

Design and Implementation of Compensator for Optimizing Linear Time Invariant System Parameters

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Abstract:- Most of control systems in vogue are Linear Time Invariant (LTI) systems and are characterized by several parameters such as settling time, over shoot and rise time etc. Optimization of these parameters is required to meet the desired system response and smooth operation of the system. In this paper, an optimization based compensator is designed and implemented for a closed loop LTI system and the system response is observed both in time and frequency domain. The compensator is designed using optimization technique so that the control system meets the system specifications. The LTI system model consisting of gain, limited integrator and a delay unit is considered for optimization. The controller is tuned using Ziegler-Nichols open loop tuning algorithm to optimize mainly settling time, rise time and percentage of over shoot. The feasible and optimal solutions within the specified tolerances of these parameters are obtained and presented.

Keywords:- control systems, controller, closed loop, optimization, stability

I. INTRODUCTION

A system is considered to be Linear Time Invariant (LTI) system if it satisfies the requirement of time-invariance and linearity. LTI systems mainly finds their applications in control theory, circuits, signal processing, NMR spectroscopy, seismology, and other technical areas [1-2]. These systems are designed to study the response to an arbitrary input signal. The optimization of these systems is required for faithful operation of the system. Basically, optimization is the selection of the best design within available means. An optimization problem is mathematically defined by design variables, objective function, and constraints. The design variables represent the parameters that describe the design [3]. The objective function is the criterion that is minimized, while the constraints impose additional restrictions to the design and optimization is to find the design variable values that will minimize the objective function while satisfying the constraints. The performance characteristics of an LTI system are shown in fig. 1.

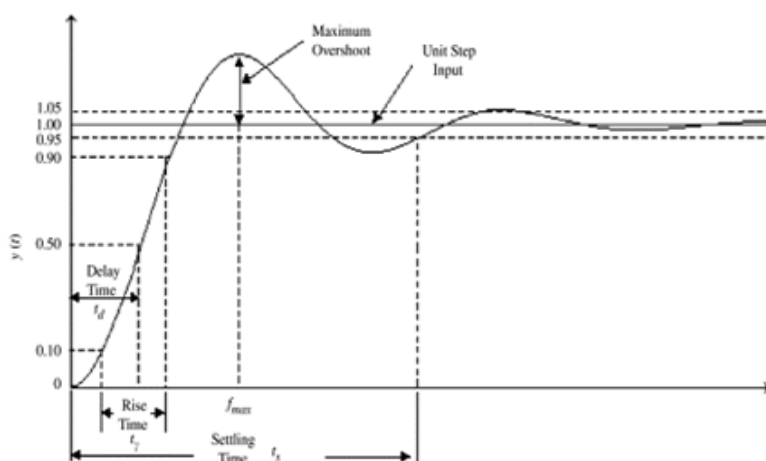


Fig.1: step response of an LTI system

In the fig.1, the desired performance characteristics of an LTI system is specified in terms of the transient response to a unit step signal. The transient response of the system depends upon the initial conditions, so it is necessary to start with necessary standard initial conditions. The various time domain specifications are delay time, peak time, settling time, rise time and percentage of over shoot etc. In this paper, a compensator using optimization technique has been designed and implemented to optimize various system parameters of an

LTI system. The LTI system has following specifications: natural frequency (ω_0) = 1; damping ratio (ζ) = 0.707; gain= 0.5; time delay = 1 sec. The optimized system response is observed in time domains.

II. SYSTEM DESIGN

The LTI system model considered for simulation is shown in fig.2.

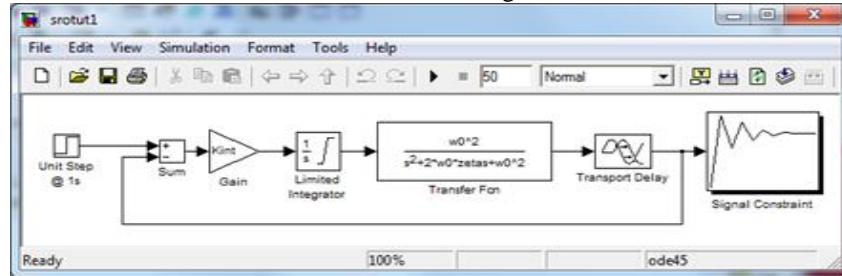


Fig.2: Linear system model

As shown in the fig.2, the linear model consists of a gain unit, a limited integrator and a delay unit. The gain unit represents the system gain which determines the stability of the system and the limited integrator operates on the signal to limit the system steady state error. Ziegler-Nichols open loop tuning algorithm has been used in tuning the compensator. In this algorithm, controller settings are based on a second-order model with a time delay that approximates the system parameters. This method uses the Chien-Hrones-Resnick (CHR) setting with 20% overshoot. The Zeigler Nichols Open-Loop Tuning Method is a way of relating the process parameters delay time, process gain and time constant to the controller parameters, controller gain and reset time. It has been developed for second-order-lag processes followed by delay [4]. The transfer function of the desired system to be optimized is given by:

$$\frac{-0.19414(s - 2)}{(s^2 + 0.409s + 0.1136)(s^2 + 3.519s + 3.418)} \quad (1)$$

The system behavior is studied when a step input signal is applied to the system.

