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Optimized Tuning of PI Controller for a Spherical Tank Level System Using New Modified Repetitive Control Strategy

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Abstract—This paper proposes a new tuning method of the PI controller based on New Modified Repetitive Control Strategy (NMRCS) approach. A non linear spherical tank liquid level system is considered here. The NMRC incorporates the idea of Repetitive Control (RC) which accomplishes perfect asymptotic set point tracking. The process dynamics of the level process in spherical tank are described by the differential equation and worst case model parameters are identified by influencing the step test technique. By utilizing relay feedback technique, the periodic reference signal of NMRCS is generated. From the input and output chattering signals of the NMRCS, optimized PI controller parameters are identified using Recursive Least Squares (RLS) fitting technique. Simulation results are endowed to demonstrate the efficiency of the proposed tuning method. A proof of robustness of the NMRCS is also analyzed.

Keywords—RLS, ZNTR, MRC, NMRC, Spherical Tank

I. INTRODUCTION

Proportional-Integral (PI) controllers have remained as the most commonly used controllers in the industry since its inauguration many decades ago. The main reason behind its popularity among engineers is its simplicity in tuning the parameters to achieve satisfactory performance in industrial applications. Many tuning approaches have evolved in tuning the controller since 1942 when Ziegler and Nichols [1] pioneered a unified systematic tuning approach in tuning the PI controller.

The Internal Model Principle [2] has played a major role in the development of the RC. According to this principle, the output tracks a class of reference signals without error only if the generator for references is integrated in the stable closed-loop system. The main benefit of repetitive control is that the tracking error decreases with increasing number of trials. In most cases the repetitive controller affect the stability of the system. To guarantee the stability of the repetitive control system, a NMRC is considered. The main idea associated with this paper is to use NMRC as a mechanism to derive the ideal control signal for processes with significant dead time to track a periodic reference sequence. This reference sequence, in the case of the usual RC applications to robotics and motion systems where there is little time delay, can be the natural repetitive signal for the control system to execute the repetitive operations. In the case of process control applications, the frequency of the reference and repetitive sequence is chosen to be at the ultimate frequency [3]. The novelty of RC is provoked by a power supply regulation problem in proton synchrotron accelerator [4]. Recently a variety of successful applications of RC have sprung up, including high-speed motion tracking problem [5, 6], speed control of DC motor [7].

This paper is organized as follows. In section 2, the mathematical model and controller parameter of spherical tank level system are summarized. The structure of Modified Repetitive Control Strategy and New Modified Repetitive Control Strategy are detailed in section 3 and 4. Simulation results are analyzed in section 5 to exemplify the better performance of the NMRC in closed loop. Concluding remarks are given finally in section 6.

II. DYNAMIC MODEL OF THE SPHERICAL TANK LEVEL SYSTEM

Figure 1 shows the spherical tank level system, in which the control input f_{in} is being the in- flow rate (m^3/s) and the output x is the fluid level (m) in the spherical tank.

Let, r, d_0 and x_0 be the radius of spherical tank, thickness (diameter) of pipe and initial level. Assume ‘r $_{surface}$’ is radius on the surface of the fluid which varies with respect to the level of fluid in the tank.

The dynamic model of the spherical tank is given as

$$\frac{\delta}{\delta t}\left[\int_0^{x_1} A(x)\,\delta x\right] = f_{in}(t) - a\sqrt{2g(x - x_0)} \qquad (1)$$

where $A(x)$ is the area of cross section of tank (i.e) $A = \pi\,(2rx - x^2)$

a is the cross sectional area of the pipe (i.e) $a = \pi\left(\frac{d_0}{2}\right)^2$

Rewrite the above equation at time, t + δt

$$A(x)\,\delta x = f_{in}\,\delta t - a\sqrt{2g(x - x_0)}\,\delta t \qquad (2)$$

Combine the equations (1) and (2) , we have

$$\frac{\delta x}{\delta t} = \frac{\text{fin}\ \delta t - \frac{\pi . d_0^2}{4}\sqrt{2g\,(x-x_0)}}{\pi\,(2rx - x^2)\,)} \qquad (3)$$

By applying $\lim_{d \to 0}$ in equation (3), we have $\frac{\delta x}{\delta t} = \frac{dx}{dt}$

Therefore

$$\frac{dx}{dt} = \frac{\text{fin}\ \delta t - \frac{\pi . d_0^2}{4}\sqrt{2g\,(x-x_0)}}{\pi\,(2rx - x^2)} \qquad (4)$$

Equation (4) represents the dynamic model of the spherical tank level system.

