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Unifying All Classical Force Equations

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Abstract:- It can be demonstrated that all classical force equations can be derived from one another by means of a new definition of discrete electric and magnetic fields for localized massive particles and that all of them amount to Newton's F=ma fundamental acceleration equation.

Keywords:- Gravitational force, Electrostatic force, Newton, Coulomb, Lorenz

I. FORCE OF GRAVITY INVERSELY PROPORTIONAL TO THE SQUARE OF THE DISTANCE

It has long been established that all planets moving about the Sun follow elliptical orbits having the Sun at one focus (Kepler's first law) and that a line joining any planet to the Sun sweeps equal areas in equal times (Kepler's second law). Kepler also established that the square of the period (T) of any planet about the Sun is proportional to the cube of the planet's mean distance (r) to the Sun (his third law). These laws however are only descriptive and offer no theoretical explanation as to the cause of these regularities.

It was Newton who later introduced the concept of "force" and confirmed the general soundness of his classical gravitational theory by deriving Kepler's three laws from his gravitational equations. Georges Gamow, Nobel Prize winner for his contribution to relativistic theory, clearly summarizes in his popularization work "Gravity" how Newton proceeded ([1], Chapters 2, 3 and 4). Note that we find a similar although much less complete demonstration of Kepler's third law in Halliday and Resnick "Physics" [2, p 402].

It is obvious from analyzing Kepler's first and second laws that the motion of any planet about the Sun can mathematically be simplified at the limit as if it was circular at a distance from the Sun equal to the mean radius of the elliptical orbit. This is what allowed Newton to associate the centripetal acceleration of circular motion (v^2/r) to orbital motion, where v is the velocity of an orbiting body of mass m and whose radius of the theoretical circular orbit is r.

$$F = ma = m\frac{v^2}{r} \qquad (1)$$

Newton's basic postulate was that each planet and the Sun must be attracted to each other with a force proportional to the product of the their masses and inversely proportional to the square of the distance separating them, a relation that mathematically can be represented by equation

$$F = G\frac{Mm}{r^2} \qquad (2)$$

Where M represents the mass of the Sun, m the mass of a planet and r the mean radius of this planet's orbit, G being a constant that was to be experimentally determined.

As explained by Gamow, Newton's insight was that the centripetal acceleration multiplied by the mass of a planet should be equal to the gravitational force of attraction, which implied that equations (1) and (2) were equivalent and could be equated:

$$F = ma = \frac{mv^2}{r} = \frac{GMm}{r^2} \qquad (3)$$

On the other hand, given that the length of a circular orbit is 2πr, the period (T) of one revolution will be given by

$$T = \frac{2\pi r}{v} \qquad \text{hence} \qquad v = \frac{2\pi r}{T} \qquad (4)$$

Substituting (4) in equation (3), we obtain

$$\frac{m}{r}\left(\frac{4\pi^2 r^2}{T^2}\right) = \frac{GMm}{r^2} \quad \text{and simplifying:} \quad 4\pi^2 r^3 = GMT^2 \qquad (5)$$

Which clearly establishes, as Newton demonstrated, that the cube of the mean radius (r) of an orbit is proportional to the square of the orbiting body's period (T), which is very precisely Kepler's third law. Infinitesimal calculation would also show that the same law applies to elliptical orbits.

But equation (5) allows much more than only confirming Kepler's third law. It actually allows calculating G from the now well known parameters set of the Earth orbit, which will allow us to confirm the experimentally determined value of gravitational constant G! So, isolating G in equation (5), we obtain the following equation:

$$G = \frac{4\pi^2 r^3}{M T^2} \tag{6}$$

The latest values of Earth orbit parameters as obtained from the **CRC Handbook of Chemistry and Physics** are M representing the estimated mass of the Sun (M=1.9891E30 kg), r representing the mean radius of the Earth orbit (r=1.4959787E11 m) and T representing the time for the Earth to complete one orbit, which is one year (T=3.15581E7 s). The reader can do the calculation for himself for verification. The value of the Gravitational Constant having been experimentally established by various means as G=6.673 E-11 Newton • m^2/kg^2, we obtain from calculation:

$$G = \frac{4\pi^2 r^3}{M T^2} = \frac{4\pi^2 (1.4959787E11)^3}{1.9891E30 \times (3.15581E7)^2} = 6.672024824 E-11 N \bullet m^2/kg^2 \tag{7}$$

We observe that the value mathematically calculated for G is of course very close to the experimentally obtained value, since the experimental error margin is rated at 0.003 E-11 Nm^2/kg^2.

