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Wavelet Transform: A Recent Image Compression Algorithm

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Abstract: *- Digital imaging has had an enormous impact on scientific and industrial applications. Uncompressed images require considerable storage capacity and transmission bandwidth. The solution to this problem is to compress an image for desired application. There are two different compression categories : lossless and lossy . Lossless compression preserves the information in the image. Thus an image could be compressed and decompressed without loosing any information. Lossy compression is useful in areas such a video conferencing, fax or multimedia applications and is the focus of this paper. Discrete cosine and wavelet transforms, the work horses of JPEG-93 And JPEG-2000, respectively. This paper focus on the use of the two-dimensional wavelet transform over images, which are represented as a two dimensional array of numbers.*

Keywords: *- lossless compression, loosy compression, wavelet transform, Discrete cosine transform, JPEG-2000*

I. INTRODUCTION

Data compression refers to the process in which a given information is represented with reduced number of data points.

A digital image can be represented by a two dimensional (2-D) array i.e., a matrix, each of whose element f (i,j) corresponds to the value of the (i,j)th pixel in the image. If each pixel represents a shade gray in monochrome images , we need to allocate only one byte or 8 bit per pixel (bpp). With $2^8 = 256$ combinations, one can represent numbers ranging from 0 (black) to 255 (white). An uncompressed say 800 × 800 pixel image, will need 64×10^4,bytes $\cong$ 0.64 MB. If 1000 images are to be stored, we need 640 MB!. Obviously we then need to compress the image, i.e. represent the same image with reduced number of bits, possibly with some loss, without changing the size of the image.

II. TYPES OF REDUNDANCY

Image can be compressed, primarily by eliminating the following types of redundancies:

A. Spatial Redundancy:- In most of the natural images, the value of the neighbouring pixels are strongly correlated i.e., the value of any given pixel can be reasonably predicted from the value of its neighbours. Thus the information carried by individual pixel is relatively small. This type of redundancy can be reduced , if the 2-D array representation of the image, is transformed into a format which keeps the differences in the pixel values. Wavelet transform has a capability to effectively capture variation at different scales. [2,3] Hence, wavelets are ideally suited for this purpose.

B. Coding Redundancy:- For a given image stored as a matrix with values ranging from 0 to 255, one can find out the number of times each digit occur in the image . This frequency distribution of pixel values from 0 to 255 is called histogram. In typical images, a few pixel values have greater frequency of occurance, as compare to others. Hence , if the same code word size is assigned to the more probable gray scale value than the less probable ones. This redundancy can be reduced using Huffman coding.

C. Psychovisual Redudancy:- This takes into account the properties of human vision. Human eyes do not respond with equal sensitivity to all visual informations. Certain information has less relative importance as compare to other informations in normal visual processing. This information is said to be psychovisually redundant. Broadly speaking, an observer searches for distinguishing features like edges or textural regions and mentally combines them into recognizable groupings. The brain then relates these groupings, with its prior knowledge, in order to complete the image interpretation process. This redundancy can be overcome by process of thresholding and quantization of the wavelet coefficients.

III. LOSSY IMAGE ENCODER

Typical lossy image compression system contains four closely related components as shown in figure 1 .They are (a) source encoder (b) thresholder (c) quantizer (d) entropy encoder. Compression is accomplished by first applying a linear transform to decorrelate the image data, the ttransformed coefficient are then thresholded and quantized and finally, the quantized value are entropy coded. The decompression is achieved by applying the above four operation in the reverse order.

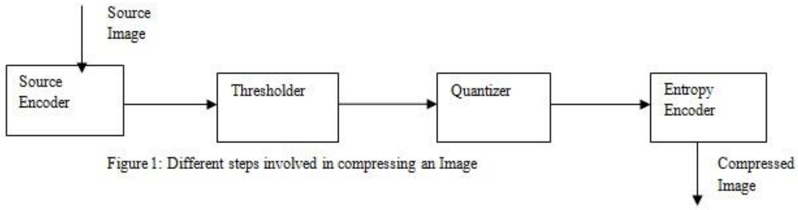

Figure 1: Different steps involved in compressing an Image

A. Source Encoder:- This refers to the linear transforms, which are used to map the original image into some transformed domain. Popular transform techniques used discrete Fourier transform (DFT), discrete cosine transform (DCT) or discrete wavelet transform (DWT). Each transform has its own advantages and disadvantages. In this article we describe DWT. JPEG- 2000 will be totally based on DWT.

