

A Design of Synchronization Control Based on Three-Dimensional Chaotic System

Zhang Liming, Pan Yi, Yang Shuangchun
Liaoning Shihua University, Fushun, Liaoning, China 113001

Abstract:-In the practical application environment, the parameters of the chaotic system will often produce floating, at the same time, if the chaotic synchronization are applied to parameter modulation method of secure communication system, we also need to solve the synchronization problem about not matching between the drive system and response system parameters. In this paper, we research about three dimensional chaotic system synchronization control design, and apply linear system resilient control thought to three dimensional chaotic system synchronization, the design of the controller for the uncertainty of the controller itself has certain tolerance.

Keywords:-chaotic system; synchronization; Lyapunov functional; resilient control

I. INTRODUCTION

Chaos synchronization and its application in secure communication aroused extensive attention from the physics community, due to its extreme sensitivity and strong random for initial value in many fields of science technology, chaotic signal shows potential application value. At present, some scholars are studying flexible control problems of linear systems and continuous chaotic system, and having designed relevant flexible controller to solve the problem in the impact of controller uncertainty, and make it have tolerance for the impact of controller uncertainty. However, there are no results for such a controller design problem in three-dimensional chaotic system synchronization. Based on the previous relevant design ideological, and with my own ideas, this paper designs the corresponding controller for three-dimensional chaotic system, and realizes chaotic system synchronization.

II. SYSTEM DESCRIPTION AND SYNCHRONIZATION PROBLEM

This paper uses feedback synchronization method to realize three-dimensional chaotic system synchronization, so as to realize synchronization of two chaotic systems.

Considering multivariable feedback, giving chaotic system model, as follows:

$$\begin{cases} \dot{x} = -ax + by + yz \\ \dot{y} = cx - dy - xz \\ \dot{z} = ex - fz + gxy \end{cases} \quad (1)$$

Among them, $X = (x, y, z)^T \in R^3$ is System state, a, b, c, d, e, f, g is System parameter.

Using linear output feedback to realize driving system and response system synchronization. Considering uncertainty influence about the controller, response system equation has following form:

$$\begin{cases} \dot{x}_1 = -ax_1 + by_1 + y_1z_1 + (k_1 + \Delta k)(x - x_1) \\ \dot{y}_1 = cx_1 - dy_1 - x_1z_1 + (k_2 + \Delta k)(y - y_1) \\ \dot{z}_1 = ex_1 - fz_1 + gx_1y_1 + (k_3 + \Delta k)(z - z_1) \end{cases} \quad (2)$$

Among them, the feedback gain form is $(k + \Delta k)$, two kinds of cases: $\|\Delta k\| = 0$ and $\|\Delta k\| \leq \alpha$.

In synchronization system design, system model itself is uncertainty, and interference to drive signal in the transmission process is also an important problem. In order to outstand problems and convenient to analy, only considering the controller itself uncertainty conditions. Definiting system error term: $e_1 = x - x_1, e_2 = y - y_1, e_3 = z - z_1$, when $t \rightarrow \infty, e \rightarrow 0$, two systems achieve the synchronization, transform synchronization problem into error system zero stabilization problem.

III. SYNCHRONIZATION CONTROLLER DESIGN

A Precise k

The entire document should be in Times New Roman or Times font. Type 3 fonts must not be used. Other font types may be used if needed for special purposes.

Recommended font sizes are shown in Table 1. Chaotic synchronization, when k can realize accurately, $\|\Delta k\| = O(\|\bullet\|)$ is Euclidean norm)

$$\begin{cases} \dot{e}_1 = -ae_1 + be_2 + ze_2 + y_1e_3 - k_1e_1 \\ \dot{e}_2 = ce_1 - de_2 - ze_1 - x_1e_3 - k_2e_2 \\ \dot{e}_3 = ee_1 - fe_3 + g(y_1e_1 + x_1e_2) - k_3e_3 \end{cases} \quad (3)$$

Among the form, $e_1 = x - x_1, e_2 = y - y_1, e_3 = z - z_1$.

