

Design, Modeling and Analysis of Involute Spur Gears by Finite Element Method

D.SasiKanth¹, Tippa Bhimasankara Rao²

¹PG Student, Department of Mechanical Engineering, Nimra Institute of Science and Technology

² HOD, Department of Mechanical Engineering, Nimra Institute of Science and Technology, Vijayawada, AP, INDIA

Abstract:- Gearing is one of the most critical components in mechanical power transmission systems. In the transmission of motion or power between the shafts, with definite velocity ratio and for a positive drive, gears are employed. Generally for medium power transmission spur gears with involute profile is used. In transmitting the power, the gears are subjected to number of stresses and the failure of the gear is mainly caused due to bending and pitting. The former can be avoided by providing high strength material i.e. material having high static strength but the pitting failure can be avoided only by proper surface hardening of the gear teeth. Most of the gear failures occur mainly due to contact failures. Pitting occurs at the pitch point on the surface of the tooth. The stresses developed are called as Contact stresses. The major objective of this project is to find the region of the surface distortion due to wear at vicinity of the pitch line. In this project both gear and pinion are modeled using Pro E (wildfire 2.0) as it offers versatile design schema and robust compatibility with analysis packages. After modeling, the pair is exported to ANSYS. The contact stresses were examined using 2-D FEM models and ANSYS on the spur gear tooth with full depth involute and it is found how the surface failure takes place on the pitch surface. The results of the two dimensional FEM analyses from ANSYS are presented.

INTRODUCTION

Gearing is one of the most critical components in a mechanical power transmission system, and in most industrial rotating machinery. It is possible that gears will predominate as the most effective means of transmitting power in future machines due to their high degree of reliability and compactness. AGMA (American Gear Manufacturers Association) is a representative of gear manufacturers, whose standards in gear design and testing are quite popular. The teeth of spur gears are cut in the straight which are parallel to axis. A gearbox as usually used in the transmission system is also called a speed reducer, gear head, gear reducer etc., which consists of a set of gears, shafts and bearings that are factory mounted in an enclosed lubricated housing. Speed reducers are available in a broad range of sizes, capacities and speed ratios. Their job is to convert the input provided by a prime mover (usually an electric motor) into an output with lower speed and correspondingly higher torque.

Gears analyses in the past were performed using analytical methods, which required a number of assumptions and simplifications. As computers have become more and more powerful, people have tended to use numerical approaches to develop theoretical models to predict the effect of whatever is studied. This has improved gear analyses and computer simulations. Numerical methods can potentially provide more accurate solutions since they normally require much less restrictive assumptions. The model and the solution methods, however, must be chosen carefully to ensure that the results are accurate and that the computational time is reasonable. In this project, first, the finite element models and solution methods needed for the accurate calculation of spur gear contact stresses were determined. Then, the contact stresses calculated using ANSYS 8.0.

I. PROBLEM STATEMENT

The project concerned with designing the spur gears i.e., both gear and pinion in PRO E and we transfer that assembly into ANSYS and perform contact analysis at pitch point of meshing of the gears.

II. ABOUT SPUR GEAR

A spur gear is one of the most important ways of transmitting a positive motion between two shafts lying parallel to each other, have involute teeth that are parallel to the shaft and can have internal or external teeth. They cause no external thrust between gears. They are inexpensive to manufacture. They give lower but satisfactory performance. They are used when shaft rotates in the same plane. They are among the most cost-

effective type of gearing. They are also used to create large gear reductions. The most commonly used gears are the straight spur gears. In the straight spur gears, the whole width of the teeth comes in contact at the same time.

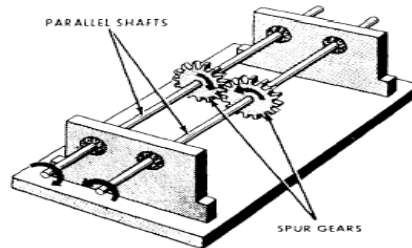


Fig.1 Spur gears coupling two parallel shafts

When two spur gears of different sizes *mesh* together, the larger gear is called a *wheel*, and the smaller gear is called a *pinion*. In a simple gear train of two spur gears, the *input motion* and force are applied to the *driver gear*. The *output motion* and force are transmitted by the *driven gear*. The driver gear *rotates* the driven gear without slipping.

