International Journal of Engineering Research and Development
e-ISSN: 2278-067X, **p**-ISSN: 2278-800X, www.ijerd.com
Volume 6, Issue 4 (March 2013), PP. 01-10

From Classical to Relativistic Mechanics via Maxwell

André Michaud
SRP Inc Service de Recherche Pédagogique Québec Canada

Abstract:- It can be shown that Newton's non relativistic equations can be upgraded to full relativistic status by means of integrating the magnetic aspect of massive particles derived from de Broglie's hypothesis on the internal structure of localized photons and from Paul Marmet's remarkable exploration of the relation between the magnetic aspect of electrons and the contribution of this magnetic aspect to the electron rest mass and relativistic mass, which he termed the "magnetic mass". The outcome is a complete relativistic equations set, one of which is the first to allow velocity calculation of all existing scatterable particles, from photons at the speed of light to velocity and relativistic mass of all massive particles from total rest to infinitesimally close to asymptotic velocity of light and related infinite mass.

Keywords:- Relativistic equations, classical equations, Marmet, Maxwell, magnetic mass

I. CONTRIBUTION OF THE MAGNETIC ASPECT OF AN ELECTRON TO ITS MASS

Causalist physicist Paul Marmet made a remarkable discovery regarding the relation between the magnetic aspect of electrons and the contribution of this magnetic aspect to the electron rest and relativistic masses. In a paper that he published in 2003 ([2]), Paul Marmet obtained the following definition of current by quantizing the charge in Biot-Savart's equation and doing away with the time element as he replaced dt by dx/v, since the velocity of current is constant at any given instant:

$$I = \frac{dQ}{dt} = \frac{d(Ne)}{dt} = \frac{d(Ne)v}{dx} \tag{1}$$

Where "e" represents the unit charge of the electron and N represents the number of electrons in one Ampere.

Note that although starting with Section 8 of his article ([2]), Marmet exposes a personal hypothesis obviously subject to discussion, the first part, from Section 1 to Section 7, is a flawless mathematical demonstration whose implications are an enormous progress to further advance the understanding of the electromagnetic structure of elementary particles. The reader should also be aware that due to some transcription error, in view of the fact that only one charge is being considered and that at any instantaneous velocity being considered, the **B** field has the exact intensity related to that velocity, which Marmet clearly explains by the way; his equation (7) should read:

$$\mathbf{B}_i = \frac{\mu_0 e^- v}{4\pi r^2}$$

By substituting the resulting value of I in the scalar version of the Biot-Savart equation,

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi r^2}\sin(\theta)\,dx\,, \quad \text{he obtained} \quad d\mathbf{B} = \frac{\mu_0 v}{4\pi r^2}\sin(\theta)\, d(Ne) \tag{2}$$

Without going into the detail of his derivation, which is very clearly laid out in his paper ([2], Equations (1) to (26)), let us only mention that the final stage of this development consists in spherically integrating the electron magnetic energy, whose density is mathematically deemed to vary radially from a minimum limit corresponding to r_e to a maximum limit located at infinity.

$$M = \left\{\frac{\mu_0 e^2 v^2}{2(4\pi)^2 c^2 r^4}\right\} 2\pi \int_0^{\pi} \sin(\theta)\, d\theta \int_{r_e}^{\infty} r^2 dr \tag{3}$$

The electron classical radius r_e is the mandatory lower limit in such an integration to infinity, due to the simple fact that integrating any closer to r = 0 would accumulate more energy than experimental data warrants. This constraint seems to be the only reason for the existence of this "classical radius" of the electron in fundamental physics. After integrating, we finally obtain:

$$M = \frac{\mu_0 e^2 v^2}{8\pi r_e c^2} = \frac{m_e}{2}\frac{v^2}{c^2} \tag{4}$$

Which very precisely corresponds to the total mass of the magnetic field of an electron moving at velocity v, from which can be concluded that the invariant magnetic field of the electron at rest corresponds to a mass of?

$$M = \frac{\mu_0 e^2}{8\pi r_e} \tag{5}$$

Which is exactly half the mass of an electron, the other half of which being made up of what could be termed its "electric" mass ([1]).

He discovered by the same token that **any instantaneous "magnetic mass" increase of an electron is a direct function of the square of its instantaneous velocity**, even though its charge remains unchanged.

When this velocity is small with respect to the speed of light, the following classical equation is obtained, allowing to clearly determine the contribution of the magnetic component to the rest mass of the electron, a contribution that corresponds in the present model, to a discrete LC oscillation of that energy between magnetostatic space and normal space, as clarified in a separate paper (**[1], Section 8**):

$$\frac{\mu_0 e^2}{8\pi r_e}\frac{v^2}{c^2} = \frac{m_e}{2}\frac{v^2}{c^2} \tag{6}$$

Where "r_e" is the classical electron radius (2.817940285E-15 m), and "e" represents the charge of the electron (1.602176462E-19 C).

