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# A Monotone Process Replacement Model for a Two Unit Cold Standby Repairable System

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**Abstract:-** In this paper a two unit cold standby repairable system with two monotone processes exposing to exponential failure law, is studied under the assumption that each component after repair is not **as good as new**. Under this assumption we study an optimal replacement policy N in which we replace the system when the number of failures of component 1 reaches N. We determine an optimal repair replacement policy N* such that the long run average lose is minimized. We derive an explicit expression of the long-run average lose and the corresponding optimal replacement policy can be determined analytically. Numerical results are also established to highlight the theoretical results.

**Keywords:-** Renewal process, Geometric process, α-series process, Repair replacement policy, Renewal cycle, Renewal reward theorem.

## I. INTRODUCTION

In the earlier days , most repair replacement models assume that a failure system after repair will yield a function system which is "as good as new" and the repair times are neglected, so that the successive operating times forms a renewal process. These types of models may be called as perfect repair models.

Barlow and Hunter introduced [1] a minimal repair model in which a minimal repair does not change the age of the system. Thereafter an imperfect repair model was developed by Brown and Proschan [3] under which a repair with probability 'p' as perfect repair and with probability '1-p' as minimal repair. Many others worked in this direction and developed corresponding optimal replacement polices by Black et.al [2], Park [10], Kijima [6], McCall [9] , Nakagawa [11] ,Stadje and Zukerman [13] and so on.

In general, for a deteriorating system, it is reasonable to assume that the successive working times are stochastically decreasing while the consecutive repair times after failures are stochastically increasing, due to the ageing and accumulated wearing many systems. To model such simple repairable deteriorating system Lam [7,8] first introduced a geometric process repair model under the assumptions that the system after repair is not 'as good as new.' Under these assumptions, he considered two kinds of replacement polices -one based on the working age T of the system and other based on the number of failures N of the system. Later Zhang [15] developed a bivariate replacement policy (T, N) to generalize Lam's work. Other replacement policies under geometric process repair model were reported by Stadje and Zuckerman [13], Stanley [14], Zhang [16] ,Zhang and Wang [17,18] , and so on.

All these research works discussed above are related to one component repairable system. However, in many practical applications, the standby techniques are usually used for improving the reliability or raising the availability of the system. Zhang [16] applied the geometric process repair model to a single cold standby repairable system with one repairman and studied a replacement policy N and corresponding optimal replacement policy $N^*$ is determined such that the long-run-average cost per unit time is minimum. Later, Zhang [17] applied the geometric process repair model to a two unit cold standby repairable system with one repairman and studied a replacement policy N.

Braun et.al [4] studied some important properties of monotone processes and proved that alpha series processes is more appropriate to model the up times. On this understanding, in this chapter we have proposed to develop two monotone processes maintenance model and obtained an optimal replacement policy N.

The objective of this chapter is to determine an optimal replacement policy N for two unit cold standby systems with one repairman using two monotone processes exposing to Weibull failure law. It assumed that the successive working times $\{X_n, n=1, 2\ldots\}$ of a system form a decreasing α-series process while the consecutive repair times $\{Y_n, n=1, 2\ldots\}$ form an increasing geometric process. Under these assumptions we studied a replacement policy N and corresponding optimal replacement policy $N^*$ is determined such that the long-run-average cost per unit time is minimized.

In modeling of these deteriorating systems we utilize definitions given in Lam [7] .

**Definition 1:** Given two random variables X and Y, if P(X>t) > P(Y>t) for all real t, then X is called stochastically larger than Y or Y is stochastically less than X. This is denoted by $X >_{st} Y$ or $Y <_{st} X$ respectively.

**Definition 2:** Assume that $\{Y_n, n=1,2,\ldots\}$, is a sequence of independent non-negative random variables. If the distribution function of $X_n$ is $F_n(t) = F(a^{n-1}t)$ for some a > 0 and all n=1,2,3,…., then $\{Y_n, n=1,2,\ldots\}$ is called a geometric process, '**a**' is the ratio of the geometric process.