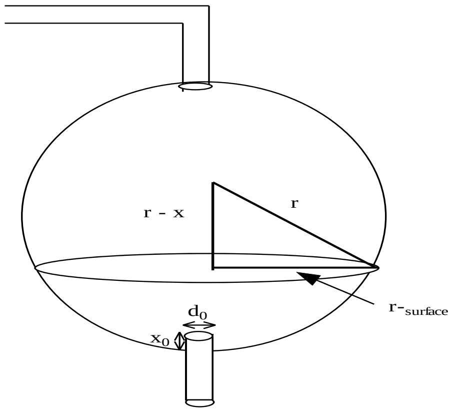

Fig. 1 Spherical Tank Level System

III. IDENTIFICATION OF MODEL PARAMETERS AND CONTROLLER SETTINGS

The spherical tank level system is kept at a steady state of different operating point of 20%, 40%, 60% and 80%. A step size of 5% level for each operating point is applied and the variation of level against time for each operating point is recorded separately until a new steady state is attained. From the recorded data, the model parameters such as process gain (K_p) time constant (τ_p) and delay (t_d) are computed and tabulated in table 1. From the table, the worst case model parameters such as larger process gain (K_p), smaller time constant (τ_p) and larger delay (t_d) are considered.

***Table I*:** Identification of Worst case Model parameters

Operating Point (%)	K_p	τ_p	t_d
20	0.864	**96.45**	**17.85**
40	1.23	219	8
50	1.38	252.75	7.75
60	1.52	258.9	8.9
80	**1.76**	174.5	13.25

The identified worst case model parameters for the spherical tank system is given by

$$G(s) = \frac{1.76}{96.45s+1}\, e^{-17.85s} \qquad (5)$$

Based on the worst case model parameters, PI mode controller settings (Kc = 2.763 and (Ki =0.046) are computed by considering Z-N [9-10] open loop tuning rule (ZNTR).

IV. REPETITIVE CONTROL

The basic RC is a model free approach to achieve a better system performance of systems over a finite time interval. It is proposed by Inoue et al. [4] for use in the control of proton synchrotron magnetic supply. It is later developed to be used in applications that required repetitive operations such as pick and place operations in robotics [8]. The main idea associated with the use of the RC is to enhance the system performance by using the information from the previous cycle in the next cycle over a period of time until the performance achieved is considered to be satisfactory. Figure 2 shows the usual RC configuration.