As strange as this may seem, Marmet's demonstration seems to imply that only the magnetic half of an electron's mass is involved in accelerating, and that the other half, corresponding in this model to the constantly unidirectional energy localized within electrostatic space, would then have no role to play during the acceleration of an electron! But of course, things are not so simple, and as we know, energy can be represented in a number of ways.

Guided however by Marmet's clean conclusion and the LC relation explored for both photon and electron in separate papers ([1]) and ([3]), we will explore how a moving electron's energy can be represented as a ratio of the constantly unidirectional kinetic energy sustaining its motion (within normal space) and the constantly unidirectional kinetic energy located in electrostatic space over a representation of the magnetic energy making up the corresponding velocity dependant instantaneous magnetic mass of the particle.

We will thus define a ratio of the unidirectional energies (fixed electric energy of the rest mass plus added kinetic energy) over the magnetic energies (fixed magnetic energy of the rest mass plus added magnetic energy).

II. NEWTON’S NON-RELATIVISTIC KINETIC EQUATION

We will start our derivation from Newton's elementary kinetic equation

$$E_K = \frac{1}{2}mv^2 \text{ or, in context : } E_K = \frac{m}{2}\frac{v^2}{} = m_m v^2 \tag{7}$$

Where m/2 would be equal to m_m (Marmet's "magnetic mass"). Note that we could as well have proceeded from equating $2E_K$ to mv^2, but to keep the focus on Marmet's magnetic mass approach, we will rather divide the mass by two here.

III. THE MAGNETIC COMPONENT OF AN ELECTRON’S MASS

On the other hand, we established in a previous paper (**[1], Section VIII, equation (26)**) the discrete LC oscillation equation for the electron at rest from that for the photon, which was clarified in a separate paper (**[3]**), that very clearly identifies the magnetic component of the electron's mass:

$$E = m_e c^2 = \left[\frac{hc}{2\lambda_C}\right]_Y + \left(2\left[\frac{(e')^2}{4C_C}\right]_X \cos^2(\omega t) + \left[\frac{L_C i_C{}^2}{2}\right]_Z \sin^2(\omega t)\right) \tag{8}$$

As equation (8) was being established (see paper **[1]**), it became clear that the unidirectional energy present in electrostatic space (Y-space) amounts to half the total energy making up the total rest mass of the electron, which leaves the amount of energy oscillating between magnetostatic space (Z-space) and normal space (X-space) to make up the other half of the electron rest mass.

So let's reduce equation (8) to an inertial instantaneous form involving the energy present in electrostatic space and that at maximum in magnetostatic space (thus at zero in normal space):

$$E = m_e c^2 = \left[\frac{hc}{2\lambda_C}\right]_Y + \left[\frac{L_C\, i_C{}^2}{2}\right]_Z \tag{9}$$

Where subscript ($_C$) refers of course to the electron Compton wavelength.

Although the magnetic field of the electron will be treated here as if it was mathematically static at maximum to make more obvious the relation between the electron and its carrier-photon, the reader must keep in mind that its discrete LC oscillation between magnetostatic and normal spaces nevertheless remains permanently active at the frequency of the electron energy (**[1]**).

The same will of course be true of the magnetic field of the electron carrier-photon and that permanently LC oscillates between magnetostatic and electrostatic spaces at its own frequency (**[3]**). This carrier-photon is actually the added energy that we will introduce a little further on (equation (14)) that propels the electron at the related velocity.

The consequence of the interaction due to this difference in frequencies between the electron energy and its carrier-photon energy (named "Zitterbewegung") is explored in (**[6], Sections 25.11.1** and **25.11.2**).

IV. THE ELECTRON REST MAGNETIC MASS

Since mass can be calculated by dividing the energy making up this mass by the square of the speed of light:

$$m_e = \frac{E}{c^2} \qquad \text{(from } E=mc^2\text{)} \tag{10}$$

we can of course also calculate the rest magnetic mass of the electron by dividing the magnetic energy obtained in equation (9) by the square of the speed of light:

$$m_m = \frac{E}{2c^2} = \frac{L_C\ i_C{}^2}{2\ c^2} \tag{11}$$

Since Newton's kinetic energy equation is being used as a starting point, the mass that must be considered initially is of course the electron rest mass, whose conversion to equivalent energy is made with $E=m_0c^2$. We will discover as we go, how to render this mass relativistic by means of the LC equation that we just defined for the electron rest mass, to finally end up with relativistic form $E=\gamma m_0c^2$.

V. CLASSICAL ELECTRON KINETIC ENERGY AS A RATIO

Substituting now in Newton's kinetic equation (7) this form of the magnetic mass, we obtain

$$E_K = m_m v^2 = \frac{L_C\ i_C{}^2}{2\ c^2} v^2 \tag{12}$$

Isolating the velocities ratio, we obtain the following form:

$$E_K = \frac{L_C\ i_C{}^2}{2} \frac{v^2}{c^2} \tag{13}$$

Which is a form identical to that defined by Marmet and that we posed as equation (6).

