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Replacement Problem for a Deteriorating Cold Standby System Using Two Monotone Processes

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Abstract:- This paper studies a cold standby repairable system consisting of two identical components namely component 1, component 2 and one repairman is studied. Assume that each component after repair is not 'as good as new' and also the successive working times form a decreasing α -series process, the successive repair time's form an increasing geometric process and both the processes are exposing to Weibull failure law. Under these assumptions we study an optimal replacement policy N in which we replace the system when the number of failures of component 1 reaches N. We determine an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. We derive an explicit expression of the long-run average cost and the corresponding optimal replacement policy N* can be determined analytically. Numerical results are provided to support the theoretical results.

Keywords:- Convolution, Geometric process, Monotone Processes-α -series process, Repair replacement policy, Renewal Process.

I. INTRODUCTION

For maintenance of simple repairable systems many replacement models were developed under the assumption that the system after repair is "as good as new". This leads to a perfect repair model. But it is not always true for deteriorating systems due to ageing and accumulated wear. In this direction Barlow and Hunter [1] developed a minimal repair model in which the minimal repair does not change the age of the system.

Brown and Proschan [3] studied an imperfect repair model under which the repair will be perfect repair with probability'p' and with probability '(1-p)' as a minimal repair. Much research work have been carried out by Block et al [2] and others have also worked in this direction. It is reasonable to assume that the successive working times of the deteriorating systems after repair will become shorter and shorter, while the consecutive repair time of the system will become longer and longer. Finally it can't work any longer, neither can it be repaired. To model such a deteriorating repairable system Lam [5,6] proposed a geometric process repair model in which he studied two kinds of replacement policies, one based on the working age T of the system and the other based on the failure number N of the system. He provided an explicit expression for the long run average cost per unit time under these two kinds of policies and also proved optimal policy N* is better than the optimal policy T*. Stadje and Zuckerman [9] presented a general monotone process to generalize Lam's work. Latter much research work has been carried out by using geometric process to generalize Lam's work and corresponding optimal replacement policies were developed by Wang and Zhang[11,12]. Zhang at al [15] determined an optimal replacement policy for a deteriorating production system with preventive maintenance by generalizing Lam's[5,6] work. Many optimal replacement policies were also developed for cold standby repairable systems using geometric processes.

Zhang [10] considered a cold standby repairable system consisting of two identical components and one repairman. He developed two kinds of repair replacement policies, one based on the working age T of component 1 under which the system is replaced when working age of component 1 reaches T and the other based on failure number N of component 1 under which the system is replaced when the failure number of component 1 reaches N. He derived an explicit expression for long-run average cost per unit time of the system under these two kinds of policies.

However the geometric process is more useful model for deteriorating system, Braun et al [2] introduced an alternative model, the α -series process ,which contributes these characteristics. Further more Braun et al [4] explained the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The α -series process grows either as a polynomial in time or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the α -series process does. Braun et al [2] also presented that both the increasing geometric

process and the α -series process have a finite first moment under certain general conditions. However the decreasing geometric process usually has an infinite first moment under certain conditions. Thus the decreasing α -series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system. Based on this understanding the present paper studies a cold standby repairable system consisting of two identical components namely component 1, component 2 and one repairman is studied. Assume that each component after repair is not 'as good as new' and also the successive working times form a decreasing α -series process, the successive repair time's form an increasing geometric process and both the processes are exposing to exponential failure law. Under these assumptions we study an optimal replacement policy N in which we replace the system when the number of failures of component 1 reaches N. We determine an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. We derive an explicit expression of the long-run average cost and the corresponding optimal replacement policy N* can be determined analytically. Numerical results are provided to support the theoretical results.
In modeling these deteriorating systems, the definitions according to Lam [5, 7], are given below.

Definition 1. Given two random variables X and Y, if P(X>t) > P(Y>t) for all real t, then X is called stochastically larger than Y or Y is stochastically less than X. This is denoted by $X >_{st} Y$ or $Y <_{st} X$ respectively.

Definition 2. Assume that $\{Y_n, n=1,2,\ldots\}$, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t) = F(a^{n-1}t)$ for some a > 0 and all n=1,2,3,…., then $\{Y_n, n=1,2,\ldots\}$ is called a geometric process, '**a**' is the ratio of the geometric process.