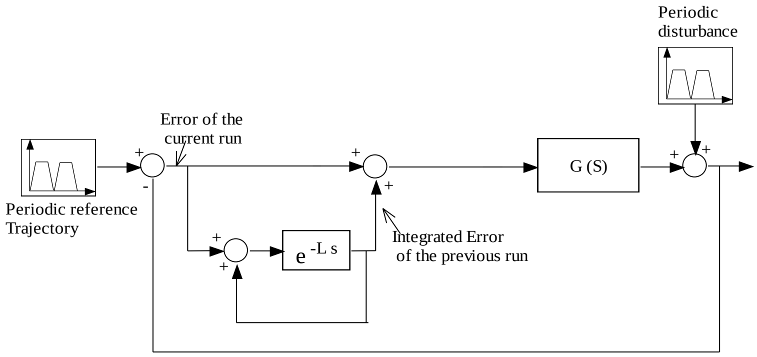

Fig. 2 Basic Repetitive Control Strategy

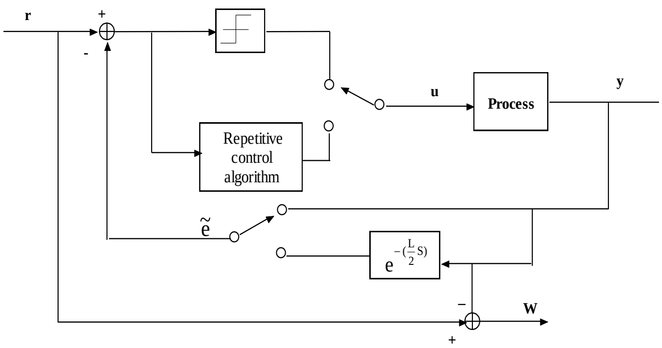

Fig. 3. Repetitive control algorithm with relay feedback.

An RC approach is used to design PI controllers [9].While this configuration works well for robotic and servo control applications with a moderately small time delay, it will fail in the area of process control applications and requirements due to the typical presence of time delay and large phase lag. In order for the RC scheme to be applicable to processes with long dead time, the basic form is modified by adding a time delay block to the feedback path as shown in figure 3.

The relay feedback configuration as shown in figure 3 is first applied to the process to obtain the repetitive excitation signal. Then the process is switched to RC mode. Since the ultimate frequency, y and u is out of phase by π, the additional delay block $e^{(-L/2)\,s}$ (where L is period of reference input) is introduced to align the phase of ẽ and u to remain in phase so that the RC remains valid even in the occurrence of large delay. Once the suitable tracking performance is accomplished through the Iterative Learning Control Strategies, the signals W (error signal) and U (control signal) are attained and exercised to find the optimum PI controller parameters by using recursive least square algorithm (RLS). Here P-type update law is adopted for the RC.

V. NEW MODIFIED REPETITIVE CONTROL (NMRC)

The modeling of spherical tank is uncertain for high frequency signals. Due to uncertainty, noise will have a great influence on the response, which affects the stability of the process. To overcome this problem, a low-pass filter (Q) and learning filter (L_f) is added to the existing RC control loop and to ensure system stability. This structure is known as Modified Repetitive Control Strategy (MRCS) is proposed by Hara et al [10] as shown in figure 4. The Key factors such as Learning filter (L_f) and Robustness filter (Q) in the learning control strategy are identified using Zero Phase Error Tracking Control (ZPETC) technique and frequency method respectively. To improve the stability of MRCS a New Modified Repetitive Control Strategy (NMRCS) is implemented and as shown in Figure 5. Where V is the stable rational factor (0 < V< 1).

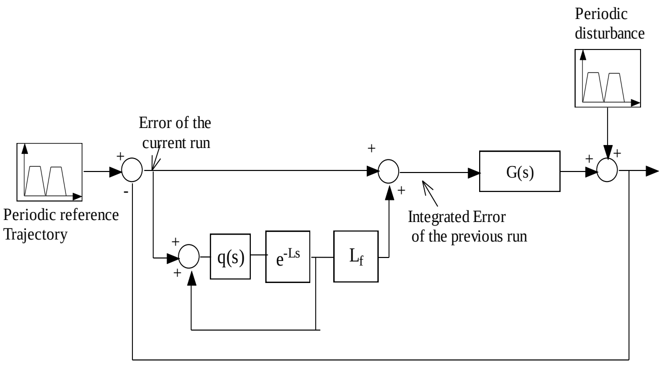

Fig.4 Modified Repetitive Control Strategy

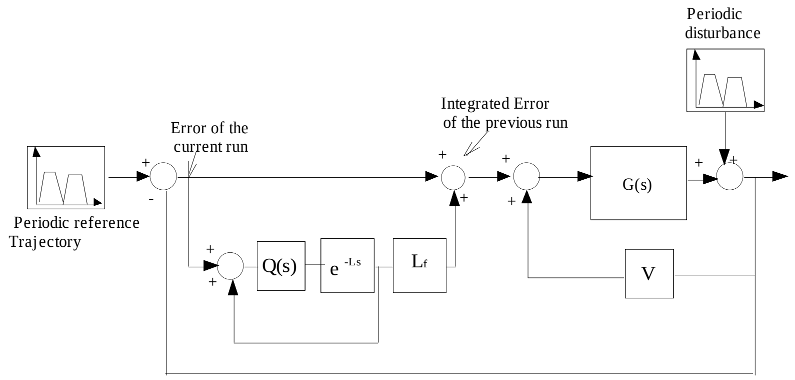

Fig 5 New Modified Repetitive Control Strategy

A. Design procedure and guidelines

1) Learning filter design:

The Learning filter (L_f) is nothing but the inverse of process-sensitivity (T) = $\frac{GC}{1+GC}$ i.e L_f = KT $^{-1}$. Due to the unstability and non-proper characterisitics of inverse complementary sensitivity, L_f can not be act as a filter. This problem is overcome by adapting Zero Phase Error Tracking Controller (ZPETC) algorithm [11].The evaluation of ZPETC method is done by comparing the bode plot of the original inverse complementary sensitivity and the approximated inverse complementary sensitivity as shown in figure 6. It seems that the magnitude and phase plots of both the cases are same. In this the phase plot, the phase caused by the delay has been taken into account.

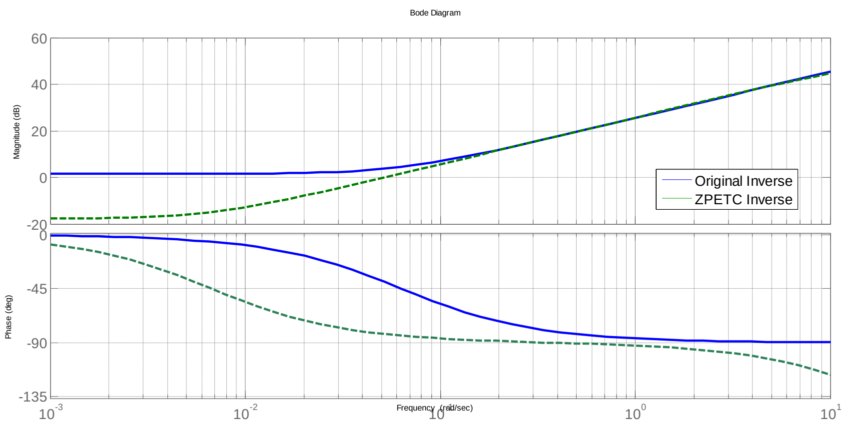

Fig.6 Bode plot of Inverse Complementary sensitivity

2) Low pass filter design:

A first order continuous time low pass filter is considered here. i.e Q(s) = $\frac{\omega_c}{s+\omega_c}$, where ω_c is the cut-off frequency in rad /sec. The cut-off frequency is obtained from the Bode plot of the spherical tank system (refer figure. 7). In this study, it is found to be 0.01

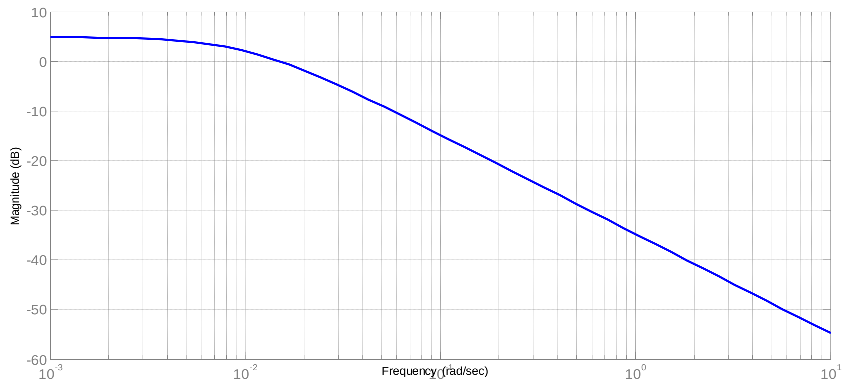

Fig.7 Magnitude plot of the spherical tank system

3) Rational factor design:

Using an optimization technique the value of stable rational factor (V) has been chosen corresponding to minimum tracking error and to enhance the stability of NMRCS. The identified value of V for spherical tank system is 0.1.