VI. RATIO OF KINETIC ENERGY OVER MAGNETIC MASS ENERGY

We know also that the unidirectional kinetic energy calculated with Newton's equation is not part of the energy making up the rest mass of the electron. Consequently, it is extra kinetic energy. But, in this model, the only form of kinetic energy contributing to maintain a velocity in normal space has been defined in a recent paper as the discrete LC equation for the photon (**[3], Equation (16)**).

$$E = \left(\frac{hc}{2\lambda}\right)_X + \left[2\left(\frac{e^2}{4C}\right)_Y \cos^2(\omega t) + \left(\frac{L\ i^2}{2}\right)_Z \sin^2(\omega t)\right] \tag{14}$$

an equation that we will reduce to its inertial version just like we did with the discrete LC equation for the electron at rest.

$$E = \frac{hc}{2\lambda} + \frac{L\ i^2}{2} \tag{15}$$

In equation (13), let us substitute for E_K the expression for kinetic energy corresponding to the unidirectional kinetic energy calculated ($hc/2\lambda$) for a photon

$$\frac{hc}{2\lambda} = \frac{L_C\ i_C{}^2}{2} \frac{v^2}{c^2} \tag{16}$$

Let us now give equation (16) the form of a ratio of unidirectional kinetic energy over the electron magnetic energy.

$$\frac{hc/2\lambda}{(L_C\ i_C{}^2/2)} = \frac{v^2}{c^2} \tag{17}$$

VII. RECTIFYING THE UNBALANCED VELOCITY EQUATION

We immediately notice that the equation comes out as an un-squared energy ratio in opposition to a squared ratio of the velocity of the particle over the speed of light, which appears mathematically untenable, but that could not possibly have come to Newton's attention since the knowledge that mass is equivalent to a particle's rest energy divided by the square of the speed of light was unknown in his time. But from the knowledge accumulated since Newton, we will now explore how this relation can be rectified to become mathematically correct.

Before we proceed however, let us confirm the identity of this mathematically unbalanced equation with the initial Newtonian kinematics energy equation. We will use for that purpose an energy very well known in fundamental physics, which is the mean energy induced at the classical rest orbit of the Bohr atom. So, by means of the know parameters of the Bohr atom, let us first verify if we still obtain the classical velocity of the electron with equation (17).

Let us first calculate the various variables of the equation.

Product hc is of course the product of two fundamental constants, that is, the speed of light (c=299792.458 m/s) and Planck's constant (h=6.62606876E-34 J·s)

$$hc= 1.98644544\text{ E-25 J}\cdot\text{m} \tag{18}$$

We will of course use here the energy induced at the Bohr radius, which is 4.359743805 E-18 Joules, so the wavelength to be used turns out to be:

$$\lambda_B=hc/E=4.556335256\text{ E-8 m} \tag{19}$$

See paper (**[3], Section 6.4, equation (12)**) for calculation of the inductance (L) of an energy. We will of course use in so doing the electron Compton wavelength, which is λ_C = 2.426310215 E-12 m, the fine structure constant (α = 7.297352533E-3) and the magnetic permeability constant of vacuum (μ_0 = 1.256637061E-6):

$$L_C = \frac{\mu_0 \lambda_C}{8\pi^2\,\alpha} = 5.29177208\;\;4\,\text{E-18 Henry} \tag{20}$$

See the same paper, **Equation (14)** for calculation of the current associated to that inductance for the electron, where "e" is the unit charge of the electron (1.602176462E-19 Coulomb)

$$i_C = \frac{2\pi e c}{\lambda_C} = 123.3840209\;\;\text{Amperes} \tag{21}$$

Isolating the velocity in equation (17) we obtain

$$v = \frac{c}{i_C}\sqrt{\frac{H}{\lambda_B L_C}} = 2187691.252\text{ m/s} \tag{22}$$

Which is very precisely the classical velocity of the electron on the Bohr orbit, that we have now calculated with an electromagnetic version of Newton's classical kinetic energy equation. Considering again equation (17) let us analyze the implications.

We observe that despite having used the absolute wavelength (λ_B) of an energy of 4.359743805 E-18 Joules, which is the energy induced at the Bohr orbit, this energy allows obtaining the exact classical velocity of the electron with equation (22) stemming from equation (17), despite the fact that the classical kinetic equation (7), and also the converted kinetic equation derived from electromagnetism (16) use only half that energy to calculate the classical velocity:

$$E = \frac{hc}{2\lambda_B} = \frac{mv^2}{2} = 2.179871902\text{ E}-18\text{ Joules} \tag{23}$$

Before proceeding further let us observe that in equation (17) repeated here for convenience, the unidirectional kinetic energy ($hc/2\lambda$) is clearly separated from the magnetic energy ($L_C i_C^2/2$) in the ratio:

$$\frac{hc/2\lambda}{(L_C\;i_C{}^2/2)} = \frac{v^2}{c^2} \tag{17}$$

We observe immediately that in this equation, the magnetic energy ($L_C i_C^2/2$) of the electron remains constant by definition since it is opposed the constant squared velocity of light while the "carrying energy" apparently can vary in relation to the square of the velocity. But, we know from equation (14) for the energy of a photon that as its kinetic energy ($hc/2\lambda$) varies, its magnetic energy ($Li^2/2$) will vary in equal proportion!