II. ELECTROSTATIC FORCE INVERSELY PROPORTIONAL TO THE SQUARE OF THE DISTANCE

Let us now examine another well known force equation, that allows calculation of the force at the ground state orbit of the Bohr atom, that is the Coulomb equation as applied to the isolated hydrogen atom, where k is the Coulomb constant, that resolves to $1/4\pi\varepsilon_0$ where ε_0 is the electrostatic permittivity constant of vacuum

$$F = k\frac{e^2}{r^2} = \frac{e^2}{4\pi\varepsilon_0 r^2} = 8.238721806 E-8 N \tag{8}$$

Why refer here to the Bohr atom? Simply because it is commonly used in numerous textbooks to compare the electrostatic force (the Coulomb force) to the Gravitational force, and by this means, "prove" that the gravitational force is immensely weaker than the electrostatic force.

It is indeed customary in introductory physics textbooks, for example the well known "Physics" by Halliday and Resnick [(1), p 1192] and so many others, to equate this equation with Newton's basic classical mechanics force equation F=ma already mentioned as equation (1) in this paper as also being equated to the gravitational equation to prove Kepler's third law, to demonstrate that F=ma gives the very same force as the Coulomb electrostatic force equation.

Using the known mass of the electron (m=9.10938188E-31 kg), the classical radius of the electron Bohr ground state orbit (r=5.291772083E-11 m), and the classical velocity of the electron on that Bohr ground state orbit (v=2187691.253 m/s), let us replay here this very well documented calculation.

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2} = ma = m\frac{v^2}{r} = 9.10938188E-31 \frac{(2187691.253)^2}{5.291772083\ E\text{-}11} = 8.238721809 E-8 N \tag{9}$$

And we effectively observe that the force calculated is exactly the same as with the Coulomb equation.

III. QUESTIONABLE TRADITIONAL RATIO OF ELECTROSTATIC VS GRAVITATIONAL FORCES

However, quite a few textbooks [(3), p 465], routinely give the following example to demonstrate that the electrostatic force (from the Coulomb equation) is immensely more intense than the gravitational force.

From the two force equations

$$F_e = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad \text{and} \quad F_g = G\frac{Mm}{r^2} \tag{10}$$

Where M is deemed to represent the mass of the proton and m the mass of the electron, a ratio is established by dividing term for term the electrostatic force equation by the gravitational force equation that seems to reveal that the gravitational force is 39 orders of magnitude less intense than the electrostatic force:

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi \varepsilon_0 GMm} = 2.269\ E39 \tag{11}$$

Now how can these authors even consider such a proposition while at the same time asserting on the other hand in total contradiction that F=ma provides the exact same force in both cases, a fact that we just verified ourselves with the gravitational equation (3) and the electrostatic equation (9)!

Indeed, we just verified how equations (3) to (6) from Gamow's book show how Newton demonstrated that his gravitational equation allows deriving Kepler's third law when equated to F=ma, and we just verified how F=ma is used in textbooks to calculate the same force as the Coulomb force equation when applied to the classical Bohr ground state orbit in the hydrogen atom!

IV. THE MASS OF THE SUN IS EMBEDDED INTO GRAVITATIONAL CONSTANT (G)

Reexamining equation (7) will give us the key to this apparent paradox.

We observe that G, the so-called universal gravitational constant, in the traditional form allowing deriving Kepler's third law, makes use of 3 variables whose sizes, although appropriate for astronomical purposes, are way out of range when dealing with atomic scale values, which is precisely what is traditionally being done with ratio (11), which led to the defective and utterly false comparison that can be found in so many textbooks.

Indeed, we observe that using standard G to calculate the force acting on an electron inside a hydrogen atom amounts to calculating the energy of an electron orbiting the Sun at Earth orbit since the mass of the Sun, the radius of the Earth's orbit and the time for the Earth to complete one revolution about the Sun are directly embedded into G!