**Obviously:**

if a>1, then $\{Y_n, n=1,2,\ldots\}$ is stochastically decreasing, i.e, $Y_n >_{st} Y_{n+1}$ , n=1,2,…;

if 0<a<1, then $\{Y_n, n=1,2,\ldots\}$ is stochastically increasing, i.e, $Y_n <_{st} Y_{n+1}$ , n=1,2,;

if a=1, then the geometric process becomes a renewal process.

**Definition 3**: Assume that $\{X_n, n=1,2,\ldots\}$, is a sequence of independent non-negative random variables. If the distribution function of $X_n$ is $F_n(t) = F(k^{\alpha}t)$ for some $\alpha > 0$ and all n=1, 2, 3… then $\{X_n, n=1, 2\ldots\}$ is called an $\alpha$ series process, $\alpha$ is called exponent of the process. Braun *et. al* [4].

**Obviously:**

if $\alpha$ >0, then $\{X_n, n=1,2,\ldots\}$ is stochastically decreasing, i.e, $X_n >_{st} X_{n+1}$ , n=1,2,…;

if $\alpha$ <0, then $\{X_n, n=1,2,\ldots\}$ is stochastically increasing, i.e., $X_n <_{st} X_{n+1}$ , n=1,2,;

if $\alpha$ =0, then the $\alpha$ series process becomes a renewal process.

## II. THE MODEL

In this section, we developed a model for two component cold standby repairable system with one repairman using two monotone processes and exposing to Weibull failure law in such a way that the long-run average cost per unit time is minimized with the following assumptions.

**ASSUMPTIONS**

1) At the beginning, both the components are new and component 1 is in working state while the other component 2 is in cold standby state.
2) The two components appear alternatively in the system. i.e., when the working component fails immediately the standby component begins to work and the failed one is repaired by the repairman. Whenever the repair of the failed one is completed, it becomes cold standby. If one fails and the other is still under repair, it must wait for repair and the system breaks down.
3) A component in the system is replaced some time by an identical one and the replacement time is negligible.
4) The components after repair are not 'as good as new'. The time interval between the completion of the (n-1)$^{th}$ repair and the completion of the n$^{th}$ repair on component i is called n$^{th}$ cycle of component i for i=1, 2 and n=1,2,… .
5) A component in the system can't produce the working reward during cold standby and no cost is incurred during the waiting period for repair.
6) Let $X_n^{(i)} \, and \; Y_n^{(i)}$ , for i=1, 2 and n=1, 2 …are all S-independent.
7) Let $X_n^{(i)}$ be working time follow decreasing α-series processes exposing to Weidbull failure law and $Y_n^{(i)}$ be the repair time follow an increasing geometric processes exposing to Weibull failure law of component i in the n$^{th}$ cycle , for i=1, 2 and n=1, 2, …. .
8) Let $E\left(X_1^{(i)}\right) = \lambda$ and $E\left(Y_1^{(i)}\right) = \mu$, for i=1, 2.
9) Let $F(k^{\alpha} x) = F_n(x)$ and $G(b^{n-1} y) = G_n(y)$ be the distribution functions of $X_n^{(i)}$ and $Y_n^{(i)}$ respectively for i=1,2 and n=1,2,3,… where α>0 and 0<b<1.
10) Let the repair cost rate of each component is $C_r$, the working reward per unit time of each component is $C_w$ and the replacement cost of the system is C.

In the next section, we find an optimal solution for policy N based on the assumptions of the model and determine an optimal solution for N such that the long-run average cost is minimum.

## III. OPTIMAL SOLUTION

We consider an optimal replacement policy N under which the number of failures of component 1 reaches N. According to the assumptions of the model, two components appear alternatively in the system. When the number of failures of component 1 reaches N, component 2 may be in the repair state of the (N-1)$^{th}$

cycle or in the cold standby state in the N$^{th}$ cycle. Naturally, a reasonable replacement policy N should be that component 1 can't be repaired any more when the number of its failures reaches N and component 2 works until failure in the N$^{th}$ cycle.