Constructing Lyapunov function:

$$L = \frac{1}{2}(e_1^2 + e_2^2 + \frac{1}{g}e_3^2) \quad (4)$$

L derivatives t :

$$\begin{aligned} \dot{L} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \frac{1}{g}\dot{e}_3e_3 \\ &= (-ae_1 + be_2 + ze_2 + y_1e_3 - k_1e_1)e_1 \\ &\quad + (ce_1 - de_2 - ze_1 - x_1e_3 - k_2e_2)e_2 \\ &\quad + \frac{1}{g}(ee_1 - fe_3 + g(y_1e_1 + x_1e_2) - k_3e_3)e_3 \\ &= -\frac{1}{g}\left(\frac{e}{2}e_1 - e_3\right)^2 - \left(\frac{y_1}{2}e_1 - e_3\right)^2 - \left(\frac{y}{2}e_1 - e_3\right)^2 \\ &\quad - \left(\frac{b}{2}e_2 - e_1\right)^2 - \left(\frac{c}{2}e_2 - e_1\right)^2 - \left(a + k_1 - \frac{e^2}{4g} - \frac{y^2 + y_1^2}{4}\right)e_1^2 \\ &\quad - \left(d + k_2 - \frac{b^2 + c^2}{4}\right)e_2^2 - \frac{1}{g}(f + k_3 - 2g - 1)e_3^2 \end{aligned}$$

Therefore, if $\dot{L} < 0$ is established, only making e_1^2, e_2^2, e_3^2 coefficient negative, that is satisfied:

$$a + k_1 - \frac{e^2}{4g} - \frac{y^2 + y_1^2}{4} > 0$$

$$d + k_2 - \frac{b^2 + c^2}{4} > 0$$

$$f + k_3 - 2g - 1 > 0$$

The error system (3) will be stable in the zero balance of state space, chaotic synchronization can be realized. When it satisfies the follows:

$$k_1 > \frac{e^2}{4g} + \frac{y^2 + y_1^2}{4} - a$$

$$k_2 > \frac{b^2 + c^2}{4} - d$$

$$k_3 > 2g + 1 - f$$

That is,

$$k_1 > \frac{e^2}{4g} + \frac{y^2 + y_1^2}{4} - a$$

$$k_2 > \frac{b^2 + c^2}{4} - d$$

$$k_3 > 2g + 1 - f$$

$\dot{L} < 0$.

According to Lyapunov stability theorem, the error system is asymptotically stable, namely drive system and response system achieve synchronization.

B Imprecise k

Assume Euclidean norm have bounded and the upper bound is known, namely $\|\Delta k\| \leq \alpha$. Among it, α is known normal number.

Drive system and response system produce the error system, which can be expressed as follows:

$$\begin{cases} \dot{e}_1 = -ae_1 + be_2 + ze_2 + y_1e_3 - (k_1 + \Delta k)e_1 \\ \dot{e}_2 = ce_1 - de_2 - ze_1 - x_1e_3 - (k_2 + \Delta k)e_2 \\ \dot{e}_3 = ee_1 - fe_3 + g(y_1e_1 + x_1e_2) - (k_3 + \Delta k)e_3 \end{cases} \quad (5)$$

Among the form, $e_1 = x - x_1, e_2 = y - y_1, e_3 = z - z_1$.

Constructing Lyapunov function:

$$L = \frac{1}{2}(e_1^2 + e_2^2 + \frac{1}{g}e_3^2) \quad (6)$$

L derivates time t :

$$\begin{aligned} \dot{L} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \frac{1}{g}\dot{e}_3e_3 \\ &= (-ae_1 + be_2 + ze_2 + y_1e_3 - k_1e_1)e_1 \\ &\quad + (ce_1 - de_2 - ze_1 - x_1e_3 - k_2e_2)e_2 \\ &\quad + \frac{1}{g}(ee_1 - fe_3 + g(y_1e_1 + x_1e_2) - k_3e_3)e_3 \\ &= -\frac{1}{g}\left(\frac{e}{2}e_1 - e_3\right)^2 - \left(\frac{y_1}{2}e_1 - e_3\right)^2 - \left(\frac{y}{2}e_1 - e_3\right)^2 \\ &\quad - \left(\frac{b}{2}e_2 - e_1\right)^2 - \left(\frac{c}{2}e_2 - e_1\right)^2 - \left(a + k_1 + \Delta k - \frac{e^2}{4g} - \frac{y^2 + y_1^2}{4}\right)e_1^2 \\ &\quad - \left(d + k_2 + \Delta k - \frac{b^2 + c^2}{4}\right)e_2^2 - \frac{1}{g}(f + k_3 + \Delta k - 2g - 1)e_3^2 \end{aligned}$$