How could these astronomical values possibly counterbalance the mass of the proton, the radius of the Bohr rest orbit and the time for one electron revolution in the Bohr atom!

Considering equation (7) again, it seems rather simple to correct this problem by using in the definition of G values for central mass, orbit radius and time for one Bohr electron orbit that are coherent with the atomic scale to which the Coulomb equation applies.

V. EMBEDDING THE PROTON MASS INTO THE GRAVITATIONAL CONSTANT

Let us first establish the values required to calculate a G specific to the hydrogen atom. First, we have the known Bohr radius r_o = 5.291772083E-11 m. From the know frequency (6.57968391E15 Hz) of the energy (27.21138345 eV) induced at Bohr's radius by the Coulomb force, we can calculate time T taken by the electron to circle once the Bohr model rest orbit:

$$T = 1\ \text{sec} / 6.57968391\text{E}15\ \text{Hz} = 1.519829851\text{E-}16\ \text{sec}. \tag{12}$$

The effective mass of the proton being rated as M = 1.67262158E-27 kg, let us now calculate a value of G that applies to the effective mass of the proton:

$$G_p = \frac{4\pi^2 {r_o}^3}{M_p T^2} = 1.514172983\text{E}29\ \text{N} \bullet \text{m}^2/\text{kg}^2 \tag{13}$$

VI. REPAIRING THE CLASSICAL RATIO DISCONNECT

Let's now recalculate the force at the Bohr radius from the gravitational equation with this corrected value of G:

$$F_g = G_p \frac{M_p m_e}{{r_o}^2} = 8.238721759\text{E}-8\,\text{N} \tag{14}$$

So, we observe that the so-called "universal" gravitational constant G, may not be as universal as is generally believed! We just verified, contrary to the obviously faulty demonstration made in numerous textbooks, that we now obtain the very same force with the gravitational equation (14) using a logically valid value for G that can be calculated with the Coulomb equation (8). If we recalculate ratio (11) with this correctly amended value of G, we finally obtain 1 as a result, *which can only mean the electrostatic force and the gravitational force can only the same force*.

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi \varepsilon_0 G_p M_p m_e} = 1 \tag{15}$$

VII. SIMPLIFYING THE CENTRAL MASS OUT OF THE FORCE EQUATION

Now, if we resolve G to its detailed definition (13) in the gravitational force equation (14), we obtain:

$$F = \frac{4\pi^2 r_o{}^3}{M_p T^2} \frac{M_p \bullet m_e}{r_o{}^2} \tag{16}$$

We immediately notice that central mass M can be cancelled out since both occurrences of M coherently represent the same central mass of the system (here the mass of the proton). We can see also that the radius r of the orbit will also considerably simplify since the radius built into the definition of G happens to be the same as that of the gravitational force equation; in this case, the radius of the Bohr rest orbit.

$$F = \frac{4\pi^2 r_o m_e}{T^2} = 8.238721759 E - 8 N \tag{17}$$

Interestingly, it seems that to calculate the force acting between an orbiting body and a central mass with the classical gravitational equation, the central mass of the system is not even required! Only the length of the orbit ($\lambda=2\pi r$), the time taken to complete one orbit, that is, the inverse of the frequency ($1/f$), and the mass of the orbiting body are required.

Let us multiply both sides of equation (17) by **r** to obtain the energy induced at the distance separating both bodies

$$E = Fr = \frac{4\pi^2 r_o{}^2 m_e}{T^2} = \frac{(2\pi r)^2 m}{T^2} = (\lambda f)^2 m = 4.359743805 E - 18 \text{ J} \tag{18}$$

Let us now apply the same multiplication by r to the Coulomb equation (8):

$$E = Fr = \frac{e^2}{4\pi \varepsilon_0 r^2} = 4.359743805 E - 18 \text{ J} \tag{19}$$

So we observe that both equations (18) and (19) yield the correct energy induced at the Bohr rest orbit.

We thus have here the mathematical proof that gravity applies inside atoms in the very same manner as it does in the Solar System when logical and proper values are used in defining G.