According to the renewal reward theorem (see Ross [12]), the long-run average cost per unit time of the system under policy N is given by:

$$C(N) = \frac{\textit{The}\text{ expected cost incurred in a renewal cycle}}{\textit{The}\text{ expected length of renewal cycle}}, \tag{3.1}$$

Where the length of a system in a renewal cycle under policy N is:

$$L = \sum_{k=1}^{N} X_k^{(1)} + \sum_{k=1}^{N-1} Y_k^{(1)} + \sum_{k=2}^{N} (Y_{k-1}^{(2)} - X_k^{(1)}) I_{\left\{Y_{k-1}^{(2)} - X_k^{(1)} > 0\right\}} + \sum_{k=1}^{N-1} (X_k^{(2)} - Y_k^{(1)}) I_{\left\{X_k^{(2)} - Y_k^{(1)} > 0\right\}} + X_N^{(2)}, \tag{3.2}$$

where the first, second, third, fourth and the fifth terms refers to working age, repair time, waiting for repair time, standby time of component 1 and working age of component 2 in the N$^{th}$ cycle respectively. The total time duration when the component 1 is in cold standby is called standby time.

The expected length of a renewal cycle is

$$\begin{aligned} E(L) = &\sum_{k=1}^{N} E\left(X_k^{(1)}\right) + \sum_{k=1}^{N-1} E\left(Y_k^{(1)}\right) + \sum_{k=2}^{N} E\left[(Y_{k-1}^{(2)} - X_k^{(1)}) I_{\left\{Y_{k-1}^{(2)} - X_k^{(1)} > 0\right\}}\right] \\ &+ \sum_{k=1}^{N-1} E\left[(X_k^{(2)} - Y_k^{(1)}) I_{\left\{Y_k^{(2)} - X_k^{(1)} > 0\right\}}\right] + E(X_N^{(2)}) \end{aligned} \tag{3.3}$$

Where I is the indicator function, such that

$$I_A = \begin{cases} 1 & \text{if event A occurs} \\ 0 & \text{if event A does't occurs.} \end{cases}$$

According to the assumptions of the model, convolution and Jacobian transformations, the probability density function of $Y_{k-1}^{(2)} - X_k^{(1)}$ and $X_k^{(2)} - Y_k^{(1)}$ are respectively.

$$g(u) = \int_0^\infty f\left(v, u+v\right) dv,$$

Where $X_k^{(1)} = v,\ \mathrm{Y}_{k-1}^{(2)} = u + v,\ $ such that $\ u = Y_{k-1}^{(2)} - X_k^{(1)}$, (3.4)

and

$$g(v) = \int_0^\infty f\left(u+v, u\right) du, \tag{3.5}$$

$$\textit{where} \quad \mathrm{X}_k^{(2)} = u + v;\ \ \mathrm{Y}_k^{(1)} = u \quad \textit{such that } v = X_k^{(2)} - Y_k^{(1)}. \tag{3.6}$$

Therefore, by definition of mathematical expectation we have:

$$E\left[Y_{k-1}^{(2)} - X_k^{(1)}\, I_{\{Y_{K-1}^{(2)} - X_K^{(1)} > 0\}}\right] = \int_0^\infty u.g\left(u\right) du. \tag{3.7}$$

$$E\left[X_k^{(2)} - Y_k^{(1)} I_{\{X_K^{(2)} - Y_K^{(1)} > 0\}}\right] = \int_0^\infty v g(v) dv. \tag{3.8}$$

Now the expected length of working time can be obtained as follows:

Let $X_k^{(i)} \sim W\ \ (x : \eta_1, \beta_1),\ \ \textit{for}\ \ k = 1,2,3,....,\ \ \textit{and}\ \ i = 1,2.$

Then the distribution function of $X_k^{(i)}$, for k=1,2,3,….and i=1,2 is :

$$F_k(x) = F(k^\alpha x) = 1 - e^{-\left(\frac{k^\alpha x}{\eta_1}\right)^{\beta^1}} \ ; x > 0, \beta_1 < 1, \eta > 0 \tag{3.9}$$

