

Improved Decoding Algorithms Useful In Modified Low Density Parity Check Code for Optical Fibre Communication

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Abstract:- Low-density parity-check (LDPC) codes were first introduced by Gallager. These are linear block codes. Their parity-check matrix contains only a few 1's in comparison to the amount of 0's so these codes are called as low density parity check codes. It is a very powerful code for forward error correction system. The LDPC code can be modified and with which the quality factor can be improved because the presently used LDPC code in optical fibre communication systems is not meeting the requirements of the high speed fibre optics communication systems. This modification is beneficial for the high speed fibre optic communication systems. We explain here a sum product decoding algorithm and modified sum product decoding algorithm of low density parity check code useful in Optical fibre communication. BP and MS modified decoding algorithm also use for a better performance in high speed fibre optic communication.

Keywords:- Low density parity check codes (LDPC), Sum-Product (SP), Belief propagation (BP), Min-sum decoding (MS), Bit error rate (BER), additive white Gaussian noise (AWGN).

I. INTRODUCTION

LDPC codes are discovered by Gallager in early 60's so they are also called as Gallager codes [1]. But at that time, computing power of this code was not enough to show their effectiveness, therefore LDPC codes have been forgotten until recently. They are rediscovered by Mackay and Neal with excellent functioning using the decoding algorithm based on sum-product algorithm [2]. LDPC codes are distinguished by binary parity-check matrices. In each matrix, every row has a fixed number (j) of 1's and every column also has a fixed number (k) of 1's. In progress study Gallager introduced irregular codes which are specified by the parity-check matrices that have both non-uniform column weights and non-uniform row weights. These codes were improved codes than regular codes.[4]

In encoding of LDPC codes we choose certain variable nodes to place the message bits on. After that we calculate the missing values of the other nodes. By solving the parity check equations we can calculate the missing values. For this operations the whole parity-check matrix is use and this may add complexity in the block length. Now a days more clever methods are used by which encoding can be done in much shorter time. Those methods can use the tanner graph parity-check matrix. [1]

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Fig 1: Parity check matrix

Graphical representation of LDPC codes plays a very important role in the development of the decoding algorithms.

The graphical representation for LDPC code is called as tanner graph. This code not only provides the complete representation of the code, they also help to describe the decoding algorithm. Tanner graph is a representation of the LDPC code, it also known as a bipartite graph for the decoding process. That means that the nodes of the graph are separated into two distinctive sets and edges are only connecting nodes of two different types. The two types of nodes in a Tanner graph are called variable nodes (v-nodes) and check nodes. The Tanner graph with bit nodes and check nodes is shown in Fig. 1. A check node can be connected to bit nodes where the elements of a row are '1' in the parity check matrix H. Similarly, a single bit node can be connected to check nodes where the elements of a column are '1' in the parity check matrix H.

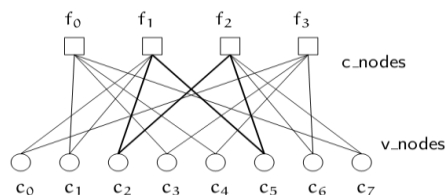


Fig 2: Tanner graph corresponding to the parity check matrix in Fig 1. The marked path $c_2 \rightarrow f_1 \rightarrow c_5 \rightarrow f_2 \rightarrow c_2$ is an example for a short cycle. Those should usually be avoided since they are bad for decoding performance. [3]

In LDPC codes, iterative decoding algorithms are widely used for decoding. The decoding algorithm message passing algorithm and the sum-product algorithm are used in a wide range of areas from artificial intelligence, signal processing to digital communications. [4]

Soft decision, hard decision or hybrid decision decoding can be used to decode LDPC codes. Soft decision decoding based on BP is one of the most powerful decoding methods for LDPC codes. BP decoding gives good performance, because of floating point computations it can become too complex for hardware implementation. By approximating the calculation at the check nodes with a simple minimum operation, the min-sum (MS) algorithm reduces the complexity of BP [17]. Ultimate performance of MS is often much worse than that of BP but MS is hardware efficient. [18], [19]

The recent studies have verified that soft-decoding of low-density parity-check (LDPC) code requires high quantisation resolution which is difficult to achieve for high-speed fibre-optic communication systems. The powerful forward error correction code for fibre optic communication is a low-density parity-check (LDPC) code. In this paper we describe the decoding algorithm of Low density parity check code. The modification in this code effectively improves the performance at quantisation resolution as low as two-bit or one-bit. By proper implementation of this technique, the Q-factor improvement of 2-dB can be obtained in comparison to RS code. [5]

A codeword vector $v = (v_1, \dots, v_k)$ is created with the help of original information vector $u = (u_1, \dots, u_k)$ ($v = uG$, where G is the generator matrix). We have the property that $v \cdot H^T = 0$, where H^T is the transposed matrix of H . If h_i denotes the i^{th} row of H , then we have

$$v \cdot h_i^T = 0, i = 1, 2, \dots, J \quad (1.1)$$

Here h_i is a row vector: $h_i = (h_{i,1}, \dots, h_{i,n})$. Equation (1.1) therefore can be written as:

$$\sum_{j=1}^n v_j h_{i,j} = 0, i = 1, 2, \dots, J [6]$$

II. LDPC DECODER

A. Decoding algorithms of LDPC

Basically two decoding algorithms are used in LDPC code for a better performance which is SP and the MS decoding algorithms. The decoding performance of LDPC code is also verified by experiment where 2-dB coding gain over RS (255, 239) code can be obtained at the same redundancy with the quasi-cyclic LDPC code studied in the simulation. The experimental data are used for processing timing synchronisation, frame synchronisation and channel estimation for each data frame before decoding in the experiment. [5]

With respect to the FEC schemes employed in optical communication systems LDPC codes provide a significant system performance enhancement. In the area of error control coding during the last few years ignited by the excellent bit-error-rate (BER) performance of the turbo decoding algorithm demonstrated by Berrou et al [7]. Low density parity check codes (LDPC) is a prime examples of codes on graphs. LDPC codes perform nearly as well as earlier developed turbo codes in many channels [such as additive white noise Gaussian (AWGN) channel, binary symmetric channel and erasure channel]. Error performance has improved the theory of codes on graphs [8]. Pearl developed the sum product algorithms and graphical models in the expert systems literature [9].