It must be realized that in Newton's time, experimentally verifiable velocities were so low with respect to the minimal velocities that would have revealed the slightest increase in relativistic mass, that it was impossible for Newton to even suspect such a possibility. Moreover, electric charges and electrostatic induction were still totally unknown.

Now let us hypothesize that the calculated unidirectional kinetic energy ($hc/2\lambda$) in Newton's kinetic equation would be part of some sort of "carrier-photon" that would be associated to the electron and that the measured velocity would be due to the fact that this carrier-photon could not move faster on account of the handicap of having to metaphorically "carry the electron on its back" on top of its own electromagnetic component, assuming that such a carrier-photon would display a discrete LC oscillating electromagnetic component just like a "normal" photon, and that it could thus also be described by the same LC equation as the photon

$$E = \frac{hc}{2\lambda} + \left[2\left(\frac{e^2}{4C_\lambda}\right)\cos^2(\omega t) + \frac{L_\lambda\;i_\lambda{}^2}{2}\sin^2(\omega t)\right] \tag{24}$$

and its inertial form

$$E_\lambda = \frac{hc}{2\lambda} + \frac{L_\lambda\;i_\lambda{}^2}{2} \tag{25}$$

We can thus isolate the "magnetic" component of this carrier-photon as we did for the electron (Equation (11)), complementary to the unidirectional kinetic energy corresponding to the velocity calculated with equation (22), and make the hypothesis that if we were to add this "magnetic energy" postulated for the carrier-photon to that of the electron in equation (17), we could possibly become more conform to Marmet's conclusion. So let's add this assumed missing half of the carrier-photon's energy, that is ($L_\lambda i_\lambda^2/2$), to our equation

$$\frac{hc/2\lambda}{(L_C\;i_C{}^2/2)+(L_\lambda\;i_\lambda{}^2/2)} = \frac{v^2}{c^2} \tag{26}$$

We can now observe that the magnetic mass of the electron will henceforth increase with the velocity, although we still have an un-squared energy ratio opposing a squared velocity ratio.

We now have the complete energy of the carrier-photon included in our equation. Now, considering again equation (9) with respect to equation (26), we observe that the "internal" unidirectional kinetic energy (electrostatic) of the electron rest mass present in equation (9), that is ($hc/2\lambda_C$), is not represented in equation (26) but certainly needs to be included since it makes up half the rest mass of the electron. So let us include it in our equation in a manner that will not change the current relation, that is, by adding it to, and subtracting it from, the unidirectional energy of the carrier photon:

$$\frac{hc/2\lambda + hc/2\lambda_C - hc/2\lambda_C}{(L_C\ i_C{}^2/2) + (L_\lambda\ i_\lambda{}^2/2)} = \frac{v^2}{c^2} \tag{27}$$

This self-cancelling insertion may seem at first glance totally useless, but let us consider that the squared velocity ratio on the other side of the equal sign reveals that a quadratic relation has to be involved on the energy side, and this indicates that the apparently self-canceling half of the electron energy statically captive in electrostatic space must play a role in determining the actual velocity due to its inertia.

Now, experimental evidence first brought to light by Kaufmann ([7]) shows that the complete mass of an electron is involved in transverse interaction, so we will double the value of the energy of the representation of its magnetic component ($L_C i_C{}^2/2$) to take that fact into account energy wise, and act similarly on its kinetic component ($hc/2\lambda_C$) to maintain equilibrium.

$$\frac{hc/2\lambda + hc/\lambda_C - hc/\lambda_C}{(L_C\ i_C{}^2) + (L_\lambda\ i_\lambda{}^2/2)} = \frac{v^2}{c^2} \tag{28}$$

Finally, Marmet's final mathematically demonstrated conclusion (his equation 23) was that "***the magnetic energy around individual electrons increases as the square of the electron velocity, just as the increase in relativistic mass***". In clear, this means that the increase in magnetic mass must also be squared. So, as a final touch, let us square the kinetic to magnetic energy ratio to finally come into harmony with the corresponding already squared velocities ratio.

$$\frac{\left(hc/2\lambda + hc/\lambda_C\right)^2 - \left(hc/\lambda_C\right)^2}{\left((L_C\ i_C{}^2) + (L_\lambda\ i_\lambda{}^2/2)\right)^2} = \frac{v^2}{c^2} \tag{29}$$

VIII. GENERAL RELATIVISTIC VELOCITIES EQUATION FROM CARRYING ENERGY

Resolving the kinetic energy quadratic and simplifying the kinetic energy representation will now give equation

$$\frac{(hc)^2\left(4\lambda + \lambda_C\right)}{\lambda_C \lambda^2\left((2L_C\ i_C{}^2) + (L_\lambda i_\lambda{}^2)\right)^2} = \frac{v^2}{c^2} \tag{30}$$

