleration problems, choose which equation to use based on the given, missing and required variables of the problem.

S. No.	Kinematic equations of motion	Variables found in equation	Variables not in equation of motion
1.	$a = (v - u) \div t$	a, u, v, t	Displacement (**S**)
2.	$S = ut + \frac{1}{2} at^2$	S, u, a, t	Final velocity (**v**)
3.	$S = (v^2 - u^2) \div 2a$	S, u, v, a	Time (**t**)
4.	$S = vt - \frac{1}{2} at^2$	S, v, a, t	initial velocity(**u**)
5.	$S = \frac{1}{2} (u + v) \times t$	S, u, v, t	Acceleration (**a**)

Table-1. The Five Key Equations of Accelerated Motion.

Our first task is to determine which of the five equations of accelerated motion to use. Usually, you can solve a problem using only one of the five equations. Second task is to identify which equation contains all the variables for which we have given variables, missing variable and the unknown variable that we are asked to calculate. After identifying the correct equation, you can use it to solve the numerical.

For e.g. (1) A sports car starts from rest. If the car accelerates at a rate of 10 m/s^2 for 15s, what is the displacement of the car?
Sol: - Given: u = 0 m/s, a = 10 m/s^2, t = 15s.

Missing: final velocity (v).

Required: Displacement (S).

Analysis: In table 1, we see that equation-3 has all the given variables, missing variable and required variable. So, Equation-3 will allow us to solve for the unknown variable.

Solution from Second equation of motion,

$$S = ut + ½ at^2$$
$$S = (0m/s × 15s) + (½ × 10 × 15s × 15s)$$
$$S = (0m) + (1125m)$$

Displacement (S) = 1125meter.

IV. IMPORTANCE AND EDUCATIONAL BENEFITS

Equations of motion are very important to calculate displacement, final velocity, initial velocity, acceleration and time. Equations $S = ut + ½ at^2$, $S = (v^2 – u^2) ÷ 2a$, $S = vt – ½ at^2$ and $S = ½ (u + v) t$ are derived from each other.

$$\mathbf{S = ut + ½ at^2}$$
Put t = (v – u) ÷ a in above equation, we get
$$S = u [(v – u) ÷ a] + ½ a [(v – u) ÷ a]^2$$
$$S = [(uv – u^2) ÷ a] + ½ [(v – u)^2 ÷ a]$$
$$S = [(uv – u^2) ÷ a] + [(v^2 + u^2 – 2uv ÷ 2a)]$$
$$S = [2uv – 2u^2 ÷ 2a] + [v^2 + u^2 – 2uv ÷ 2a]$$
$$S = 2uv – 2u^2 + v^2 + u^2 – 2uv ÷ 2a$$
$$\mathbf{S = (v^2 – u^2) ÷ 2a}$$
Put v = u + at in above equation, we get
$$S = [(u + at)^2 – u^2] ÷ 2a$$
$$S = (u^2 + a^2t^2 + 2uat – u^2) ÷ 2a$$
$$S = (a^2t^2 + 2uat) ÷ 2a$$
$$\mathbf{S = ut + ½ at^2}$$
Put u = v – at in above equation, we get
$$S = (v – at) t + ½ at^2$$
$$S = vt – at^2 + ½ at^2$$
$$\mathbf{S = vt – ½ at^2}$$
Put u = v – at in above equation, we get
$$S = (u + at) t – ½ at^2$$
$$S = ut + at^2 – ½ at^2$$
$$\mathbf{S = ut + ½ at^2}$$
Put a = (v – u) ÷ t in above equation, we get
$$S = ut + ½ [(v – u) ÷ t] t^2$$
$$S = ut + ½ [(v – u)] t$$
$$S = ut + ½ (vt – ut)$$
$$S = ut + ½ vt – ½ ut$$
$$S = ½ vt + ½ ut$$
$$\mathbf{S = ½ (v + u) t}$$
Put v = u + at in above equation, we get
$$S = ½ [u + at + u] × t$$
$$S = ½ [2u + at] × t$$
$$S = ½ [2ut + at^2]$$
$$\mathbf{S = ut + ½ at^2}$$

Importance of Second equation of motion

1. A truck is starts from rest after 20 seconds. The truck gets its final velocity with acceleration of $2m/s^2$, calculate the distance covered.

Sol: - Solution from Third equation of motion,
$$S = (v^2 – u^2) ÷ 2a$$
$$S = [v^2 – (0)^2] ÷ (2 × 2m/s^2)$$
$$S = v^2 ÷ 4m/s^2$$

Solution from Fourth equation of motion,

$$S = vt - \frac{1}{2} at^2$$
$$S = [v \times (20sec)] - \frac{1}{2} [2m/s^2 \times (20)^2]$$
$$S = [v \times (20sec)] - \frac{1}{2} [2m/s^2 \times 400]$$
$$S = (v \times 20sec) - 400meter$$

Solution from Fifth equation of motion,

$$S = \frac{1}{2} (v + u) \times t$$
$$S = \frac{1}{2} (v + 0meter) \times 20sec$$
$$S = \frac{1}{2} (v \times 20sec)$$

Here we are not able to find exact distance in numeric when final velocity is not given. However we can get answer from above equations by using one more equation [$v = u + at$] but it is too lengthy and consume time.