III. INVOLUTE GEAR

In gear drives, the attainment of a non fluctuating velocity ratio is dependent on the profile of the teeth. *Friction* and wear between two gears is also dependent on the tooth profile. There are a great many tooth profiles that will give a constant velocity ratio, and in many cases, given an arbitrary tooth shape, it is possible to develop a tooth profile for the mating gear that will give a constant velocity ratio. However, two constant velocity tooth profiles have been by far the most commonly used in modern times. They are the *cycloid* and the *involute*. The cycloid was more common until the late 1800s; since then the involute has largely superseded it, particularly in drive train applications. The cycloid is in some ways the more interesting and flexible shape; however the involute has two advantages: it is easier to manufacture, and it permits the center to center spacing of the gears to vary over some range without ruining the constancy of the velocity ratio. Cycloidal gears only work properly if the center spacing is exactly right. Cycloidal gears are still used in mechanical clocks. The **involute gear** profile is the most commonly used system for gearing today. In an involute gear, the profiles of the teeth are *involutés of a circle*. (The involute of a circle is the spiraling curve traced by the end of an imaginary taut string unwinding itself from that stationary circle.) In involute gear design, *all* contact between two gears occurs in the *same* fixed, flat plane (the Plane of Action), even as their teeth mesh in and out. Further, the contacting surfaces are always perpendicular to the plane of contact, so the dominant contact forces (in a well lubricated system) are always parallel to the plane. This way, the moment arms are kept constant. This is key to minimizing the torque/speed variations which produce vibration and noise in lower quality gears. Note that the involute profile does not prevent the teeth from scraping each other every time they mesh, and this is the dominant source of wear. It is not possible to design a gear tooth profile which rolls through the mesh without friction. Service life is often managed by using hard materials and constant lubrication. When friction wear is a critical issue, chain drives can help reduce maintenance requirements. The disadvantages of the non-involute gearing are lower transverse contact ratio and great sensitivity to the center distance accuracy.

IV. GEAR MATERIALS

- Gears are manufactured from a wide variety of materials, both metallic as well as non-metallic. While selecting a gear material its cost, strength, wear resistance, noise level etc. has to be compromised. The tensile strength of various gear materials is given in Table
- Cast Irons are one of the most commonly used gear materials. Its low cost, ease of casting, good machinability, and high wear resistance and good noise abatement property makes it a logical choice. The primary disadvantage of cast iron as a gear material is its low tensile strength.
- Steels. Steel gears are usually made of plain carbon steels or alloy steels. They have the advantage, over cast iron, of higher strength without undue increase in cost.
- Non-ferrous metals. Copper, zinc, aluminium and titanium are materials used to obtain alloys that are useful gear materials. The copper alloys, known as bronzes, are the most widely used. They are preferred where corrosion resistance is important and also where large sliding velocities exist.
- Non-metallic. Raw hide, nylon, various types of plastics, and so on, have been used for gears. The advantages obtained by using these materials are quiet operation, internal lubrication, dampening of shock and vibration, and manufacturing economy. Their primary disadvantages are lower load carrying

capacity and low heat conductivity, which results in heat distortion of the teeth. Recently thermoplastic resins, with glass-fiber reinforcement and a lubricant as additives, have been used as gear materials.

V. SPUR GEAR IN MESHING

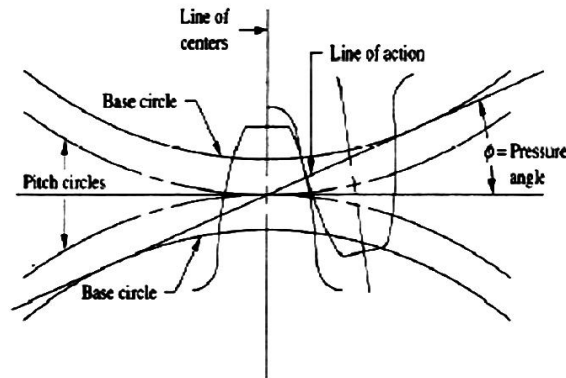


Fig.2 Mating of Spur gear

VI. GOVERNING EQUATIONS

There are two theoretical formulas, which deal with these two fatigue failure mechanisms.

- One is the Hertzian equation, which can be used to calculate the contact stresses.
- The other is the Lewis formula, which can be used to calculate the bending stresses.

The Hertzian equation will be used to investigate surface pitting and scoring by obtaining the magnitude of the contact stresses.