A few test runs with any value of λ will show that **this equation traces a relativistic velocities curve identical to that of the famous Special Relativity equation**. Let us verify this conclusion for the well known energy of the Bohr rest orbit to clearly establish the procedure. First, we need the values of the L and i variables for the magnetic inductance of the carrier-photon energy whose wavelength (λ_B= 4.556335256 E-8 m) we determined at equation (19).

$$L_\lambda = \frac{\mu_0 \lambda_\lambda}{8\pi^2 \alpha} = 9.93734748\ \ 4E-14\ \ \text{Henry} \tag{31}$$

and

$$i_\lambda = \frac{2\pi\, ec}{\lambda_\lambda \alpha} = 0.90767404\ \ 9\ \ \text{Ampere} \tag{32}$$

Having already calculated the inductance values for the electron magnetic energy (L_C and i_C) at equations (20) and (21) from the Compton wavelength (λ_C=2.426310215 E-12 m), we are now ready to proceed.
Isolating the velocity in equation (30), we now obtain

$$v = hc^2\sqrt{\frac{4\lambda + \lambda_C}{\lambda_C \lambda^2 (2L_C i_C{}^2 + L_\lambda i_\lambda{}^2)^2}} = 2{,}187{,}647561\ \text{m/s} \tag{33}$$

which is the exact relativistic velocity associated to the Bohr rest orbit energy.

Equation (33) is rather complex however. But it can be hugely simplified if we replace the inductance variables with their definitions (Equations 20, 21, 31 and 32) and give it the generic form required for graphing the relativistic velocities curve for the electron, we finally obtain:

$$f(x) = c\,\frac{\sqrt{4ax + x^2}}{2a + x} \tag{33a}$$

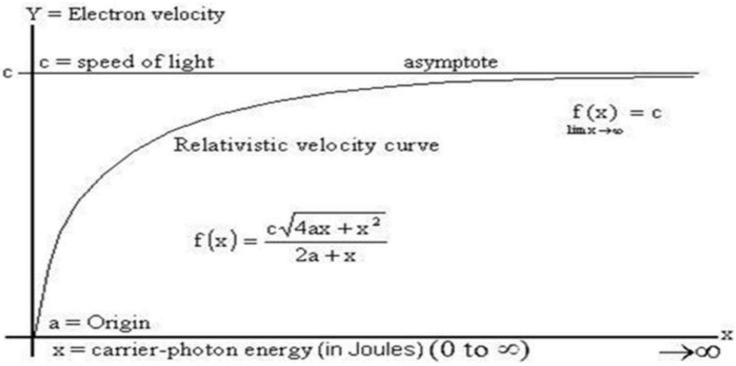

Fig.1: Relativistic velocity curve from varying carrying energy

From observation, we can see that the relativistic velocity curve is in fact the positive segment of a hyperbolic curve (continuous with no vertical asymptote and increasing without limit) and consequently answers all the criteria of an increasing exponential function. So the interested reader could attempt the exercise of converting equation (33a) to the exponential form.
In relation with equation (33a), that can also be written:

$$v = c\frac{\sqrt{4EK + K^2}}{2E + K} \tag{33b}$$

where E is the rest mass energy of the particle being considered ($E=m_0c^2$) and K is the kinetic energy that must be added to allow relativistic velocity v, there also is need to calculate the corresponding relativistic mass. This can of course be achieved by using the traditional Special Relativity gamma factor:

$$m_r = \frac{m_0 c}{\sqrt{c^2 - v^2}} \tag{34}$$

But this method requires that the particle's velocity be known in advance. Equation (33b) is particularly important considering that it allows calculating the ***electron g factor*** from first principles (since equation (33b) is drawn from the LC equation for the electron which is itself in complete agreement with Maxwell's equations) to replace the current arbitrary ad hoc value (See separate paper (**[5]**).

IX. RELATIVISTIC MASS FROM CARRYING ENERGY

This model also allows determining a particle's velocity directly from the kinetic energy that we wish to add to the rest mass:

$$m_r = m_0 + \frac{K}{2c^2} \tag{35}$$

Verification will show that both (34) and (35) provide exactly the same relativistic mass, so that

$$m_r = \frac{m_0 c}{\sqrt{c^2 - v^2}} = m_0 + \frac{K}{2c^2} \tag{36}$$

So from (36) we can now directly calculate the associated kinetic energy if we know the relativistic velocity of a particle

$$K = 2m_0c^2\left(\frac{c}{\sqrt{c^2 - v^2}} - 1\right) \quad \text{that is} \quad K = 2m_0c^2(\gamma - 1) \tag{37}$$

Alternately, from equation (35), kinetic energy K can be obtained from any known relativistic mass with

$$K = 2c^2(m_r - m_0) \tag{38}$$

So we have at our disposal four new equations, (33b), (35), (37) and (38) that allow separately calculating the three variables that determine all possible states of free motion for massive particles.

They could consequently logically be seen as belonging to a relativistic version of Newtonian mechanics, which in this model amounts to a subset of the electromagnetic mechanics of particles that we are establishing from Maxwell's theory as amended from deBroglie's hypothesis.

Note that the standard SR equation for relativistic kinetic energy is $K = m_0c^2(\gamma - 1)$.