III. OPTIMIZATION OF CONTROLLER

The compensator is designed using optimization technique so that the closed loop system meets the design specifications when the system is excited with a unit step input. The simulation is carried out using Mat Lab Single Input Single output (SISO) design tool. The controller is optimized with following parameters:

- A maximum 2-second rise time
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- maximum 30% overshoot
- A maximum 45-second settling time

The controller is designed and optimized as follows:

1. The linear model as shown in fig.2 is created and imported into a SISO Design Task.
2. SISO design Task is created with design and analysis plot.
3. Optimization based tuning is selected as the Design Method and the controller elements are selected and configured which are required to tune during the response optimization.
4. The design requirements are selected and added to the design task which the system is to be satisfied.
5. Then controller is optimized. The optimization progress results appear under response optimization. Finally, the Compensator containing the new, optimized compensator element values are obtained. The optimization of various system parameters are presented in the following section. Typically, the compensator with unity gain is taken for the optimization. The lower and upper limit of the compensator gain is $-\infty$ and $+\infty$.

A. Optimization of Rise Time

The rise time is defined as the time taken by the response of the system to increase from 10% to 90% of its final value. However, this rise time depends upon the damping ratio and natural frequency of the system [5]. For a second order system, rise time is given by:

$$t_r = \frac{\pi - \frac{\tan^{-1} \sqrt{1-\zeta^2}}{\zeta}}{\omega_n \sqrt{1-\zeta^2}} \quad (2)$$

The optimization of this specification is related to a lower limit gain on a Bode Magnitude diagram. The gain margin and phase margin of the system obtained in the bode plot are 86.7 dB and 90 degree respectively in the optimization of rise time. The response of the system with and without optimization of the compensator is shown in fig.4.(a) and 4.(b).

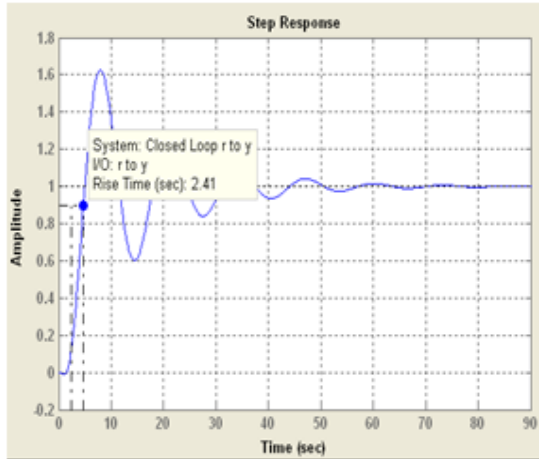


Fig.4.(a): Rise time without optimization

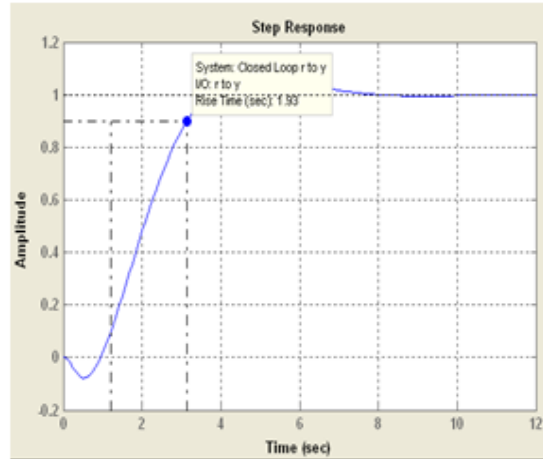


Fig.4.(b): Rise time with optimization

As shown in fig.4.(a), the rise time of the closed loop LTI system without optimizing the compensator is 2.41 second. When the compensator is optimized as shown in fig.4.(b), the rise time reduces to 1.93 second as the optimum rise time for which the system is optimized is 2 second. There is a 20 % reduction in rise time of the system.

B. Optimization of percentage of overshoot:

The transient response of a practical system often exhibits damped oscillation before reaching steady state. Maximum over shoot is defined as the maximum peak value of the response curve measured from the desired response of the system. This parameter mainly depends upon the damping ratio of the system [6] and it is given by

$$\% \text{ of overshoot} = e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \quad (3)$$

The optimization of this specification is related to the damping ratio on a root-locus diagram. The value of damping ratio chosen for the optimization of overshoot is 0.707. The percentage of overshoot with and without optimization of the controller is shown in fig.5.(a) and 5.(b).