A. How to obtain a first directly measured massive reference in the Solar System

As a side issue, this sheds an entirely new light on the manner in which the gravitational constant has traditionally been used since Cavendish to measure the masses of Solar System bodies. The fact that the Sun's mass simplifies out of the final force equation reveals that the most information we really obtain from the gravitational equation applied to the Solar System are mass ratios with only one directly calculated massive reference, that is, the mass of the Earth, first calculated by Cavendish in 1798. To obtain a first directly measured massive reference in the Solar System bodies, there would be required to compare at least one naturally orbiting Solar system mass with a man made orbiting body whose mass will have been measured before launch at ground level.

The way to do this would be to send the largest payload possible to orbit the smallest naturally orbiting satellite in the solar system that we can easily observe (Phobos or Déimos, or one of Jupiter's satellites, maybe), measure the wobble of the satellite against the translating mass of the payload, which will then allow really measuring the satellite mass, then the mass of its primary, and then from the very precise wobble ratios that we have for all other Solar System bodies, their respective masses, and finally, the true mass of the Sun and Earth.

Here is how this wobble is to be calculated. Although the gravitational equation gives the correct attractive force between two bodies orbiting each other, it does not provide the center of translation about which the two bodies orbit. Let's see how we can find that center of translation. A very simple rule of basic mechanics reveals that in a system of two captive bodies in translation about a common center, the product of the translation radius of one mass and of that mass will be equal to the product of the translation radius of the other mass by that other mass

$$Mx=m(r-x)$$

We can now redefine m_{ie} in terms of Mp, r_0 and x.

$$m = \frac{M_p x}{r_o - x}$$

We can now substitute the value of m in the gravitational force equation

$$F = \frac{G_p M_p M_p x}{r_o{}^2 (r_0 - x)} = \frac{G_{p2} M_p{}^2 x}{r_o{}^3 x - r_0{}^2 x}$$

Isolating x, and calculating, we obtain:

$$x = \frac{Fa = r_0^{\ 3}}{G_p M_p^{\ 2} + Fr_o^{\ 2}} = 2.880420459\,E - 14\,m$$

Which is the radius of the orbit that the proton traces about the common center of translation that it shares with the electron, and is a reflection of the wobble of the proton in the hydrogen atom due to the motion of the electron on its orbit, as hypothesized by Bohr. So, let's confirm by recalculating the mass of the electron with this equation for the center of translation of the system.

$$m = \frac{M_p x}{r_o - x} = 9.10938194E - 31\,kg$$

which is exactly the well known mass of the electron.

VIII. DERIVING FORCE EQUATION F=MA FROM THE GRAVITATIONAL FORCE EQUATION

As summarized at the beginning of this paper, it is current knowledge that Kepler's third law is derived by equating the fundamental force equation (1), that is F=ma, with the gravitational force equation (2). But since these force equations can be equated, it should also be possible to derive one from the other.

So looking for the simplest form, which is equation (1), we will demonstrate how it can be derived from the gravitational equation. This can be done however ONLY if the central mass M, radius r and time T defining G (respectively equations (7) and (13)) coherently are the same M, r and T values used in the complete gravitational equation (respectively equations (2) and (14)).

For simplicity's sake, we will use the parameters valid for the hydrogen atom from equations (13) and (14), but the same demonstration can be done with the regular astronomical values from equations (7) and (2).

We saw with equations (16) and (17) that once the variables making up the definition of G are integrated into the gravitational equation, equation (16) simplifies and reduces to:

$$F = \frac{4\pi^2 r\,m}{T^2} \qquad \text{which we can reorganize as} \qquad F = mr\left(\frac{2\pi}{T}\right)^2 \tag{20}$$

Multiplying and dividing equation (20) by mutually canceling occurrences of **r** allows the following transformation

$$F = mr\left(\frac{2\pi}{T}\right)^2 \frac{r}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 \tag{21}$$

Since the length of an orbit (2rπ) divided by the time taken to complete it (T) is the velocity of the orbiting body (v), we can then write

$$F = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{m}{r}(v)^2 = m\frac{v^2}{r} = ma \qquad \text{that is } F=ma \tag{22}$$

Which finalizes the demonstration.