By definition the expected length of working time is given by:

$$E\left(X_x^{(i)}\right)=\int_0^\infty xdF\left(k^\alpha x\right), \qquad i=1,2. \tag{3.10}$$

$$=\frac{\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right)}{k^\alpha}=\frac{\lambda}{k^\alpha}, \ where \ \lambda=\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right), \qquad i=1,2. \tag{3.11}$$

The expected length of repair time of component 1 can be obtained as follows:
Let $Y_k^{(i)} \sim W \ (y:\eta_2,\beta_2)$ then the distribution function of $Y_k^{(i)}$ for i=1,2, and k=1, 2, 3, …., is

$$F_k(y)=F(a^{k-1}y)=1-e^{-\left(\frac{a^{k-1}y}{\eta_2}\right)^{\beta_2}} \quad ; y>0, \beta_2<1. \tag{3.12}$$

By definition, the expected length of repair time is is given by:

$$E\left(Y_x^{(i)}\right)=\int_0^\infty ydF(a^{k-1}y) \qquad ,i=1,2.$$

$$=\frac{\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right)}{a^{k-1}}=\frac{\mu}{a^{k-1}}, \qquad where \ \mu=\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right), \qquad i=1,2. \tag{3.13}$$

The expected length of waiting time for repair can be computed as follows:
Let g(u) be the probability density function of $u=Y_{k-1}^{(2)}-X_k^{(1)}$, then by definition of probability density function and using Jacobian transformation

we have:

$$g(u)=\int_0^\infty f\left(v,u+v\right)dv,$$

where $X_k^{(1)}=v, \ \mathrm{Y}_{k-1}^{(2)}=u+v,$ such that $u=Y_{k-1}^{(2)}-X_k^{(1)}.$ (3.14)

Since $X_k^{(i)}$ *and* $Y_k^{(i)}$ are all independent, for i=1,2 and k=1,2,3,…..,n.

$$g(u)=\int_0^\infty f\left(v\right).f\left(u+v\right)dv. \tag{3.15}$$

From equations (3.14) and (3.15),we have:

$$g(u)=\int_0^\infty\left(\frac{k^\alpha}{\eta_1}\right)^{\beta^1}\beta_1 v^{\beta-1}e^{-\left(\frac{k^\alpha v}{\eta_1}\right)^{\beta^1}}\left(\frac{a^{k-2}}{\eta_2}\right)^{\beta_2}\beta_2(u+v)^{\beta_2-1}e-\left(\frac{a^{k-2}(u+v)}{\eta_2}\right)^{\beta_2}dv\cdot \tag{3.16}$$

On simplification, equation (3.16) becomes:

$$g(u)=\left(\frac{a^{k-2}}{\eta_2}\right)^{\beta_2}\beta_2 Z^{\beta_2-1}e^{-\left(\frac{a^{k-2}Z}{\eta_2}\right)^{\beta_2}}, \qquad Z>0, \ \beta_2<1. \tag{3.17}$$

Let $E\left[Y_{k-1}^{(2)}-X_k^{(1)}I_{\{Y_{K-1}^{(2)}-X_K^{(1)}>0\}}\right]=\int_0^\infty ug(u)du$ (3.18)

$$=\int_0^\infty u\left(\frac{a^{k-2}}{\eta_2}\right)^{\beta_2}\beta_2 Z^{\beta_2-1}e^{-\left(\frac{a^{k-2}Z}{\eta_2}\right)^{\beta_2}}du\,.$$

On simplification, equation (3.18) becomes:

$$E\left[Y_{k-1}^{(2)}-X_k^{(1)}I_{\{Y_{K-1}^{(2)}-X_K^{(1)}>0\}}\right]=\frac{\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right)}{b^{k-2}}=\frac{\mu}{b^{k-2}} \quad . \tag{3.19}$$

Where $\mu=\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right)$.