1. Discrete Wavelet Transform (DWT) Based Coding

Wavelet are the probing functions, which gives optimal time frequency localization of a given signal. From a practical point of view , wavelets [6,7,8] provide a basis set which allows one to represent a data set in the form of differences and averages, called the high- pass or detailed and low-pass or average coefficients, respectively. The number of data points to be averaged and the weights to be attached to each data point, depend on the wavelet one chooses to use. Usually, one takes N = 2^n (where n is a positive integer), number of data points for analysis. In case of the simplest, Haar wavelet, the level-1 hiigh- pass and low- pass coefficients are the nearest neighbour differences and nearest neighbour averages respectively, of the given set of data with the alternate points removed. Subsequently, the level -1 low pass coefficients, can again be written in the form of level- 2 high pass and low pass coefficients, having one fourth number of points of the original set. In this way with 2^n number of points, at the n^{th} level of decomposition, the low- pass will have only one point. For the case of Haar, modulo a normalization factor, the n^{th} level low- pass coefficient is the average of all the data points. In principle, an infinite choice of wavelets exist. The choice of a given wavelet depends upon the problem at hand. As one can easily imagine, wavelets are ideal to find variations at different scales present in a data set. This procedure can be easily extended to two dimensional case, for application to image processing.

2. Advantage of a Designed Wavelet Set

It is possible to construct our own basis function, for wavelets, depending upon the application in hand. Thus, if a suitable basis function is designed for a given task, then it will capture most of the energy of the function with very few coefficients.

B. Thresholder:- Once DWT is performed, the next task is thresholding, which is neglecting certain wavelet coefficients. For doing this one has to decide the value of a threshold and how to apply the same.

1. Value of the Threshold

This is an important step which affect the quality of the compressed image. The basic idea is to truncate the insignificant coefficients, since the amount of information contained in them is negligible. The question of deciding the value of threshold is a problem in itself. Ideally, one should have a uniform recipe, which would work satisfactaorily for a given set of problems, so that the procedure is automated. One such method by Donoho and co- authors [9] gives an asymptotically optimal formula called the universal threshold t:

$$t = \sigma\sqrt{(2\ln N)} \quad (1)$$

Here, σ = standard deviation of the N wavelet coefficients
The value of t should be calculated for each level of decomposition and only for the high pass coefficients. The low pass coefficients are usually kept untouched so as to facilitate further decomposition.

2. How to Apply a Threshold

There are two ways in which threshold can be applied.
(a) Hard Threshold: If x is the set of wavelet coefficients, then threshold value t is given by,

$$T(t,x) = \begin{Bmatrix} 0 & if|\,x| < t \\ (x) & otherwise \end{Bmatrix} \quad (2)$$

i.e. all the values of x which are less than threshold t are equated to zero. This condition is shown in figure 2(b).
b) Soft Threshold: In this case, all the coefficients x lesser than threshold t are mapped to zero. Then t is subtracted from all x ≥ t. This condition is depicted by the following equation:

$$T(t,x) = \begin{Bmatrix} 0 & if|\,x| < t \\ sign(x)(|x| - t) & otherwise \end{Bmatrix} \quad (3)$$

This condition is shown in figure 2(c). Usually, soft threshold gives a better peak signal to noise ratio (PSNR) as compared
to hard threshold.

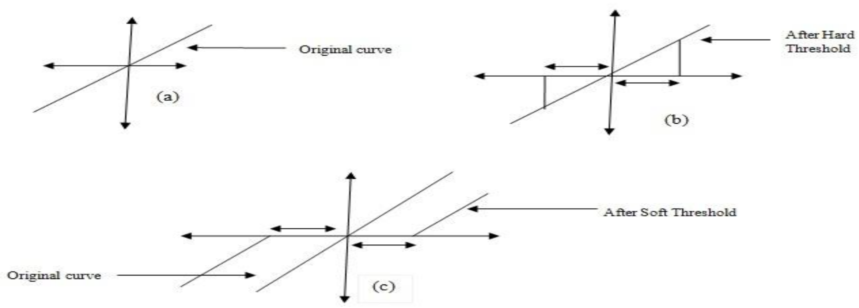

Figure 2: Different Thresholds: (a) original (b) hard and (c) soft.

C. Quantizer:- higher compression ratios can be obtained by quantizing the non-zero wavelet coefficients, before they are encoded. A quantizer is a many-to-one function Q(x) that maps many input values into a (usually much) smaller set of output values. Quantizers are staircase functions characterized by a set of numbers $\{d_i$, i = 0….,N} called decision points and a set of numbers $\{r_{i,}$ I = 0..,N-1} called reconstruction level $r_{i,}$ if x lies in the interval $(d_i, d_{(i+1)})$.