However, this latter equation provides only the unidirectional kinetic energy required to cause the related relativistic mass to move at velocity v, but not the added kinetic energy from the carrier-photon that actually makes up the related added relativistic mass.

Shouldn't it be normal for a relativistic kinetic energy equation to provide all of the kinetic energy that must be added for a particle at rest (m_0) to move at velocity v, including both the energy going into the extra relativistic mass increase plus the unidirectional energy sustaining velocity v of the increased mass? This issue is the reason why equation (37) doubles the Kinetic energy provided by the standard relativistic kinetic energy equation, so that all of the extra kinetic energy required for the rest mass to reach relativistic velocity v is represented.

X. EXPONENTIAL MASS INCREASE WITH INCREASE IN CARRYING ENERGY

One last point that deserves to be emphasized is the clear exponential relation that it suggests, since the rate of mass increase with respect to the unidirectional half of the carrying energy is proportional to the mass present at that energy level, which can be expressed by the following differential equation:

$$\frac{dm}{d(E/2)} = km \tag{39}$$

where k is the constant of proportionality. So:

$$\frac{dm}{m} = k\, d(E/2), \quad \int \frac{dm}{m} = k \int d(E/2) \quad \text{and} \quad \ln m = k\, E/2 + C \tag{40}$$

And since the mass of the electron is m_0 when the carrying energy is equal to zero, we can thus write that C= ln m_0. So, substituting this value in equation (40), we obtain

$$\ln m - \ln m_0 = k\, E/2 \quad \text{that is} \quad \ln\left(\frac{m}{m_0}\right) = k\, E/2 \tag{41}$$

Which means that:

$$\frac{m}{m_0} = \exp\left[kE/2\right] \text{ and finally } m = m_0 \exp\left[kE/2\right] \tag{42}$$

Without entering full description, a quick calculation establishes the proportionality constant at about k=1.1125E13. The interested reader could calculate this value with more precision.

Let us note also that once the kinetic energy is determined for any relativistic velocity of a massive particle, we can easily obtain its absolute wavelength from the traditional equation λ=hc/K, which then allows directly calculating the local electric and magnetic fields of the moving particle, as explained in a separate paper ([4]).

XI. RELATIVISTIC EQUATION VALID FOR PHOTONS AND MASSIVE PARTICLES

But let's now go back to equation (33a), and see what happens when we reduce the energy provided by the carrier-photon to zero (by setting x to zero):

$$f(x) = c\frac{\sqrt{4ax + x^2}}{2a + x} = c\frac{\sqrt{0 + 0^2}}{2a + 0} = c\frac{0}{2a} = 0 \tag{43}$$

We observe that velocity ($f_{(x)}$ = v) will fall to zero, which is exactly what happens when no energy is provided to an electron in excess of its rest energy. Considering again equation (33a) and this time, let's see what happens when we reduce the rest energy of the electron to zero (setting **a** to zero):

$$f(x) = c\frac{\sqrt{4ax + x^2}}{2a + x} = c\frac{\sqrt{0 + x^2}}{0 + x} = c\frac{\sqrt{x^2}}{x} = c\frac{x}{x} = c \tag{44}$$

In this case, we observe that we are left with only the energy of the carrier-photon, that is the unidirectional half of the photon's energy (hc/2λ) over the magnetic half ($L_\lambda i_\lambda^2/2$), and that the formula reduces to

$$\frac{x}{x} = \frac{\left(hc/2\lambda\right)}{\left((L_\lambda\ i_\lambda^{\ 2}/2)\right)} = \frac{v}{c} \tag{45}$$

Verification with any value of λ will show that velocity (v) will now systematically be equal to (c), the speed of light

$$v = c\frac{\left(hc/2\lambda\right)}{\left((L_\lambda\ i_\lambda^{\ 2}/2)\right)} = 299{,}792{,}45\ \ 8\, m/s \tag{46}$$

thus proving that ***the energy in excess of the rest energy of an electron definitely is a normal photon***. If we now give again to equation (45) the general form of Newton's kinetic equation (modeled after equations (12) and (13)):

$$\frac{hc}{2\lambda} = \frac{L\ i^2}{2}\frac{c^2}{c^2} = \frac{L\ i^2}{2c^2}c^2 \quad \text{that is} \quad \boxed{E = m_m c^2} \tag{47}$$

we thus prove that ***the energy of a photon moving at the speed of light can truly be represented as a magnetic mass (actually a discretely LC oscillating quantum of energy) corresponding to half its energy, that would be propelled at the speed of light by the other half of its energy***, in conformity with Newton's kinetic equation and with the discrete LC electromagnetic structure imposed on the photon by this model.

Haven't we just linked up Newton's mechanics with Maxwell's electromagnetic theory in a rather convincing manner? We now have a very special equation at our disposal (33a) that reduces to 2 more very special forms, that is (43) and (45) that together cover the whole spectrum of all existing scatterable electromagnetic particles' velocities.

Representation (43) of the equation shows an electron at rest, while representation (33a) represents an electron moving at any possible relativistic velocity, while finally representation (45) represents a photon of any energy always moving at c.