IX. DERIVING FORCE EQUATION F=MA FROM THE COULOMB EQUATION

Now, after having demonstrated that the gravitational equation reduces to F=ma, what if we similarly demonstrated that the Coulomb equation also reduces to F=ma for the electron at the Bohr orbit!

By substituting in (8) a little documented but standard definition of the electrostatic permittivity constant of vacuum ($\varepsilon_0 = 1/(4\pi c^2 \cdot 10^{-7})$), the coulomb equation can be reformulated as follows

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_o^{\ 2}} = \frac{4\pi c^2 \cdot 10^{-7}}{4\pi}\frac{e^2}{r_o^{\ 2}} = \frac{e^2 \cdot 10^{-7}}{}\frac{c^2}{r_o^{\ 2}} \tag{23}$$

From a development regarding the magnetic field of a moving electron published by Paul Marmet in 2003 [(4)], a new and very useful definition of energy was derived in a previous paper [(5), equation (11)] involving only the absolute wavelength of the energy considered as the only variable:

$$E = hf = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \tag{24}$$

Of course, if we use the electron Compton wavelength (λ_C) in equation (24) and divide the electron rest mass energy (calculated from the Compton wavelength, of course) by the square of the speed of light, we obtain a corresponding new definition of the electron rest mass:

$$m_e = \frac{E}{c^2} = \frac{e^2}{2\varepsilon_0 \alpha \lambda_C c^2} \tag{25}$$

Now, if we operate in equation (25) the same substitution already carried out in equation (23) for the electrostatic permittivity constant of vacuum (ε_0), we obtain this new definition of the electron rest mass

$$m_e = \frac{e^2}{2\varepsilon_0 \alpha \lambda_C c^2} = \frac{4\pi c^2 \cdot 10^{-7} \cdot e^2}{2\alpha \lambda_C c^2} = \frac{2\pi \cdot 10^{-7} \cdot e^2}{\alpha \lambda_C} = \frac{e^2 \cdot 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \quad (26)$$

If we compare this new definition of the electron mass with equation (23), we observe that if we wanted to use definition (26) in equation (23), we would have to multiply equation (23) by mutually cancelling integrated amplitude of the electron's energy ($\lambda_C\alpha/2\pi$), so let's proceed. From equation (23):

$$F = \frac{e^2 \cdot 10^{-7}}{} \frac{c^2}{{r_o}^2} = \left(\frac{e^2 \cdot 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \frac{\lambda_C \alpha}{2\pi} \frac{c^2}{{r_o}^2} \quad (27)$$

We now observe that the expression between parentheses in equation (27) is identical to the new definition of the electron mass derived in equation (26). So let's replace this expression by the symbol of the electron mass. From (27)

$$F = \left(\frac{e^2 \cdot 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \frac{\lambda_C \alpha}{2\pi} \frac{c^2}{{r_o}^2} = m_e \frac{\lambda_C \alpha}{2\pi} \frac{c^2}{{r_o}^2} \quad (28)$$

On the other hand, we know that the theoretical velocity of the electron at the Bohr orbit is equal to the speed of light multiplied by the fine structure constant v=αc. Since the speed of light is squared in equation (28) and that α is not squared, we need to multiply and divide the equation by mutually canceling occurrences of α so that we can convert the squared speed involving the speed of light to squared theoretical velocity of the electron on the Bohr orbit (that is classical velocity v = αc = 2,187,691.252 m/s). So, let's proceed from (28)

$$F = m_e \frac{\lambda_C \alpha}{2\pi} \frac{c^2}{{r_o}^2} = m_e \frac{\lambda_C}{2\pi\alpha} \frac{\alpha^2 c^2}{{r_o}^2} = m_e \frac{\lambda_C}{2\pi\alpha} \frac{v^2}{{r_o}^2} \quad (29)$$

Finally, a little calculation on a pocket calculator will confirm that $\lambda_C/2\pi\alpha$ restitute very precisely the Bohr radius (r_0). So we can carry out the proper substitution in equation (29) and finally obtain F=ma as initially intended

$$F = m_e \frac{\lambda_C}{2\pi\alpha} \frac{v^2}{{r_o}^2} = m_e \frac{r_o}{} \frac{v^2}{{r_o}^2} = m_e \frac{v^2}{r_o} = m_e a \quad (30)$$