To achieve best results, a separate quantizer should be designed for each scale, taking into account statistical properties of the scale's coefficient and for images, properties of the human visual system. The coefficient statistics guide the quantizer design for each scale, while the human visual system guides the allocation of bits among the different scales. For our present purpose, a simple uniform quantizer (i.e. constant step size) is used. The wavelet coefficients, after thresholding were uniformly quantized into 256 different bins. Thus the size of each bin was $\frac{x_{max} - x_{min}}{256}$, where x_{min} and x_{max} are the wavelet coefficients with minimum and maximum values respectively. To minimize the maximum error (maximum condition), centroid of each bin is assigned to all the coefficient falling in that bin.

D. Entropy Encoder:- This is the last component in the compression model. Till now, we have devised models for an alternate representation of the image, in which its interpixel redundancies were reduced. This last model, which is a lossless technique, then aims at eliminating the coding redundancies, whose notion will be clear by considering an example.

Suppose, we have a domain in an image, where pixel values are uniform or the variation in them is uniform. Now one requires 8bpp (bit per pixel) for representing each pixel with the same (or constant difference) value will introduce coding redundancy. This can be eliminated, if we transform the real values into some symbolic form, usually a binary system, where each symbol corresponds to a particular value. We will discuss a few coding techniques and analyse their performances.

1. Run Length Encoding

Run- length encoding (RLE) makes use of the fact that nearby pixel in an image will probably have the same brightnesss value. This redundancy can then be coded as follows, Original image data (8-bit)
127 127 127 127 129 129 129
Run-length encoded image data
127 4 129 2
This technique will be useful for encoding an online signal. But data explosion problem can occur and even a single data error will obstruct full decompression.

2. Differential Pulse Code Modulation

Predictive image compression techniques assume that a pixel's brightness can be predicted given the value of the preceding pixel. Differential pulse code modulation (DPCM) codes the differences between two adjacent pixels. DPCM starts coding at the top left- hand corner of an image and works left to right, until all the image is encoded as shown:

Original Image Data
86 86 86 86 88 89 89 89 89 90 90
86- 0- 0- 0 -2 -1- 0- 0- 0- 1- 0
DPCM Code
This technique will be use full for images that have larger runs of equal value pixels.

3. Huffman Coding

This is the most popular statistical data compression technique for removing coding redundancy. It assigns the smallest possible number of code symbols per source symbol and hence reduces the average code length used to represent the set of given values. The general idea is to assign least number of bits to most probable (or frequent) values occurring in an image. A Huffman code can be built in the following manner:

- Rank all symbols in decreasing order of probability of occurrence.
- Successively combine the two symbols of the lowest probability to form a new composite symbol (source reduction); eventually we will build a binary tree, where each node is the probability of all nodes beneath it.
- Trace the path to each leaf, noticing the direction at each node.

For a given frequency distribution , there are many possible Huffman codes, but the total compressed length will be the same [5,6].

IV. RESULTS

We Now briefly discuss the results obtained from our analysis. Figure 3 is the original Lena image [7]. Lena, an academic model in the image processing community, has emerged as a bench mark image for testing the efficiancy of various algorithms. This gives a chance to researchers across the globe, to compare and contrast their respective methods. The image was subjected to one level DWT, using Daubechies'-6 wavelet. The coefficients thus obtained, were thresholded using Donoho's formula (equation(1)), for both hard and soft (equations (2) and (3)) cases. The results are as shown in Figure 3. It is worth remembering that in soft thresholding, the wavelet coefficients less than or equal to the threshold are equated to zero and the remaining ones are reduced by the same amount, whereas in the hard case, the coefficients above the threshold value are not altered. The PSNR when calculated using equations (2) and (3) turns out to be approximately 42 dB for soft case and 45 dB for hard case.

Figure 3 (a): Original Lena image

Figure 3 (b): Reconstructed image after soft threshold + quantization (top) & after hard threshold + Quantization (bottom). The quality of the image is better in the one which has used soft threshold and quantization

. In conclusion, we have described the application of wavelet transform to image compression, a subject of intense current interest. The properties of images which makes them amenable for compression were pointed out. The various steps like thresholding , Quantization and their usefulness for compression were also pointed out. We dealt with the concept of entropy encoder to bring out the idea of lossless coding transparently.

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