Obviously:
if a>1, then $\{Y_n, n=1,2,\ldots\}$ is stochastically decreasing, i.e, $Y_n >_{st} Y_{n+1}$, n=1,2,…;
if 0<a<1, then $\{Y_n, n=1,2,\ldots\}$ is stochastically increasing, i.e, $Y_n <_{st} Y_{n+1}$, n=1,2,;
if a=1, then the geometric process becomes a renewal process.

Definition 3. Assume that $\{X_n, n=1,2,\ldots\}$, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t) = F(k^{\alpha} t)$ for some α > 0 and all n=1, 2, 3… then $\{X_n, n=1, 2\ldots\}$ is called a α series process, α is called exponent of the process. Braun et al [2].

Obviously:
if α >0, then $\{X_n, n=1,2,\ldots\}$ is stochastically decreasing, i.e, $X_n >_{st} X_{n+1}$, n=1,2,…;
if 0< α <1, then $\{X_n, n=1,2,\ldots\}$ is stochastically increasing, i.e., $X_n <_{st} X_{n+1}$, n=1,2,;
if α =0, then the α series process becomes a renewal process.

II. MODEL

In this section, an optimal replacement policy N for a cold standby repairable system using geometric process exposing to exponential failure law is studied under the following assumptions:

ASSUMPTIONS:

1. At the beginning two components are good. The component 1 works while component 2 is under cold standby.
2. As soon as the working component fails, it is immediately repaired by the repairman. At the same time, standby one begins to work. When the failed one has been repaired, it either begins to work again or becomes cold standby. If one fails another is still under repair, it must wait for repair and the system breaks down.
3. The replacement time is negligible.
4. Each component after repair is not 'as good as new'.
5. The time interval between the completion of the $(n-1)^{th}$ repair and the completion of the n^{th} repair on component 'i' is called n^{th} cycle of component i, for i=1, 2 and n=1,2,…
6. Let $X_n^{(i)}$ and $Y_n^{(i)}$ are all independent, for i=1, 2 and n=1, 2, 3….. .
7. Let $X_n^{(i)}$ and $Y_n^{(i)}$ be successive working time follows decreasing a α -series process, the successive repair times form an increasing geometric process respectively and both the processes are exposing to exponential failure law. Where i=1, 2 and n=1, 2, 3….. .
8. Let $F(a^{n-1} x)$ and $G(b^{n-1} y)$ be the distribution function of $X_n^{(i)}$ and $Y_n^{(i)}$ respectively, for i=1,2 and n=1,2,…. where a > 1 and 0 < b < 1.

9. $E(X_n^{(i)}) = \dfrac{\lambda}{k^{\alpha}}$ *and* $E(Y_n^{(i)}) = \dfrac{\mu}{a^{k-1}}$ *for* $i = 1,2.$

10. $E(X_1^{(i)}) = \lambda$ and $E(Y_1^{(i)}) = \mu,$ for $i = 1, 2.$.

11. The cold standby state and nearest working state have the same distribution. Similarly the waiting time for repair state and repair period have the same distribution.

12. The component in the system can't produce the working reward while in cold standby state, and no cost is incurred during waiting for repair.

13. The repair cost rate of the each component is C_r, the working reward rate of each component is C_w, and the replacement cost of the system is C.

Under these assumptions, an explicit expression for the long-run average cost per unit time and optimal solution for obtaining number of failures (N), which minimizes the long-run average cost per unit time, is discussed below.

III. OPTIMAL SOLUTION

There are two kinds of repair replacement policies: one based on the working age T of component 1 under which we replace the system when the working age of component 1 reaches T and the other based on the failure number (N) of component 1 under which we replace the system when failure number of component 1 reaches N. But here the replacement policy N is considered because it is very effective and easy to implement.

Thus number of failures of component 1 reaches N, then component 2 is either under working state or under waiting for repair state in the N^{th} cycle. Naturally, the farmer works until failure in the N^{th} cycle. The latter is not repaired any more in the N^{th} cycle, while component 1 works in the $(N+1)^{th}$ cycle.

Let T_n ($n \geq 2$) be the time between the $(n-1)^{th}$ replacement and the n^{th} replacement of the system under policy N. Clearly $\{T_1, T_2 \ldots\}$ form a renewal process and the inter arrival time between two consecutive replacements is called renewal cycle.