The belief propagation (BP) (sum product) algorithm [10] provides a powerful tool for iterative decoding of LDPC codes. LDPC codes with iterative decoding based on BP achieve a remarkable error performance that is very close to the Shannon limit [11]. For that reason LDPC codes have received significant attention recently. A LDPC code is specified by a parity-check matrix, it contains more number of zeros and only a small number of ones. Generally, LDPC codes can be divided into regular LDPC codes and irregular LDPC codes. If the weights of rows and columns in a parity check matrix are equal then an LDPC code is called as regular and it is called as irregular if not. It is proved that with properly chosen structure, irregular LDPC codes have better performance than regular ones [12].

For decoding of LDPC codes, soft decision, hard decision or

hybrid decision decoding can be used. It has been shown that one of the most powerful decoding methods for LDPC codes is soft decision decoding based on SP. Although SP decoding offers good performance, because of floating point computations it can become too complex for hardware implementation. The min-sum (MS) algorithm reduces the complexity of SP by approximating the calculation at the check nodes with a simple minimum operation [13].

To explain sum product algorithm we assume (N,K) LDPC code which is defined by a parity check matrix H. Let the received codeword of the bit node n through the channel is y_n and the transmitted codeword of the bit node n is u_n , $q_{n \rightarrow m}$, $\tilde{x} \in \{0,1\}$ represents the probability of the bit node n being 0 or 1 which the bit node n sends to the check node m. Similarly $r_{n \rightarrow m}$, $\tilde{x} \in \{0,1\}$ is the probability of the check node m being 0 or 1 which the check node m sends to the bit node n. The SP algorithm can be described as follows:

The transmitted codeword u_n is assigned to the bit node n as $L(u_n) = Lcy_n$ where $L(u_n)$ is defined as a posterior probability. Assume that the channel is AWGN. Initialization of Every position of a parity-check matrix H where $H_{m,n}=1$ is as following two equations[14]

$$\lambda_{n \rightarrow m}(u_n) = L(u_n) \quad (1)$$

$$\Lambda_{m \rightarrow n}(u_n) \quad (2)$$

High BER performance can achieve by SP algorithm but it requires high computational complexity due to compute the hyperbolic tangent and hyperbolic arc tangent functions.

This computational complexity is reduce by the modified SP algorithm has. The modified SP algorithm divides the hyperbolic tangent and hyperbolic arc tangent functions into seven regions, respectively. To represent seven regions the eight quantization values have been selected. The quantization values for the $\tanh(x)$ function shown in table I and The quantization table for the $\tanh^{-1}(x)$ function shown in table II. [14]

Table I: Quantization values for the $\tanh(x)$

X	$\tanh(x)$
$-7 < x \leq -3$	-0.99991
$-3 < x \leq -1.6$	-0.9801
$-1.6 < x \leq -0.8$	-0.8337
$-0.8 < x \leq 0$	-0.3799
$0 < x \leq 0.8$	0.3799
$0.8 < x \leq 1.6$	0.8337
$1.6 < x \leq 3$	0.9801
$3 < x \leq 7$	0.99991

Table II: Quantization values for the $\tanh^{-1}(x)$

X	$\tanh(x)$
$-0.999998 < x \leq -0.9951$	-3.03516
$-0.9951 < x \leq -0.9217$	-1.9259
$-0.9217 < x \leq -0.6640$	-1.0791
$-0.6640 < x \leq 0$	-0.3451
$0 < x \leq 0.6640$	0.3451
$0.6640 < x \leq 0.9217$	1.0791
$0.9217 < x \leq 0.9951$	1.9259
$0.9951 < x \leq 0.999998$	3.03516

By using the modified SP algorithm [15] we can produces relatively a large BER performance degradation compared with the SP algorithm [16]. But to complete the check node updates the modified SP algorithm [15] as well as the SP algorithm [16] requires high computational complexity due to the multiplications and divisions.

III. CONCLUSION

Through simulation we have studied the quantisation effect on the soft decoding of LDPC codes in fibre-optic communication systems and the decoding performances for several quantisation resolutions are compared. The decoding performance shows that low-bit quantisation may cause severe deterioration. With the help of simple modification to the decoding algorithms the improvement of the low-bit quantised soft decoding is done. Simulation results show that, the modified 2-bit quantised MS decoding can achieve about 0.6 dB

performance gain in comparison to the without modified decoding algorithm. The proposed modification scheme can also be applied to the BP and the MS decoding algorithms for a better performance.

LDPC code is also verified by experiment where 2-dB coding gain over RS (255, 239) code can be obtained at the same redundancy with the quasi-cyclic LDPC code. But Q-factor fluctuation cannot be avoided in the experiment, the experimental BER is only slightly worse than the one from the simulation. Quasi-cyclic structure of LDPC code provides a way for distributed memory storage and access of LLR information during row and column operations of the information exchange in an orderly way. The encoder and decoder implementation of such a LDPC code can more easily meet the high-speed requirement for fibre-optic communication systems.

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