XII. GENERAL RELATIVISTIC VELOCITIES EQUATION FROM WAVELENGTH

The reader may have noted that when we resolved the kinetic energy quadratic and simplified equation (29) to obtain equation (30), that this quadratic resolved to only two wavelength besides the transverse acceleration constant (the product hc, sometimes symbolized H in 3-spaces model dependant papers). Let's now convert the magnetic representations to the same form, resolve the second quadratic and simplify. From (30), we have

$$\frac{(hc)^2(4\lambda+\lambda_C)}{4\lambda_C\lambda^2\left(hc/\lambda_C+hc/2\lambda\right)^2}=\frac{v^2}{c^2}\quad \text{that is}\quad \frac{(hc)^2(4\lambda+\lambda_C)}{4\lambda_C\lambda^2(hc)^2\left(\frac{4\lambda^2+4\lambda\lambda_C+\lambda_C{}^2}{4\lambda^2\lambda_C{}^2}\right)}=\frac{v^2}{c^2} \tag{48}$$

and simplifying, we finally get a very interesting relativistic velocities equation that requires only the wavelengths of the carrying energy and that of the electron:

$$\frac{4\lambda\lambda_C+\lambda_C{}^2}{(2\lambda+\lambda_C)^2}=\frac{v^2}{c^2} \tag{49}$$

If we now give equation (49) the generic form required to trace the relativistic velocities curve for the electron, we obtain

$$f(x)=c\,\frac{\sqrt{4ax+a^2}}{2x+a} \tag{49a}$$

Let us compare equation (49a) which is function of the carrier-photon wavelength to equation (33a) which is function of the carrier-photon energy. We observe the identity of structure of both equations even though they are function of inversely related variables, an identity that let both equations calculate exactly the same relativistic velocities curve for the electron.

In addition to equation (33b), equation (49) can also serve to calculate the **electron g factor**. (See paper (**[5]**). But contrary to equation (33b), equation (49) can also be derived from the Special Relativity Theory.

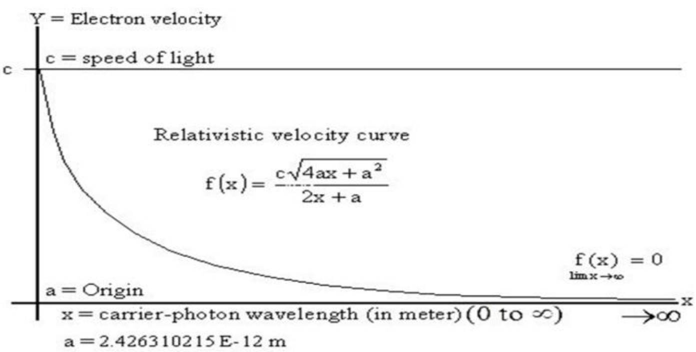

Fig.2: Relativistic velocity curve from carrying energy wavelength

A comparison of both graphs visually confirms the inverse relation, and we then obtain another hyperbolic curve, but this time reflecting a decreasing exponential function.

XIII. DERIVING THE SR RELATIVISTIC MASS EQUATION AND GAMMA FACTOR

We will now derive the famous Special Relativity relativistic $E=\gamma m_0 c^2$ equation from equation (49). But we must first isolate the 4 variables in this equation:

$$v=c\sqrt{\frac{4\lambda\lambda_C+\lambda_C{}^2}{(2\lambda+\lambda_C)^2}}=c\sqrt{\frac{\left(4\lambda^2+4\lambda\lambda_C+\lambda_C{}^2\right)-4\lambda^2}{(2\lambda+\lambda_C)^2}}=c\sqrt{\frac{(2\lambda+\lambda_C)^2-4\lambda^2}{(2\lambda+\lambda_C)^2}} \tag{50}$$

$$v=c\sqrt{1-\frac{4\lambda^2}{(2\lambda+\lambda_C)^2}}=c\sqrt{1-\left(\frac{2\lambda}{2\lambda+\lambda_C}\right)^2}=c\sqrt{1-\frac{1}{\left(\frac{2\lambda+\lambda_C}{2\lambda}\right)^2}} \tag{51}$$

and finally $$v = c\sqrt{1-\frac{1}{\left(1+\frac{\lambda_C}{2\lambda}\right)^2}} \tag{52}$$

Now, from the definition of energy derived from the work of Marmet [(2)], we can pose:

$$E = hf = \frac{e^2}{2\varepsilon_0 \alpha\lambda} \tag{53}$$

Which means that the energy in excess of the rest mass of the particle in motion can be represented by:

$$E = \frac{e^2}{2\varepsilon_0 \alpha\lambda} = \frac{e^2}{2\varepsilon_0 \alpha}\frac{1}{\lambda} \tag{54}$$

And that the energy contained in the rest mass of an electron can be represented by

$$m_0 c^2 = \frac{e^2}{2\varepsilon_0 \alpha\lambda_C} = \frac{e^2}{2\varepsilon_0 \alpha}\frac{1}{\lambda_C} \tag{55}$$