According to renewal reward theorem Ross [
8], the long-run average cost per unit time under policy N is :

$$C(N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of the renewal cycle}}. \qquad (3.1)$$

Let L be the length of renewal cycle of the system under policy N, then

$$L = \sum_{k=1}^{N+1} X_K^{(1)} + \sum_{k=1}^{N} Y_K^{(1)} + \sum_{k=2}^{N} \left[Y_{K-1}^{(2)} - X_K^{(1)} I_{(Y_{K-1}^{(2)} - X_K^{(1)} > 0)} \right]$$
$$+ \sum_{k=1}^{N} \left[X_K^{(2)} - Y_K^{(1)} I_{\{X_K^{(2)} - Y_K^{(1)} > 0\}} \right], \qquad (3.2)$$

where the first, second, third and fourth terms respectively working age, repair time, waiting for repair and cold standby time of component 1 and where I is an indicator random variable such that

$$I_A = \begin{cases} 1 & \text{if event A occurs.} \\ 0 & \text{if event A does't occurs.} \end{cases}$$

Now we find expected value of renewal cycle length L, under the assumptions of the model.

$$E(L) = E\left[\sum_{k=1}^{N+1} X_k^{(1)}\right] + E\left[\sum_{k=1}^{N} Y_k^{(1)}\right] + E\left[\sum_{k=2}^{N} (Y_{k-1}^{(2)} - X_k^{(1)}) I_{\{Y_{k-1}^{(2)} - X_k^{(1)} > 0\}}\right] + E\left[\sum_{k=1}^{N} \left(X_k^{(2)} - Y_k^{(1)}\right) I_{\{X_k^{(2)} - Y_k^{(1)} > 0\}}\right]. \qquad (3.3)$$

Now the expected length of working time can be obtained as follows:

Let $X_k^{(i)} \sim \exp(\lambda)$ *for* $k = 1,2,3,....,$ *and* $i = 1,2.$

Then the distribution function of $X_k^{(i)}$, for k=1,2,3,….and i=1,2 is :

$$F_k(x) = F(k^\alpha x) = 1 - e^{-\left(\frac{k^\alpha x}{\lambda}\right)} \; ; x > 0, \lambda > 0 \qquad (3.4)$$

.
By definition the expected length of working time is :

$$E\left(X_x^{(i)}\right) = \int_0^\infty x dF\left(k^\alpha x\right), \qquad i = 1,2. \qquad (3.5)$$

$$= \frac{\lambda}{k^\alpha}, \; where \; i = 1,2. \qquad (3.6)$$

The expected length of repair time of component 1 can be obtained as follows:
Let $Y_k^{(i)} \sim \exp(\mu)$ then the distribution function of $Y_k^{(i)}$ for i=1,2, and k=1, 2, 3, …., is

$$F_k(y) = F(b^{k-1} y) = 1 - e^{-\left(\frac{b^{k-1} y}{\mu}\right)} ; y > 0, \mu > 0 \qquad (3.7)$$

By definition, the expected length of repair time is:

$$E\left(Y_x^{(i)}\right) = \int_0^\infty y dF(b^{k-1} y) \qquad ,i = 1,2.$$

$$= \frac{\mu}{a^{k-1}}, \qquad i = 1,2. \qquad (3.8)$$

The expected length of waiting time for repair can be computed as follows:
Let g(u) be the probability density function of $u = Y_{k-1}^{(2)} - X_k^{(1)}$, then by definition of probability density function and using Jacobian transformation we have:

$$g(u) = \int_0^\infty f\left(v, u + v\right) dv,$$

where $X_k^{(1)} = v, \; \mathrm{Y}_{k-1}^{(2)} = u + v,$ such that $u = Y_{k-1}^{(2)} - X_k^{(1)}.$ (3.9)
Since $X_k^{(i)}$ *and* $Y_k^{(i)}$ are all independent, for i=1,2 and k=1,2,3,…..,n.

$$g(u) = \int_0^\infty f\left(v\right). f\left(u + v\right) dv. \qquad (3.10)$$

From equations (3.9) and (3.10) we have:

$$g(u) = \frac{k^\alpha b^{k-2} \lambda \mu}{k^\alpha \lambda + b^{k-2} \mu} e^{-b^{k-2} \mu u} \quad for \; u \geq 0. \qquad (3.11)$$