Similarly, the expected length of cold standby time can be computed as follows:

$$E\left[X_k^{(2)}-Y_k^{(1)}I_{\{X_K^{(2)}-Y_K^{(1)}>0\}}\right]=\int_0^\infty vg(v)dv\,. \tag{3.20}$$

Where $g(v)$ be the probability density function (p.d.f) of $v=X_k^{(2)}-Y_k^{(1)}$. By definition of p.d.f and using Jacobean Transformation, we have:

$$g(v)=\int_0^\infty f\left(u+v,u\right)du\,. \tag{3.21}$$

$$where \quad \mathrm{X}_k^{(2)}=u+v,\ \mathrm{Y}_k^{(1)}=u \ \text{ such that } v=X_k^{(2)}-Y_k^{(1)}. \tag{3.22}$$

Since $X_k^{(i)}$ *and* $Y_k^{(i)}$ , for i=1, 2 are all independent and form a geometric process,

$$g(v)=\int_0^\infty f\left(u+v\right).f(u)du. \tag{3.23}$$

Using equations (3.22) and (3.23), we have:

$$g(v)=\int_0^\infty\left(\frac{k^\alpha}{\eta_1}\right)^{\beta^1}\beta_1(u+v)^{\beta-1}e^{-\left(\frac{k^\alpha\ (u+v)}{\eta_1}\right)^{\beta^1}}\left(\frac{a^{k-2}}{\eta_2}\right)^{\beta_2}\beta_2u^{\beta_2-1}e-\left(\frac{a^{k-2}u}{\eta_2}\right)^{\beta_2}du \tag{3.24}$$

On simplification, equation (3.24) becomes:

$$g(v)=\left(\frac{k^\alpha}{\eta_1}\right)^{\beta_1}\beta_1Z^{\beta_1-1}e^{-\left(\frac{k^\alpha Z}{\eta_1}\right)^{\beta_1}}, \qquad Z>0,\ \beta_1<1. \tag{3.25}$$

From equations (3.8) and (3.25), we have:

$$E\left[X_k^{(2)}-Y_k^{(1)}I_{\{X_K^{(2)}-Y_K^{(1)}>0\}}\right]=\int_0^\infty vg(v)dv$$

$$=\int_0^\infty v\left(\frac{k^\alpha}{\eta_1}\right)^{\beta_1}\beta_1Z^{\beta_1-1}e^{-\left(\frac{k^\alpha Z}{\eta_1}\right)^{\beta_1}}dv,$$

$$=\frac{\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right)}{k^\alpha}=\frac{\lambda}{k^\alpha},\ \ where\ \ \lambda=\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right), \qquad i=1,2. \tag{3.26}$$

From equations (3.11), (3.13), (3.19), and (3.26), equation (3.3) becomes:

$$E(L)=\sum_{k=1}^{N}\frac{\lambda}{k^\alpha}+\sum_{k=1}^{N-1}\frac{\mu}{b^{k-1}}+\sum_{k=2}^{N}\frac{\mu}{b^{k-2}}+\sum_{k=1}^{N-1}\frac{\lambda}{k^\alpha}+\frac{\lambda}{N^\alpha} \tag{3.27}$$

Using the equations (3.1) and (3.27) ,we have:

$$C(N)=\frac{C_r E\left[\sum_{k=1}^{N-1}\left(Y_k^{(1)}+Y_k^{(2)}\right)\right]+C-C_w E\left[\sum_{k=1}^{N}\left(X_k^{(1)}+X_k^{(2)}\right)\right]}{E(L)}$$

$$C(N)=\frac{2C_r\sum_{k=1}^{N-1}\frac{\mu}{b^{k-1}}-2C_{wr}\sum_{k=1}^{N-1}\frac{\lambda}{k^{\alpha}}+C}{\frac{\lambda}{N^{\alpha}}+\sum_{k=1}^{N-1}\frac{\mu}{b^{k-1}}+\sum_{k=2}^{N}\frac{\mu}{b^{k-2}}+\sum_{k=1}^{N-1}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N}\frac{\lambda}{k^{\alpha}}}$$

$$C(N)=\frac{2C_r l_2+C-2C_w l_1}{2(l_1+l_2)} \tag{5.28}$$

This is the long-run average cost function under policy N.

Where $l_2=\sum_{k=1}^{N-1}\frac{\mu}{b^{k-1}}$, $l_1=\sum_{k=1}^{N}\frac{\lambda}{k^{\alpha}}$

$$\lambda=\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right),\qquad \mu=\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right).$$

Using C (N) we determined an optimal replacement policy N* such that the long-run average cost per unit time is minimum.

