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Operators of Two-dimensional Generalized Canonical sine Transform

S. B. Chavhan and V.C.Borkar
Department of Mathematics & Statistics, Yeshwant Mahavidyalaya, Nanded-431602 (India)

Abstract:-This paper is concerned with the definition of two-dimensional (2-D) generalized canonical sine transform it is extended to the distribution of compact support by using kernel method.

Keywords:-2-D canonical transform, 2-D canonical sine transform, 2-D fractional Fourier transform, generalized function, testing function space.

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I. INTRODUCTION

The fractional Fourier transform has been used in many applications such as optical system analysis, filter design, solving differential equation, phase retrieval, and patter recognition, etc [2],[5],[6]. The fractional Fourier transform is an extension of the fourier transform. The fractional fourier transform is extended from one dimension into the dimensions [1],[3] [4],[7].The two dimensional canonical sine transform is defined as.

$$\{2DCSST\ f(t,x)\}(s,w)=-\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

When $b\neq 0$

Notation and terminology of this paper is as per [8],[9].In this paper section 2 gives the definition of testing function space and 2-D generalized canonical sine transform, in section 3 inversion theorem is proved. Section 4 some basic properties are proved, lastly the conclusion is stated.

II. DEFINITION OF TESTING FUNCTION SPACE

An infinitely differentiable complex valued function ϕ on R^n belongs to $E(R^n)$, if for each compact set. $I\subset s_a$, $J\subset s_b$

where $s_a=\{t:t\in R^n,\quad |t|\le a,\, a>0\}$, $s_b=\{x:x\in R^n,\quad |x|\le b,\, b>0\}$

and for $I\in R^n,\ J\in R^n,$

$$\gamma_{E,k}\phi(t,x)=\sup_{\substack{-\infty<t<\infty\\-\infty<t<\infty}}\left|D_t^{\,k}D_x^{\,l}\,\phi(t,x)\right|<\infty \qquad (2.1)$$

k=0,1,2,3....and l=0,1,2,3

Thus $E\left(R^n\right)$ will denotes the space of all $\phi(t,x)\in E\left(R^n\right)$ with support contained in s_a and s_b. Note that space *E* is complete, Frechet space and E' denotes the dual space of E.

2.1.Definition two dimensional (2D) canonical sine transform [2DCSST]:

Let $E^{'}(R\times R)$ denote the dual of $E(R\times R)$.Therefore the generalized canonical sine transform of $f(t,x)\in E^{'}(R\times R)$ is defined as

$$\{2DCSST\ f(t,x)\}(s,w)=\langle f(t,x),K_{s_1}(t,x)K_{s_2}(x,w)\rangle$$

$$\{2DCST\ f(t,x)\}(s,w)$$
$$=(-1)\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

Where, $K_{s_1}(t,s)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.\sin\left(\frac{s}{b}t\right).e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}$ when $b\neq 0$

$$=\sqrt{d}\,e^{\frac{i}{2}c\left(ds^2\right)}\delta(t-ds)$$ when b = 0

$K_{s_2}(x,w)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}.\sin\left(\frac{w}{b}x\right).e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}$ when $b\neq 0$

$$=\sqrt{d}\,e^{\frac{i}{2}c\left(dw^2\right)}\delta(x-dw)$$ when b = 0

where $\gamma_{E,k}\{K_{s_1}(t,s)K_{s_2}(x,w)\}=\sup_{\substack{-\infty<t<\infty\\ -\infty<x<\infty}}\left|D_t^kD_x^lK_{s_1}(t,s)K_{s_2}(x,w)\right|<\infty$

III. INVERSION OF 2-D DIMENSIONAL CANONICAL SINE TRANSFORM

Any transform is used to solve differential equations, only if inverse of the transform is available obtain inverse of 2D canonical sine transform next theorem.

Theorem3.1: (Inversion) If $\{2DCSST\ f(t,x)\}(s,w)$ is 2-D canonical sine transform of $f(t,x)$ then inverse of transform is given by

$$f(t,x)$$
$$=-\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\,e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}\{2DCSST\ f(t,x)\}(s,w)\,dsdw$$