Let $E\left[\mathrm{Y}_{k-1}^{(2)} - X_k^{(1)} I_{\{Y_{K-1}^{(2)} - X_K^{(1)} > 0\}}\right] = \int_0^\infty u g(u) du$, (3.12)

$$= \int_0^\infty u \frac{k^\alpha b^{k-2} \lambda \mu}{k^\alpha \lambda + b^{k-2} \mu} e^{-b^{k-2} \mu u} du.$$

$$= \frac{k^\alpha \lambda}{b^{k-2} \mu \left(k^\alpha \lambda + b^{k-2} \mu\right)} \; , \; For \; k > 2. \qquad (3.13)$$

Similarly, the expected length of cold standby time can be computed as follows:

$$E\left[X_k^{(2)}-Y_k^{(1)}I_{\{X_K^{(2)}-Y_K^{(1)}>0\}}\right]=\int_0^{\infty} vg(v)dv\,. \qquad (3.14)$$

Where $g\left(v\right)$ be the p.d.f of $v=X_k^{(2)}-Y_k^{(1)}$. By definition of p.d.f and using Jacobean Transformation we have:

$$g(v)=\int_0^{\infty} f\left(u+v,u\right)du\,. \qquad (3.15)$$

$$where \quad \mathrm{X}_k^{(2)}=u+v,\ \mathrm{Y}_k^{(1)}=u \ \text{ such that } v=X_k^{(2)}-Y_k^{(1)}. \qquad (3.16)$$

Since $X_k^{(i)}$ *and* $Y_k^{(i)}$, for i=1, 2 are all independent and form a geometric process,

$$g(v)=\int_0^{\infty} f\left(u+v\right).f(u)du. \qquad (3.17)$$

Using equations (3.17) and (3.18), we get:

$$g(v)=\frac{k^{\alpha}b^{k-2}\lambda\mu}{k^{\alpha}\lambda+b^{k-2}\mu}e^{-k^{\alpha}\lambda v}\,,\quad for \quad v\geq 0 \qquad (3.18)$$

From equations (3.15) and (3.20), we have:

$$E\left[X_k^{(2)}-Y_k^{(1)}I_{\{X_K^{(2)}-Y_K^{(1)}>0\}}\right]=\int_0^{\infty} vg(v)dv$$

$$=\frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda+b^{k-1}\mu\right)} \qquad (3.19)$$

Using the equations (3.6), (3.8), (3.13) and (3.19) ,equation (3.3) becomes:

$$E(L)=\sum_{k=1}^{N+1}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N-1}\frac{k^{\alpha}\lambda}{b^{k-2}\mu\left(k^{\alpha}\lambda+b^{k-2}\mu\right)}+\sum_{k=1}^{N}\frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda+b^{k-1}\mu\right)} \qquad (3.20)$$

using equations (3.4), (3.8), and (3.20) .we have:

$$C(N)=\frac{C_r E\left[\sum_{k=1}^{N}Y_k^{(1)}+\sum_{k=1}^{N-1}Y_k^{(2)}\right]+C-C_w E\left[\sum_{k=1}^{N+1}X_k^{(1)}+\sum_{k=1}^{N}X_k^{(2)}\right]}{E(L)}$$

From equations (3.6) ,(3.8),(3.14),(3.21)and (3.22), we have:

$$C(N)=\frac{C_r\mu\left(\sum_{k=1}^{N-1}\frac{1}{b^{k-1}}+\sum_{k=1}^{N}\frac{1}{b^{k-1}}\right)-C_w\lambda\left(\sum_{k=1}^{N+1}\frac{1}{k^{\alpha}}+\sum_{k=1}^{N}\frac{1}{k^{\alpha}}\right)+C}{\sum_{k=1}^{N+1}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N-1}\frac{k^{\alpha}\lambda}{b^{k-2}\mu\left(k^{\alpha}\lambda+b^{k-2}\mu\right)}+\sum_{k=1}^{N}\frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda+b^{k-1}\mu\right)}}$$

$$C(N)=\frac{C_r\mu\left(l_3+l_4\right)-C_w\lambda\left(l_1+l_2\right)+C}{\lambda\left(l_1+l_2\right)+\left(l_5+l_6\right)} \qquad (3.24)$$

This is the long run average cost per unit time under policy N.