Therefore, if making $\dot{L} < 0$ established, only needing to make e_1^2, e_2^2, e_3^2 coefficient negative, namely satisfies the follow forms:

$$\begin{aligned} a + k_1 + \Delta k - \frac{e^2}{4g} - \frac{y^2 - y_1^2}{4} &> 0 \\ d + k_2 + \Delta k - \frac{b^2 + c^2}{4} &> 0 \\ f + k_3 + \Delta k - 2g - 1 &> 0 \end{aligned}$$

The error system (5) will be stable in the zero balance of state space, chaotic synchronization can be realized. When it satisfies follow forms:

$$\begin{aligned} k_1 &> \frac{e^2}{4g} + \frac{y^2 + y_1^2}{4} - a - \Delta k \\ k_2 &> \frac{b^2 + c^2}{4} - d - \Delta k \\ k_3 &> 2g + 1 - f - \Delta k \quad (\|\Delta k\| \leq \alpha) \end{aligned}$$

Namely,

$$\begin{aligned} k_1 &> \frac{e^2}{4g} + \frac{y^2 + y_1^2}{4} - a - \alpha \\ k_2 &> \frac{b^2 + c^2}{4} - d - \alpha \\ k_3 &> 2g + 1 - f - \alpha \end{aligned}$$

$\dot{L} < 0$. According to Lyapunov stability theorem, the error system is asymptotically stable, namely drive system and response system achieve synchronization.

IV. CONCLUSIONS

This paper applies linear system resilient control thought to three-dimensional chaotic system synchronization, the designed controller has certain tolerance for the uncertainty of the controller itself. When the upper bound of the uncertainty is known, the paper gives a design proposal of feedback gain, and does correlative research for this three-dimensional chaotic system. When the upper bound of the uncertainty is unknown, it will be a follow-up research problem.

REFERENCES

- [1]. Li Wenlin. Use part state feedback chaotic synchronization of unified chaotic systems[J]. Control and Decision, 2008, 23(5):593-596.
- [2]. Liang Xiaomin, Nie Hong. Fuzzy system stability based on the switch controller[J]. Liaoning University of Petroleum and Chemical Journal, 2007, 27(2):57-61.
- [3]. Li Aiping, Yang Dongsheng, Meng \ Ziyi, Li Ying. Chaos system fuzzy adaptive sampling synchronization control research[J]. Journal of System Simulation, 2010, 22(8):1920-1924.
- [4]. Jianwen Feng, Shaohui Sun, Chen Xu, Yi Zhao and Jingyi Wang. The synchronization of general complex dynamical network via pinning control[J]. Nonlinear Dynamics, 2012, 67(2):1623-1633.
- [5]. Guan Xinping, He Yanhui, Wu Jing. Chaotic synchronization based on the elastic controller[J]. Physics journal, 2003, (11) :2718-2722.
- [6]. Hu Chengjun, Li Chuandong. Application of chaos synchronization index the insecret communication[J]. Computer Engineering, 2012,38 (2) : 148-150.
- [7]. Deng Kuibiao, Yu Simin. A class of chaotic system dynamic output feedback control[J]. Modern Electronics Technique, 2012,35 (2) : 69-71.
- [8]. Jin Tian, Zhang Hua. Statistical approach to weak signal detection and estimation using Duffing chaotic oscillators[J]. Science China Information Sciences. 2011, 54(11):2324-2337.
- [9]. Choon Ki Ahn. Chaos Synchronization of Nonlinear Bloch Equation Based on Input-to-State Stable Control[J]. Communications In Theoretical Physics. 2010, 2:308-312.
- [10]. Sun Yong Zheng, Ruan Jiong. Synchronization between two different chaotic systems with noise perturbation[J]. Chinese Physics. 2010, 7:150-155.
- [11]. Cui Chang. Chaos synchronization and its research progress in the secure communication[J]. Modern Electronics Technique, 2010,3:52-54.
- [12]. Guan Xinping, Fan Zhengping, Chen Cailian, Hua Changchun. Chaos control and its application in secure communication[M]. BeiJing, Defense Industry Press, 2002.10:226-271.