VII. FORCES ACTING ON SPUR GEAR

Consider a pair of spur gears having their-teeth in contact at the pitch point P, as shown in Fig.1.4 The force that the tooth of one gear exerts on the meshing tooth of the other gear is normal to the tooth surface, and therefore acts along the pressure line. The normal force F_n is the force exerted by the follower gear tooth on the driver gear tooth. Obviously, the driver tooth would exert a force equal to F_n in magnitude on the follower, but acting in the opposite direction. Hence the forces acting are

- Normal force, F_n
- Tangential force, F_t
- Radial force, F_r

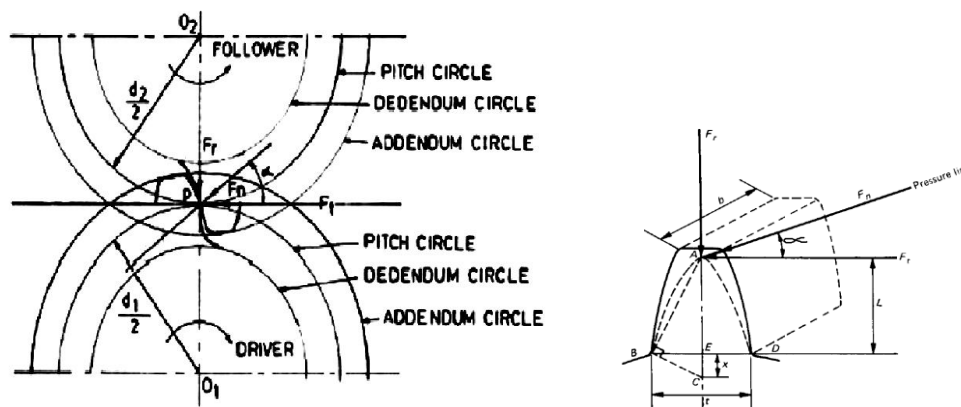


Figure.3 Forces on a Spur gear tooth

The normal force F can be resolved into tangential force F_t which transmits the power and radial component F_r which does no work but tends to push the gears apart. From the figure

$$F_t = F_n \cos \alpha$$

$$F_r = F_n \sin \alpha = F_t \tan \alpha$$

α = pressure angle.

VIII. CALCULATIONS

- **Pressure angle**

The most common pressure angles are 14.5°, 20° and 25°, with 20° pressure angles being used most often. The 20° pressure angle has a higher load bearing ability than 14.5° and it allows using fewer pinion teeth and tooth is wider at base. Therefore, the 20 pressure angle is used for design purpose.

$$\alpha = 20^\circ$$

- **Determining the module, m**

Power to be transmitted, $p = 20\text{kw}$

Speed of pinion, $N_p = 1200\text{ rpm}$

Speed of gear, $N_g = 600\text{ rpm}$

Speed ratio $i = N_p/N_g = 1200/600 = 2$

Number of teeth on pinion for minimum interference, $Z_p = 14$

Number of teeth on gear = $Z_g = 2 * 14 = 28$

The diameters of gear and pinion are 84 and 42 respectively

Module, $m = \text{diameter} / \text{no. of teeth} = 3$

The materials of gear and pinion are plain carbon steel respectively with $E = 0.2\text{ mpa}$

Lewis form factor, form factor $Y = 0.154 - 0.912 / Z$

By substituting the values Z_g and Z_p in the above equation we form factor values for gear and pinion are 0.12 and 0.09 respectively.

Since both gear and pinion are made of same material pinion is the weaker one. So design is based on **pinion**.

Beam strength, $F_t = \sigma_b b Y P_c$

Where

σ_b = bending stress,

P_c = circular pitch = $\pi m = 9.5\text{ mm}$

b = face width of tooth = $4 P_c = 38\text{ mm}$

Y = Lewis form factor = 0.09

Velocity, $V_p = \pi D_p N_p / 60 = 2.7\text{ m/sec}$

C_v = velocity factor = $3 / (3 + v) = 0.53$

Power, $P = 20\text{ KW}$

Torque, $T = 60P / (2\pi N_p) = 160\text{ kN-mm}$

Therefore, $F_t = 2T / D = 7.620\text{ KN}$

- **For contact analysis**

From data book the contact stress may be written as:

Contact stress $\sigma_c = 0.418 (P^1 * E (R_1^{-1} + R_2^{-1}))^{0.5}$

Where $P^1 = P / \text{width}$

$P = 1\text{ mpa}$,

Width = 20 mm,

$R_1 = 42\text{ mm}$,

$R_2 = 21\text{ mm}$,

$E = 0.2\text{ mpa}$.

$$\sigma_c = 0.418 (20^{-1} * 200000 (42^{-1} + 21^{-1}))^{0.5} = 11.18\text{ kpa}$$

This is approximately equal to results obtained from ANSYS.