Where $l_1=\sum_{k=1}^{N+1}\frac{1}{k^{\alpha}}$ $\qquad l_2=\sum_{k=1}^{N}\frac{1}{k^{\alpha}}$,

$l_3=\sum_{k=1}^{N}\frac{1}{a^{k-1}}$, $\qquad l_4=\sum_{k=1}^{N-1}\frac{1}{a^{k-1}}$.

$$l_5 = \sum_{k=2}^{N-1} \frac{k^{\alpha}\lambda}{b^{k-2}\mu\left(k^{\alpha}\lambda + b^{k-2}\mu\right)} \quad , \quad l_6 = \sum_{k=1}^{N} \frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda + b^{k-1}\mu\right)} .$$

Using this C (N), the optimal replacement policy N* is determined by numerical methods such that C(N*) is minimized.

The next section provides some numerical results to highlight the theoretical value of the results.

IV. NUMERICAL RESULTS AND CONCLUSIONS

For the given hypothetical values of the parameters $\alpha, b, \lambda, \mu, C_r, C_w, and\ C$, the long run average cost per unit of time is determined as follows:

Table:1

	α =0.15 b=0.85 $\lambda=10, \mu=20$ C_r=100, C_w=50,C=8000	α =0.25 b=0.85 $\lambda=10, \mu=20$ C_r=100, C_w=50,C=8000	α =0.35 b=0.85 $\lambda=10, \mu=20$ C_r=100, C_w=50,C=8000	α =0.45 b=0.85 $\lambda=10, \mu=20$ C_r=100, C_w=50,C=8000
K	C(K)	C(K)	C(K)	C(K)
2	2377.164	2472.905	2565.657	2655.211
3	2229.152	2341.361	2449.209	2552.371
4	**2198.638**	**2322.403**	**2440.187**	**2551.61**
5	2220.392	2352.166	2476.215	2592.186
6	2269.061	2405.975	2533.403	2651.099
7	2332.555	2472.179	2600.622	2717.825
8	2404.241	2544.481	2671.984	2786.945
9	2480.09	2619.142	2744.097	2855.451
10	2557.461	2693.789	2814.896	2921.604
11	2634.532	2766.853	2883.089	2984.395
12	2710.007	2837.279	2947.87	3043.256
13	2782.948	2904.358	3008.756	3097.908
14	2852.687	2967.633	3065.484	3148.26
15	2918.759	3026.832	3117.954	3194.351
16	2980.863	3081.824	3166.179	3236.307
17	3038.826	3132.59	3210.257	3274.313
18	3092.584	3179.191	3250.345	3308.593
19	3142.157	3221.752	3286.643	3339.391
20	2635.985	2639.265	2641.758	2643.667

Table:2

	α =0.55 b=0.85 $\lambda=10, \mu=25$ C_r=100, C_w=50,C=8000	α =0.55 b=0.75 $\lambda=10, \mu=30$ C_r=100, C_w=50,C=8000	α =0.55 b=0.65 $\lambda=10, \mu=35$ C_r=100, C_w=50,C=8000	α =0.55 b=0.55 $\lambda=10, \mu=40$ C_r=100, C_w=50,C=8000
K	C(K)	C(K)	C(K)	C(K)
2	1348.262	1455.299	1508.171	1561.167
3	882.5183	1028.642	1101.366	1174.045
4	660.0425	834.75	922.0494	1009.255
5	551.1883	746.1509	843.8186	941.4006
6	499.0358	707.6681	812.3532	916.9783
7	475.0522	692.5089	801.7317	**910.9182**

8	464.6269	687.5858	**799.6401**	911.6742
9	460.4536	686.757	800.5324	914.2977
10	459.0131	687.3128	802.1153	916.9135
11	**458.681**	**688.1568**	803.5643	918.9705
12	458.7438	688.9062	804.6665	920.4274
13	458.909	689.4692	805.434	921.4002
14	459.0691	689.8588	805.9414	922.026
15	459.1941	690.1155	806.2658	922.4183
16	459.2825	690.2793	806.4683	922.6596
17	459.3416	690.3814	806.5925	922.8059
18	459.3798	690.444	806.6676	922.8937
19	459.4037	690.4819	806.7126	922.9458
20	459.461	690.5574	806.7975	923.04

V. CONCLUSIONS

I) From the table1, at α =0.15 it is observed that C(4)= **2198.638.** Thus the long-run average cost per unit time at the time of 4th failure is minimum i.e., we should replace the system at the time of 4th failure. From the table2,it is observed that as 'b' increases , the optimal number of failures will also increases, while 'α ' decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

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