VI. RECURSIVE LEAST SQUARE ESTIMATION (RLS) ALGORITHM

The RLS fitting method [12] is applied to the input and output chattering signals of the NMRC- relay construct to yield the gains of the optimized PI controller.

The PI controller is described by

$$u(t) = kc\, e + ki \int_0^t e\, dt \quad (8)$$

The above equation can be written in a matrix form as

$$u(t) = \begin{bmatrix} e & \int_0^t e \end{bmatrix} \begin{bmatrix} kc \\ ki \end{bmatrix} \quad (9)$$

The equation 8 is written in the linear in the parameters form

$$u(t) = \theta(t)\, \varphi^T \quad (10)$$

Where

$$\theta(t) = \begin{bmatrix} kc \\ ki \end{bmatrix} \text{ and } \varphi^T = \begin{bmatrix} e & \int_0^t e \end{bmatrix} \quad (11)$$

The RLS algorithm with a time varying forgetting factor can be directly used here as vt and T are available, the update of θ(t) can be expressed as

$$\theta(t) = \theta(t-1) + k(t)\,\varepsilon(t) \quad (12)$$

where $\theta(t-1)$ refers to the controller settings identified during the last cycle, $\varepsilon(t)$ and $k(t)$ are the error signal and Kalman gain vector, where

$$\varepsilon(t) = u(t) - \phi^T \theta(t-1) \quad (13)$$

$$k(t) = p(t-1)\phi(\lambda I + \phi^T p(t-1)\phi)^{-1} \quad (14)$$

$$p(t) = (I - k(t)\phi^T)p(t-1)/\lambda \quad (15)$$

where λ is a forgetting factor ($0 < \lambda < 1$). There are two matrices to be initialized for the recursive algorithm and $p(0)$ and $\theta(0)$. It is usual to initialize $p(0)$ such that $P_0 = \alpha I$, where α is a large number (10^4– 10^6) and I is the identity matrix. $\theta(0)$ is set to be the gains of the PI controller before tuning. The robust control configuration, comprising of the relay and the NMRC controller, puts a high gain in the loop and ensures satisfactory closed-loop performance. Although it incurs a chattering phenomenon, the chattering signals are used to tune PI controller parameters.

VII. RESULTS AND DISCUSSION

To analyze the effectiveness of the NMRCS based PI tuning method, design parameters such as learning filter L_f , robustness filter Q and rational factor V are designed initially by considering the spherical tank level process model equation 5 and it is given by

$$L_f(s) = \frac{-383.8s+2.677}{s+20}, \quad Q(s) = \frac{0.01}{s+0.01} \text{ and } V = 0.1 \quad (16)$$

In addition to that, the learning gain (K) and it is chosen as 0.1. The relay feedback arrangement, as shown in figure.3, is first applied to the process to get the repetitive excitation signal for the NMRCS as shown in figure 8. Then the process is switched to NMRCS set up with the repetitive excitation signal.

Figure 9 and 10 shows the reference input r and process output y of MRCS and NMRCS which are 180 out of phase. After tracking performance is consummate through the repetitive control strategies, the signals W and U are attained (refer figure 11figure 12). By using the recursive least squares algorithm, the signals W and U are exercised to find the optimum PI controller parameters (Kc = 3.008 and Ki = 0.06237). Likewise PI controller settings for MRCS are computed. Controller parameters for all cases are reported in table 2.

Table II: PI Controller Parameters

Control loop	K_c	K_I
Conventional PI	2.7	0.04
MRCS	2.892	0.057
NMRCS	3.008	0.06237

Simulation run of spherical tank level system is carried out with NMRCS based PI values. Initially the tank is maintained at 40 % of operating level. After that, a step size of ±5% of level is applied to control loop. Similar test runs of MRCS based PI and ZN based PI are carried out and the responses of all the three cases are recorded in figure 13 and figure14. From the results, the performances of each control scheme are analyzed in terms of ISE and IAE and the performance indices are tabulated in table 3. The results proven that NMRC based PI controller gives better performance than the others.