In the next section, we provide numerical work to highlight the theoretical results

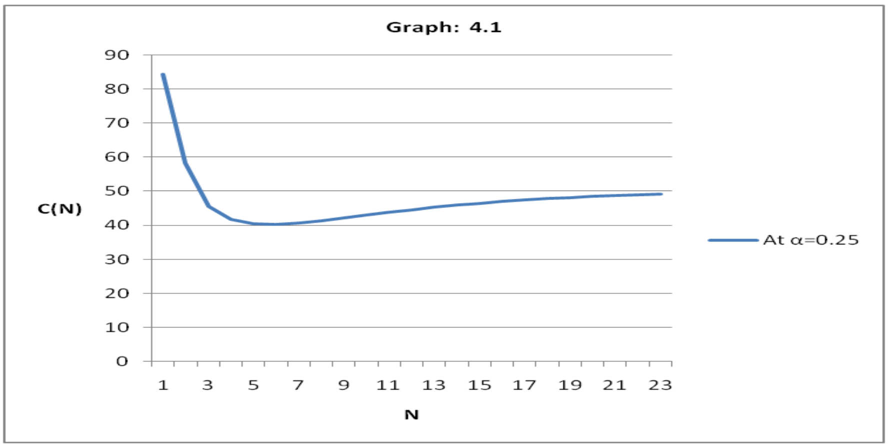
## IV. NUMERICAL RESULTS AND CONCLUSIONS
## V.

For given hypothetical values of the parameters α, b, Cw, C, Cr, λ, μ, the optimal replacement policy N* is calculated from an explicit expression in equation (5.3.28) such that the long-run average cost is minimum :

**Table: 5.4.1: Values of long-run average cost per unit of time**

| | For given hypothetical values of α =0.25,b=0.8 $\lambda=15,\mu=25$,Cr=50 Cw=25,C=5000 | For given hypothetical values of α =0.35,b =0.8 $\lambda=15,\mu=25$,Cr=50 Cw=25,C=5000 |
|---|---|---|
| (N) | **C(N)** | **C(N)** |
| 1 | 84.375 | 84.375 |
| 2 | 58.15365 | 59.51042 |
| 3 | 45.52993 | 47.06602 |
| 4 | 41.57305 | 43.17404 |
| 5 | 40.31524 | 41.92263 |
| **6** | **40.18813** | **41.76148** |
| 7 | 40.59775 | 42.10754 |
| 8 | 41.26899 | 42.69403 |
| 9 | 42.05759 | 43.38355 |
| 10 | 42.88194 | 44.10028 |
| 11 | 43.69413 | 44.80111 |
| 12 | 44.46598 | 45.4617 |
| 13 | 45.18158 | 46.06917 |
| 14 | 45.83303 | 46.61778 |
| 15 | 46.41766 | 47.10639 |

| | | |
|---|---|---|
| 16 | 46.93626 | 47.53674 |
| 17 | 47.39187 | 47.91227 |
| 18 | 47.78887 | 48.23744 |
| 19 | 48.13235 | 48.51712 |
| 20 | 48.42769 | 48.75631 |
| 21 | 48.68029 | 48.95982 |
| 22 | 48.89529 | 49.13223 |
| 23 | 49.07752 | 49.27771 |




## VI. CONCLUSIONS

i) From the table 4.1 and graph 4.1, we observe that C (6)= **40.18813** is the minimum of the long run average cost leading to the optimal policy N* = 6, which indicates we should replace the system at the end of 6th failure.

ii) We observed that the long run average cost per unit time is decreases as the value of b is increases and vice versa in case of the value of α.

iii) It is observed that for a small increase in ‘b’ , there is an increase in N* and a decrease in average long-run cost per unit time. **S**imilarl conclusion can be drawn in case of the parameter α.

iv) At different values of the parameters of the model considered the value of long-run average cost per unit time is converges to a constant value. This result is coinciding with theoretical result.

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