- **For static analysis**

Load applied on the test gear at the contact point by the master gear is 20444 N and the bending stress can be determined analytically.

The bending stress can be calculated by the Lewis formula analytically.

Bending stress, $\sigma_b = F_t / b P_c Y = 235\text{ mpa}$.

The maximum bending stress induced in the test gear is 235 mpa.

• **For dynamic (modal) analysis**

Number of modes	20
Type of extraction scheme	Subspace
Mass density (kg/mm ³)	7.86 e -6

Table.1 Dybanmic analysis

Geometric model, material properties and the boundary conditions are same as that of Static analysis.

IX. SPECIFICATIONS OF GEAR AND PINION

PROPERTY	GEAR	PINION
Pitch circle diameter	84 mm	42 mm
Diameter of addendum circle	90 mm	48 mm
Diameter of dedendum circle	76.5 mm	34.5 mm
Teeth	28	14
Module	3 mm	3 mm
Width	20 mm	20 mm
Center distance	63 mm	63 mm
Poisson's ratio	0.3	0.3
Material	Steel	Steel
Young's Modulus (mpa)	0.2	0.2
Density of Steel (kg/mm ³)	7.86 e -6	7.86 e -6
Yield strength (mpa)	6000	6000

Table.2 Specifications of gear and pinion

X. MODELLING

Modelling of gear and pinion using pro engineer with the specifications obtained above we model the gear in pro e by using many relations and parameters in pro e. when we give the parametric equation of involute it automatically draws the curve. the parametric equation of the involute profile is given as follows

- step 1:** setting parameters **step 2:** part relations sort relations – ok
- step 3:** create datum axis
- step 4:** create datum curve (adendum circle)
- step 5:** create datum curve (dedendum circle)
- step 6:** create datum curve (pitch circle)
- step7:** name circles
- step 9:** rename datum curve
- step 10:** create datum point pnt0 (pitch point)
- step 12:** create datum plane (dtm2)
- step 13:** mirror involute profile
- step 14:** create datum curve
- step 15:** create base cylinder
- step 16:** create teeth profile
- step 17:** making pattern

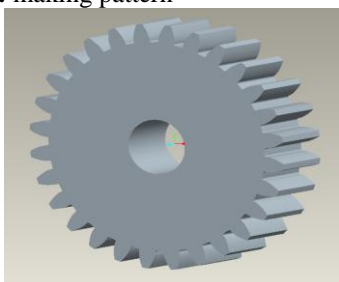


Fig.4 gear of 28 teeth

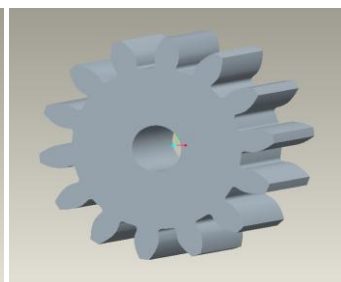


Fig.5 Gear of 14 teeth

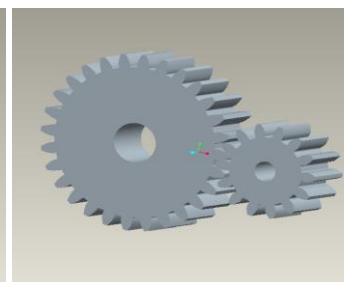


Fig.6 Gears are in mating

XI. ANALYSIS

The basic steps for performing a typical surface-to-surface contact analysis are listed below

- Create the model geometry and mesh
- Identify the contact pairs
- Designate contact and target surfaces
- Define the target surface
- Define the contact surface
- Set the element KEYOPTS and real constants
- Define/control the motion of the target surface (rigid-to-flexible only)
- Apply necessary boundary conditions
- Define solution options and load steps
- Solve the contact problem
- Review the results