We can easily observe that all terms of both equations are constants, except for the wavelengths. What is of interest to us here is that the sets of constants in both equations (54) and (55) are identical. This means that we can multiply and divide the wavelengths terms of equation (52) by mutually reducible occurrences of that constants set without changing the value of the equation. So, let's proceed from equation (52):

$$v = c\sqrt{1-\frac{1}{\left(1+\frac{\lambda_C}{2\lambda}\right)^2}} = c\sqrt{1-\frac{1}{\left(1+\frac{2\varepsilon_0\alpha\lambda_C}{e^2}\frac{e^2}{4\varepsilon_0\alpha\lambda}\right)^2}} \tag{56}$$

Substituting now the equivalent left members of equations (54) and (55) in equation (56), we obtain:

$$v = c\sqrt{1-\frac{1}{\left(1+\frac{1}{m_0c^2}\frac{E}{2}\right)^2}} \tag{57}$$

We have seen previously that only half of the energy in excess of rest mass for a particle in motion contributes to the relativistic increase in mass of that particle, so let's reformulate equation (57) according to this fact

$$v = c\sqrt{1-\frac{1}{\left(1+\frac{E/2}{m_0c^2}\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{m_0c^2+E/2}{m_0c^2}\right)^2}} \tag{58}$$

The final step of simplification now reveals that velocity of the particle can be calculated from an exponential ratio of rest mass energy over relativistic mass energy:

$$v = c\sqrt{1-\left(\frac{m_0c^2}{m_0c^2+E/2}\right)^2} = c\sqrt{1-\left(\frac{m_0c^2}{mc^2}\right)^2} \tag{59}$$

But we know that mc^2 corresponds to the total energy of the current instantaneous relativistic mass of the particle, which means that $mc^2 = E$. Substituting in equation (59), we obtain

$$v = c\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2} \tag{60}$$

Squaring and rearranging equation (60)

$$\frac{v^2}{c^2} = 1-\left(\frac{m_0c^2}{E}\right)^2 \text{ and } \left(\frac{m_0c^2}{E}\right)^2 = 1-\frac{v^2}{c^2} \tag{61}$$

Extracting the square root, we finally obtain

$$\frac{m_0 c^2}{E} = \sqrt{1 - \frac{v^2}{c^2}} \quad \text{and} \quad \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = E \tag{62}$$

Which resolves to $\gamma\, m_0 c^2 = E$, since $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ (63)

And we finally obtain the well known Special Relativity equation:

$$E = \gamma\, m_0 c^2 \tag{64}$$

An article published separately ([4]) already describes how to retro-derive equation (49) from the traditional gamma factor from Special Relativity, which means that we can henceforth seamlessly link up SR with Maxwell by means of the discrete LC equations set defined in two separate papers ([1]) and ([3]).

XIV. CONCLUSION

As demonstrated, the 3-Spaces model reveals four new equations, (equations (33b), (35), (37) and (38) in **Section IX**) that allow separately calculating the three variables that determine all possible states of free motion for massive particles.

Furthermore, **Section XI** reveals a very special equation (33a) that reduces to 2 more very special forms, that is (43) and (45) that together cover the whole spectrum of all existing electromagnetic particles' velocities. Representation (43) of the equation shows an electron at rest, while representation (33a) represents an electron moving at any possible relativistic velocity, while finally representation (45) represents a photon of any energy always moving at c.

REFERENCES

[1]. André Michaud. The Birth of Electron-Positron Pairs in the 3-Spaces Model, The General Science Journal 2010: http://www.gsjournal.net/Science-Journals/Essays/View/2270

[2]. Paul Marmet. Fundamental Nature of Relativistic Mass and Magnetic Fields, International IFNA-ANS Journal, No. 3 (19), Vol. 9, 2003, Kazan University, Kazan, Russia. (Also available over the Internet: http://www.newtonphysics.on.ca/magnetic/mass.html).

[3]. André Michaud. On the Fundamental Discrete LC Relation of Elementary EM Particles, The General Science Journal 2010: http://www.gsjournal.net/Science-Journals/Essays/View/2269.

[4]. André Michaud, Field Equations for Localized Individual Photons and Relativistic Field Equations for Localized Moving Massive Particles, International IFNA-ANS Journal, No. 2 (28), Vol. 13, 2007, p. 123-140, Kazan State University, Kazan, Russia. (Also available online at http://www.gsjournal.net/Science-Journals/Essays/View/2257).

[5]. André Michaud. On the Electron Magnetic Moment Anomaly, The General Science Journal 2009: http://www.gsjournal.net/Science-Journals/Essays/View/2259.

[6]. André Michaud. Expanded Maxwellian Geometry of Space. 4th Edition 2005, SRP Books. https://www.smashwords.com/books/view/163704.

[7]. Walter Kaufmann. Uber die "Elektromagnetische Masse" der Elektronen, Kgl. Gesellschaft der Wissenschaften Nachrichten, Mathem.-Phys. Klasse, 1903, S. 91-103.