Solution from Second equation of motion,

$$S = ut + \frac{1}{2} at^2$$
$$S = (0\ m/s \times 20sec) + (\frac{1}{2} \times 2m/s^2 \times 20sec \times 20sec)$$
$$S = 0meter + 400meter$$

Displacement (S) = 400meter

Importance of Fourth equation of motion

2. A Ferrari car is travelling with a constant velocity suddenly car driver increase the speed of the car and car get its final velocity 123m/s in 15sec with acceleration of $3m/s^2$, calculate the distance covered.

Sol: - Solution from Second equation of motion,

$$S = ut + \frac{1}{2} at^2$$
$$S = [(u \times 15s) + (\frac{1}{2} \times 3m/s^2 \times 15s \times 15s)$$
$$S = [(u \times 15s) + (337.5meter)]$$

Solution from Third equation of motion,

$$S = (v^2 - u^2) \div 2a$$
$$S = (123)^2 - (u^2) \div 2a$$
$$S = [15129m^2/s^2 - u^2] \div (2 \times 3m/s^2)$$
$$S = [15129m^2/s^2 - u^2] \div 6m/s^2$$

Solution from Fifth equation of motion,

$$S = \frac{1}{2} (v + u)\ t$$
$$S = \frac{1}{2} (123 + u)\ 15$$
$$S = \frac{1}{2} (1845 + 15u)$$
$$S = 922.5 + 7.5u$$

Here we are not able to find exact distance in numeric when initial velocity is not given. However we can get answer from above equations by using one more equation [$u = v - at$] but it is too lengthy and consume time.

Solution from Fourth equation of motion,

$$S = vt - \frac{1}{2} at^2$$
$$S = (123m/s \times 15s) - (\frac{1}{2} \times 3m/s^2 \times 15s \times 15s)$$
$$S = (1845m) - (337.5meter)$$

Displacement = 1507.5meter

Educational benefits of equations of motion

According to duke's survey, approxmately 97% students are not read '$S = vt - \frac{1}{2} at^2$' in thier whole life and the strange thing is that students are reading '$S = \frac{1}{2} (u + v)\ t$' but no one know that this is also an equation of motion. So now we are discuss the educational benefits that why these equations [$S = \frac{1}{2} (u + v)\ t$ and $S = vt - \frac{1}{2} at^2$] should be considered as equation of motion?

Reason first: - Basically, there are only three equations of motion. These are first equation of motion [$v = u + at$], second equation of motion [$S = ut + \frac{1}{2} at^2$] and the third equation of motion [$S = (v^2 - u^2) \div 2a$] which are derived from velocity-time graph. In the same manner fourth equation of motion[$S = vt - \frac{1}{2} at^2$] and fifth equation of motion [$S = \frac{1}{2} (u + v) \times t$] is also derived from velocity-time graph. Hence, the total equations of motion are five.

Reason Second: - Fourth equation of motion is the only equation of motion that calculates distance, when initial velocity is not given. In same manner, Fifth equation of motion is the only equation of motion that calculates distance, when acceleration is not given.

Reason Third: - Fourth & Fifth equation of motion can be derived graphically as well as algebraically same as for other equations of motion.
Reason Fourth: - After deep calculations and many derivations, only five different equations are possible from the velocity-time graph for a body moving with a constant acceleration.
Reason Fifth: - Usually we go with three equations of motion, which are actually five in number; this fact is confusing the students. They cannot understand the difference between laws of motion & equations of motion. So if we consider both equations [$S = vt - \frac{1}{2} at^2$ and $S = \frac{1}{2} (u + v) t$] as Fourth & Fifth equation of motion then the total number of equations of motion will be five & therefore a great confusion will be ended. This will help to make easier concepts in the mind of a physics student.
Reason Sixth: - because these equations are, the last two equations of motion so both of them should be considered as equations of motion, so that people can gain complete knowledge about equations of motion.

V. SUMMARY AND CONCLUSIONS

Equations of motion are consists of five kinematical quantities and only five different equations of motion are derived from velocity-time graph. All the five equations of motion are derived from each other but they are different in properties i.e. all the five equations of motion are important.

The easiest and fastest way to solve the numerical regarding equations of motion is to choose the right equation of motion from Table-1. These five equations of motion are very useful for researchers and students especially undergraduate because these equations of motion are teaching to the students of Ninth standard in India.

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