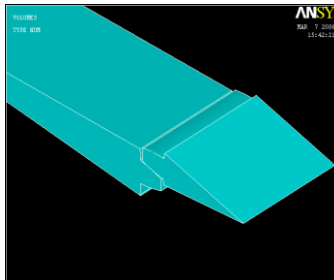


Fig.7 Gear assembly in ansys

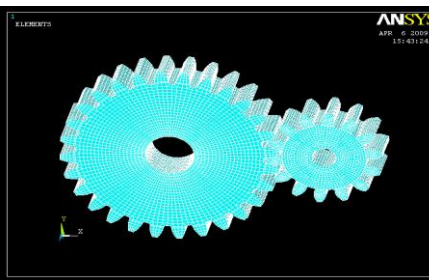


Fig.8 Meshing of gear assembly

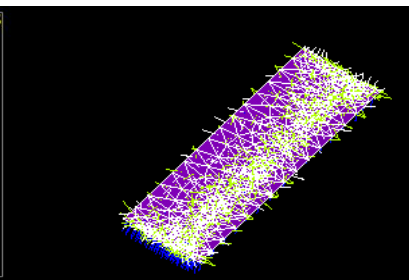


Fig.9 contact pair

XII. RESULTS & DISCUSSION

Contact analysis

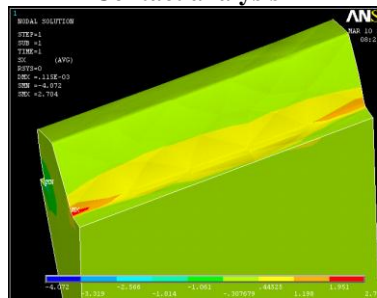


Fig.10 Direct stress in x-direction

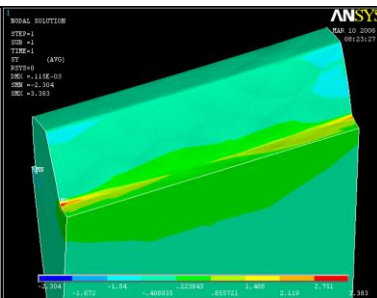


Fig.11 Direct stress in y-direction

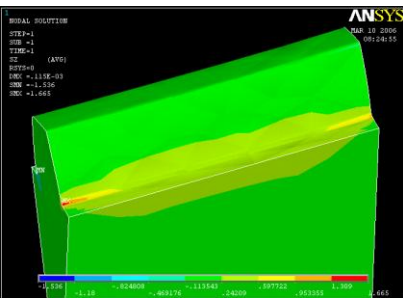


Fig.12 Direct stress in z-direction

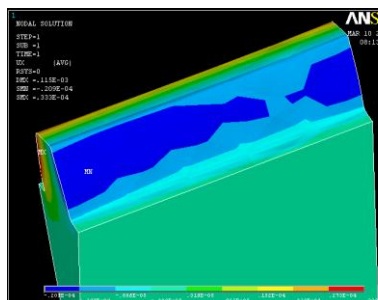


Fig.13 Distribution of vonmises stress

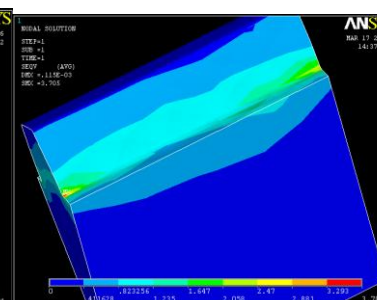


Fig.14 Displacement along x-direction

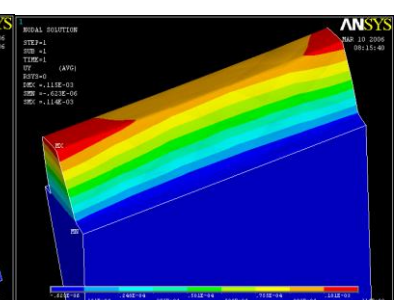


Fig.15 Displacement along y-direction

XIII. STATIC ANALYSIS

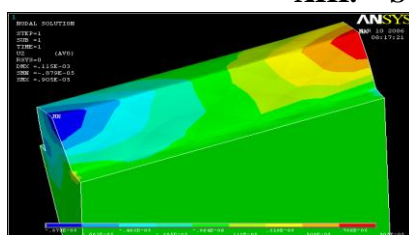


Fig.16 Displacement along z-direction

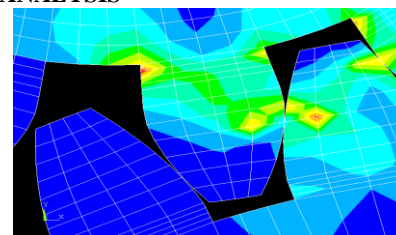


Fig.17 s_{xx} (bending) stress contour

XIV. DYNAMIC ANALYSIS

The output from Modal analysis consists of

- Natural frequencies are determined corresponding to each mode shape. First 20 modes are presented in the Table 6.3.
- Modes of vibration of gears are predominantly torsional in nature about the axis of rotation.