Proof: The two dimensional canonical sine transform of $f(t,x)$ is given by

$$\{2DCSST\ f(t,x)\}(s,w)$$
$$=-\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$F(s,w)=\{2DCSST\ f(t,x)\}(s,w)$$

$$F(s,w)$$
$$=-\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$F(s,w)\sqrt{2\pi ib}\sqrt{2\pi ib}\,e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}$$
$$=(-1)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$C_1(s,w)=F(s,w)\sqrt{2\pi ib}\sqrt{2\pi ib}\,e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}$$

and $g(t,x)=e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}.e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}.f(t,x)$

$$C_1(s,w)=(-1)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right).\sin\left(\frac{w}{b}x\right).g(t,x)\,dxdt$$

$$C_1(s,w)=\{2DCCCTg(t,x)\}\left(\frac{s}{b},\frac{w}{b}\right)$$

Where $\{2DCSSTg(t,x)\}\left(\frac{s}{b},\frac{w}{b}\right)$ is 2D Canonical sine-sine transform of *g(t,x)*.two dimensional Canonical sine -sine transform *g(t,x)* with argument $\therefore\frac{s}{b}=\eta$ and $\frac{w}{b}=\xi$ therefore $\frac{ds}{b}=d\eta$ and $\frac{dw}{b}=d\xi$

$$\therefore C_1(s,w)=\{2DCSSTg(t,x)\}(\eta,\xi)$$

By inversion formulae $g(t,x)=(-1)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_1(s,w).\sin(\eta t).\sin(\xi x)\,d\eta\,d\xi$

$$g(t,x)=(-1)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(s,w)\sqrt{2\pi ib}\sqrt{2\pi ib}\,e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}\sin(\eta t).\sin(\xi x)\,d\eta\,d\xi$$

$$f(t,x) = -\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}} e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right) e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2} \left\{2DCSST\ f(t,x)\right\}(s,w)\,ds\,dw$$

IV. PROPERTIES OF TWO DIMENSIONAL CANONICAL SINE TRANSFORM $[2DCSST]$

Separability (4.1): If $f(t,x) = f_1(t).f_2(x)$ then

$$\left\{2DCSST\ f(t,x)\right\}(s,w) = \left\{CST\ f_1(t)\right\}(s).\left\{CST\ f_2(x)\right\}(w)$$

Linearity property (4.2): If C_1, C_2 constant and f_1, f_2 are functions of t and x , then

$$\left\{2DCSST\left[c_1 f_1(t,x)+c_2 f_2(t,x)\right]\right\}(s,w) = c_1\left\{2DCSST\ f_1(t,x)\right\}(s,w)+c_2\left\{2DCSST\ f_2(t,x)\right\}(s,w)$$

Scaling Property (4.3) : If $\left\{2DCSST\ f(t,x)\right\}(s,w)$ is canonical sine-sine transforms of $f(t,x)$ then

$$\left\{2DCSST\ f(pt,qx)\right\}(s,w) = \frac{1}{pq}\exp\left[\frac{i}{2}\left(\frac{d}{b}\right)\left(\left(p^2-1\right)\left(\frac{s}{p}\right)^2+\left(q^2-1\right)\left(\frac{w}{q}\right)^2\right)\right]\left\{2DCSST\ f(u,v)\right\}\left(\frac{s}{p},\frac{w}{q}\right)$$

Shifting property (4.4): If $\left\{2DCSST\ f(t)\right\}(s,w)$ is canonical sine-sine transform of $f(t,x)$ then

$$\left\{2DCSST\ f(t-p,x-q)\right\}(s,w) = e^{\frac{i}{2}\left(\frac{a}{b}\right)\left(p^2+q^2\right)}\left[\cos\left(\frac{s}{b}p\right)\cos\left(\frac{w}{b}q\right)\left\{2DCSST\ f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)\right.$$

$$-\sin\left(\frac{s}{b}p\right)\sin\left(\frac{w}{b}q\right)\left\{2DCCCT\ f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)$$

$$-i\cos\left(\frac{s}{b}p\right)\sin\left(\frac{w}{b}q\right)\left\{2DCSCT\ f(u,v)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}(s,w)$$

$$-i\sin\left(\frac{s}{b}p\right)\cos\left(\frac{w}{b}q\right)\left\{2DCCST\ f\left(u,v\right)e^{i\left(\frac{a}{b}\right)(up+vq)}\right\}\left(s,w\right)\Big]$$

Addition theorem (4.5): If $\left\{2DCSST\ f\left(t,x\right)\right\}\left(s,w\right)$ and $\left\{2DCSST\ g\left(t,x\right)\right\}\left(s,w\right)$ are canonical sine-sine transform of $f\left(t,x\right)$ and $g\left(t,x\right)$ then

$$\left\{2DCSST\left[\ f\left(t,x\right)+g\left(t,x\right)\right]\right\}\left(s,w\right)$$
$$=\left\{2DCSST\ f\left(t,x\right)\right\}\left(s,w\right)+\left\{2DCSST\ g\left(t,x\right)\right\}\left(s,w\right)$$

V. CONCLUSION

In this paper two-dimensional canonical sine is generalized in the form the distributional sense, we have inversion theorem for this transform is proved some properties of generalized 2-D canonical sine transform are discussed.

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