Fig.5.(a): Peak overshoot without optimization

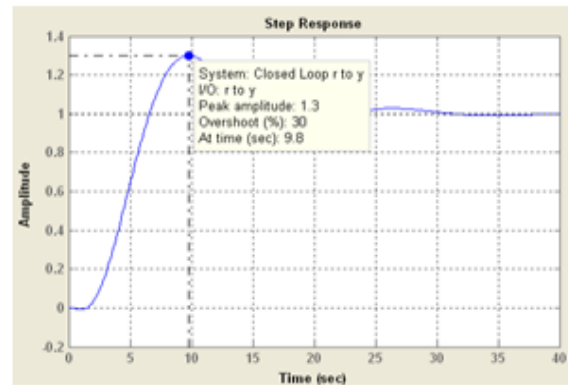


Fig.5.(b): Peak overshoot with optimization

The peak overshoot of the system without optimizing the compensator is shown in fig5.(a). It is seen that the peak overshoot of the system response is 62.6%. As shown in fig.5.(b), the peak overshoot of the system

when the compensator is optimized is obtained as 30% as desired for the system. The peak amplitude of the system response with and without optimization are 1.3 and 1.63 respectively.

C. Optimization of Settling Time

It is seen that the response of the second order system has two component such as decaying exponential component and sinusoidal component. The decaying exponential component either dampens or reduces the oscillations produced by sinusoidal component. Hence the settling time is decided by the exponential component [7]. The settling time can be found by equating exponential component to percentage of tolerance errors. The settling time is defined as the time required for the step response to settle within a certain percentage of its final value. A frequently used figure is 2% or 5% in which case the settling time is approximately. Settling time includes a very brief propagation delay, plus the time required for the output to slew to the vicinity of the final value, recover from the overload condition associated with slew, and finally settle to within the specified error.

$$T_s = - \frac{\ln(\text{tolerance fraction})}{\text{damping ratio} \times \text{natural frequency}} \quad (4)$$

Where T_s is settling time. Generally settling time within 2% i.e., 0.02 of tolerance band is considered for the simulation. This specification is represented on a root-locus diagram as a constraint on the real parts of the poles of the open loop system in the optimization. The settling time is optimized for 45 second and is added to the design requirement. The response of the system with and without optimization of the compensator is shown in fig.6.(a) and fig.6.(b).

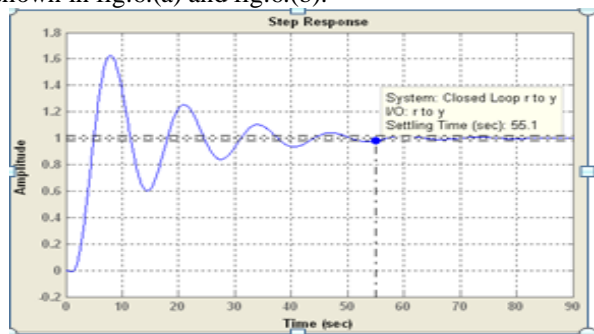


Fig.6.(a): Settling time without optimization

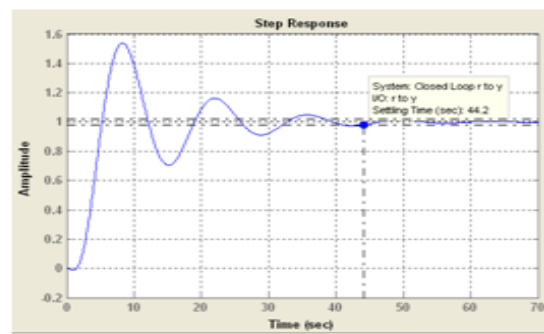


Fig.6.(b): Settling time with optimization

It is seen from fig.6.(a) that the settling time of the system without optimizing the compensator is found to be second. When the compensator is optimized for a maximum of 30%, its value is found as 30%. This shows that the optimization of the compensator meets the system specification and an optimal solution is obtained.

IV. CONCLUSIONS

The Optimization based compensator for LTI system is designed to optimize rise time, percentage of overshoot and settling time. The response of the system without and with optimization of the compensator is studied in time domain along with the specifications in the frequency domain. It is seen that Controller with Optimization exhibits better performance in terms of settling time and % of Overshoot. The optimized value of settling time and % of overshoot are 44.2 second and 30 % respectively. The simulation result shows the feasible and optimal solutions within the specified tolerances of these parameters are obtained without causing the system into the verge of instability.

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