Mode shape	Frequency (rad/sec)	Frequency (cycles/sec)	Mode shape	Frequency (rad/sec)	Frequency (cycles/sec)
1	1.753007 E+03	2.789997 E+02	11	1.702287 E+04	2.09274 E+03
2	4.322812 E+03	6.879969 E+02	12	1.907776 E+04	3.036319 E+03
3	4.599040 E+03	7.319600 E+02	13	1.913540 E+04	3.045493 E+03
4	5.474730 E+03	8.713303 E+02	14	1.971885 E+04	3.138352 E+03
5	6.557745 E+03	1.043697 E+03	15	2.008289 E+04	3.196292 E+03
6	6.729461 E+03	1.071027 E+03	16	2.070703 E+04	3.295626 E+03
7	1.192885 E+04	1.898535 E+03	17	2.128512 E+04	3.387633 E+03
8	1.330001 E+04	2.116763 E+03	18	2.138203 E+04	3.403056 E+03
9	1.370918 E+04	2.181884 E+03	19	2.150865 E+04	3.423208 E+03
10	1.683074 E+04	2.678696 E+03	20	2.230238 E+04	3.549534 E+03

Table.3 Frequencies corresponding to mode shape

XV. CONCLUSION

- **Result's summary & conclusion**

The contribution of the project work presented here can be summarized as follows:

It was shown that an FEA model could be used to simulate contact between two bodies accurately by verification of contact stresses between two cylinders in contact and comparison with the Hertzian equations. Effective methods to estimate the tooth contact stress using a 2D contact stress model and to estimate the root bending stresses using 2D FEA model are proposed. The analysis of gear contact stress and the investigation of 2D solid bending stresses are detailed. The development of a new numerical method for FEA modeling of the whole gear body which can rotate in mesh including the contact problem is presented. On performing result analysis it is found that σ_x (Direct stress in x-direction) is maximum around root fillet towards the end of gear rather than around pitch line as analysis is performed for the gear with perfect surface without any pits. As working of gear progresses, it develops pits around pitch line and consequently maximum σ_x may occur in the vicinity of it. One more interesting observation is that σ_x varies distinctively about pitch line as if it is junction between two stress regions. Because of this maximum distortion shall present around pitch line making it as very important region as far as contact failure is concerned. On observing Fig 10, it is found that σ_y (Direct stress in y-direction) is also maximum around root fillet perhaps due to smoother flank and face. After some functioning the region of maximum stress could move towards pitch line. The distribution of σ_z also presents same picture of previous normal stresses. It is also observed that out of three normal stresses, y is maximum why because the component of contact load is more along y- direction conforming analytical formulae and theories. On performing analysis on the distribution of Vonmises stress (Fig 6.4) it can be concluded that pitch line acts as a boundary between two distinct stress regions which makes it distortion prone. More over on pitting, the region of maximum stress may move towards pitch line making it more vulnerable to contact failure. It is observed that the stresses obtained from ANSYS are near by the results determined using contact stress formulae. If material and geometric non linearities are considered the results given by ANSYS could be very much near to the results given by contact stress formulae. Bending failure can be avoided by selecting suitable material and it is taken care of by Lewis equation but when it is considered wear failure there is no separate procedure to find it except Hertz formulae. We had found out these stresses using ANSYS. Finally while designing the gears wear analysis should also be taken as important factor.

REFERENCES

- [1]. www.cs.cmu.edu/People/rapidproto/mechanisms/references.html
- [2]. Mechanical Engg design – Joseph E. Shigley.
- [3]. Design data book – P.S.G. College of technology.
- [4]. Design data book – K.Mahadevan & K.Balaveera Reddy.
- [5]. Analysis and approximation of contact problem
- [6]. By Mircea Sofonia and Weimin Hann
- [7]. Elastic contact analysis by boundary elements
- [8]. By Susumu Takahashi
- [9]. “A comprehensive analysis of fillet and contact stresses in straight spur gears” By M.A. Alfares
- [10]. International Journal of Computer Applications in Technology 2001 - Vol. 14, No.1/2/3 pp. 109-116