So, we have here the mathematical proof that gravitation does apply within atoms when using the proper values just as it does in the Solar System, which is demonstrated by successfully reducing both the gravitational equation and the coulomb equation to the same fundamental acceleration equation F=ma which is traditionally used to prove on one hand the conformity of the gravitational equation with Kepler's third law, and on the other hand, to prove that the Coulomb equation is in agreement with classical mechanics.
We have now demonstrated the complete identity of the following three classical equations

$$F = G\frac{Mm}{r^2} = k\frac{e^2}{r^2} = ma \quad (31)$$

X. RELATING THE LORENTZ FORCE EQUATION TO F=MA

But there is more! We will see that F=ma can also be derived from the Lorentz force equations. To do so, we will start from the localized electric and magnetic fields of the Bohr ground state electron carrying energy F=ev**B** and F=eα**E** . The definitions of these two fields for localized particles are derived from the Marmet paper in a previous paper on discrete fields [(5), equation (34)].

B. Deriving F=ma from F=evB

From these definitions, we can write:

$$F = ev\mathbf{B} = ev\frac{\mu_0 \pi e c}{\alpha^3 \lambda^2} \quad (32)$$

In a procedure similar to that used for equation (23), we will now replace μ_0 by its π-related definition (μ_0=4π 10^{-7})

$$F = ev\frac{\mu_0 \pi e c}{\alpha^3 \lambda^2} = \frac{e^2 10^{-7}}{} \frac{4\pi^2 v c}{\alpha^3 \lambda^2} \quad (33)$$

We previously determined in equation (26) that the rest mass of an electron can be represented by the following equation

$$m_e = \left(\frac{e^2 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \quad (34)$$

where λ_C is the electron Compton wavelength.

On the other hand, since the above force equation (33) applies to the Bohr ground state energy, the energy involved is thus 4.359743805E-18 J with an absolute wavelength of 4.556335254E-8 m. A little calculation will show that when this energy is multiplied by the square of the fine structure constant (α^2), we recuperate the electron Compton wavelength ($\lambda_C=\lambda\alpha^2$). So, let's substitute this value in force equation (33)

$$F = \frac{e^2 10^{-7}}{} \frac{4\pi^2 vc}{(\lambda\alpha^2)\alpha\lambda} = \frac{e^2 10^{-7}}{} \frac{4\pi^2 vc}{\lambda_C \alpha\lambda} = \left(\frac{e^2 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \frac{2\pi vc}{\lambda} \quad (35)$$

Comparing now the force equation (35) with the mass equation (34), we observe that the mass equation presently is a subset of the force equation. So, let's replace that subset with the rest mass symbol of the electron (m_e)

$$F = \left(\frac{e^2 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \frac{2\pi}{\lambda} \frac{vc}{} = m_e \frac{2\pi}{\lambda} \frac{vc}{} \quad (36)$$

Now, to obtain the Bohr radius from the absolute wavelength of the Bohr ground state energy one needs to multiply the amplitude of that wavelength by the fine structure constant alpha (α), and to introduce this occurrence of alpha, we need to multiply and divide the equation by mutually reducible occurrences of alpha.

Let us remember that the energy involved is 4.359743805E-18 J with an absolute wavelength of 4.556335254E-8 m. Let us recall here that the absolute wavelength of a quantum of energy is defined as the wavelength that this quantum possesses when it moves at the speed of light (λ=hc/E).

So, let's proceed.

$$F = m_e \frac{2\pi}{\lambda\alpha} \frac{v\alpha c}{} = m_e \frac{v\alpha c}{r_0} \quad (37)$$

Finally, as mentioned previously, it easy to verify that the speed of light multiplied by the fine structure constant (αc) restitutes the classical velocity of the electron on the Bohr ground orbit. So, let's operate this last substitution:

$$F = ev\mathbf{B} = m_e \frac{v\alpha c}{r_0} = m_e \frac{v^2}{r_0} = m_e a \quad (38)$$

Which is the proof that the Lorentz magnetic force equation is just another form of the same fundamental Newton acceleration equation F=ma, just like the gravitational force equation and the Coulomb force equation.

C. Deriving F=ma from F=eαE

Let's now have a look at the last remaining classical force equation

$$F = e\alpha\mathbf{E} \quad (39)$$

Which is obtained from equality F=e**E**=ec**B** and that gives the intensity of the electric field of the carrying energy of an electron ([6], **Chapter 25**), having the classical velocity associated with the electron on the Bohr ground state classical orbit (since $v_{Bohr} = \alpha c$). So multiplying both members of the equation by fine structure constant α reduces the force associated to the electric field to the intensity applying at the Bohr ground orbit radius. We determined in a previous paper fields [(5), equation (40)] that the definition of the electron magnetic field derived from the Marmet paper allows redefining the electric field as

$$\mathbf{E} = \frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} \quad (40)$$

So, substituting for **E** in equation (39) we obtain

$$F = e\alpha\mathbf{E} = \frac{e\alpha}{} \frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} \quad (41)$$

Reducing ε_0 to its internal components ($1/4\pi c^2 10^{-7}$), we obtain

$$F = \frac{e\alpha}{} \frac{\pi e}{\varepsilon_0 \alpha^3 \lambda^2} = \frac{e^2 10^{-7}}{} \frac{4\pi^2 \alpha c^2}{\alpha^3 \lambda^2} \quad (42)$$

Since we already established that $\lambda_C=\lambda\alpha^2$, we can substitute as follows

$$F = \frac{e^2 10^{-7}}{\alpha^3 \lambda^2} \frac{4\pi^2 \alpha c^2}{} = \frac{e^2 10^{-7}}{\lambda_C \alpha \lambda} 4\pi^2 \alpha c^2 \quad (43)$$

But we also established with equation (26) that

$$m_e = \left(\frac{e^2 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \quad (44)$$

so we can operate the following substitution

$$F = \frac{e^2 10^{-7}}{} \frac{4\pi^2 \alpha c^2}{\lambda_C \alpha \lambda} = \left(\frac{e^2 10^{-7}}{} \frac{2\pi}{\lambda_C \alpha} \right) \frac{2\pi \alpha c^2}{\lambda} = m_e \frac{2\pi \alpha c^2}{\lambda} \quad (45)$$

We also established that the Bohr radius (r_0) is equal to the amplitude of the absolute wavelength of the Bohr ground state energy (4.359743805E-18 J with absolute wavelength 4.556335254E-8m calculated with λ=hc/E) multiplied by alpha (α), and to allow this reduction, we see that we need to multiply and divide the equation by mutually reducible occurrences of α. So, let's proceed

$$F = m_e \frac{2\pi \alpha c^2}{\lambda} = m_e \frac{2\pi}{\lambda \alpha} \frac{\alpha^2 c^2}{} = m_e \frac{\alpha^2 c^2}{a_0} \quad (46)$$

Finally, we know that multiplying the speed of light by α restitutes the classical Bohr ground state velocity, so we finally obtain

$$F = m_e \frac{\alpha^2 c^2}{r_0} = m_e \frac{v^2}{r_0} = m_e a \quad (47)$$

This completes the demonstration.

Let us take good note here that all of the classical force equations ultimately resolve to a single equation involving a mass being accelerated.

XI. CONCLUSION

So we have now demonstrated the complete identity of the five following equations since the first four, respectively dealing with the gravitational force, the electrostatic force, the Lorentz magnetic field force and the Lorentz electric field force can be converted to the last one, which is the classical force equation dealing with free fall acceleration F=ma.

$$F = G_p \frac{M_p \bullet m_e}{r_o^2} = k \frac{e^2}{r_o^2} = ev\mathbf{B} = e\alpha\mathbf{E} = m_e a = 8.238721759\,E-8\,N \quad (48)$$

And we observe that all these classical force equations ultimately resolve to dealing with a mass being accelerated and that consequently, ***only one force is at play for all of these equations***.

Let us note that the identity between all of these equations is made possible only by means of the electromagnetic transverse acceleration equation defined in ([6], **Section 9.2**).

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