To test the robustness of the NMRC controller, simulation runs is carried out at another operating level of 60%. The responses are traced in figure 15 and figure 16 and their performance indices are tabulated in table 4. From the table, it is observed that NMRCS based PI mode gives superior performance than the MRCS based PI mode and ZN based PI mode.

Table III: Performance Indices of Spherical tank level process at operating range of 40%

Controller	**Set point change (+5)**		**Set point change (-5)**	
	ISE	**IAE**	**ISE**	**IAE**
ZN-PI	667.6	316.4	643.6	296
MRC-PI	633.6	305.5	609.6	285.6
NMRC-PI	610.2	296.3	587	277

Table IV: Performance Indices of Spherical tank level process at operating range of 60%

Controller	**Set point change (+5)**		**Set point change (-5)**	
	ISE	**IAE**	**ISE**	**IAE**
ZN-PI	661.1	315.8	673.9	325.4
MRC-PI	629.6	304.3	641.5	313.8
NMRC-PI	606.8	294.8	618.3	304.2

VIII. CONCLUSION

In this paper, a new method is developed and implemented for the design of the PI controller based on a New Modified Repetitive Control approach. This control structure is more appropriate to the system having a large delay. The proposed method requires the input of periodic reference trajectories which is obtained from a relay feedback test. By using this test, the periodic reference signal is generated and utilized as the input of NMRCS control loop. From the input and output signals of the NMRCS, the optimized PI controller parameters are identified using recursive least square fitting technique. The NMRC based PI is implemented in a level control of spherical tank system. A comparison of this structure with other control strategies such as conventional and MRCS is also made in this work. Simulations results are furnished to illustrate the efficiency of the proposed method.

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REFERENCES

[1]. J.B.Ziegler and N. B. Nichols, Optimum settings for automatic controllers, ASME Transactions. 64 (1942) 759-768.

[2]. B.A.Francis,W.M.Wonham, Internal Model Principle for Linear Multivariable Regulators, Journal of Applied Mathematics and Optimization. (1975) 170-194.

[3]. Kok Kiong Tan, Kok Yong Chua, Shao Zhao, Su Yang, Ming Tan Tham, Repetitive control approach towards automatic tuning of Smith predictor controllers, ISA Transactions. 48 (2009) 16-23.

[4]. T.Inouc, S.Iwai, and M.Nakano, High accuracycontrol of a Proton synchrotron magnet power supply, in proc.8th IFAC World congress. 3 (1981) 3137-3142.

[5]. H.Fujimoto, F.Kawalrami,and S.Kondo, Multir-ate repetitive control and applications verification of switching scheme by HDD and visual servoing, in Proc. American Control Conference. 4 (2003) 2875-2880.

[6]. H.Fujimoto and Y.Hori, High speed robust visual servoing based on intersample estimation and multirate control,7th international Workshop on Advanced Motion Control. (2002) 104-109.

[7]. M.Vijayakarthick, S.Sathishbabu and P.K.Bhaba, Real time implementation of Modified Repetitive Control Strategy in a DC motor, IEEE proceedings on ICARCV. (2010) 109-113.

[8]. K.Kaneko, R.Horowitz, Repetitive and adaptive control of robot manipulators with velocity estimation, IEEE Transactions on Robotics and Automation. 13 (1997) 204-217.

[9]. Kok Kiong Tan, Kok Yong Chua, Shao Zhao, Su Yang, Ming Tan Tham. Repetitive control approach towards automatic tuning of smith predictor controllers, ISA Transactions. 48 (2009) 16-23.

[10]. S.Hara,Y.Yamamoto,T.Omata and M.Nakano, Repetitive Control System -A New Type Servo System for Periodic Exogenous Signals, IEEE Transactions on Automatic Control. 33 (1998) 659-668.

[11]. M.Tomizuka, Zero-Phase Error Tracking Algorithm for Digital Control, Journal of Dynamic Systems, Measurement and Control, Trans. of ASME. 109(1) (1987) 65-68.

[12]. K. K. Tan, R. Ferdous and S. Huang, Closed-loop automatic tuning of PID controller for nonlinear systems, Chemical Engineering Science. 57 (2002) 3005-3011.

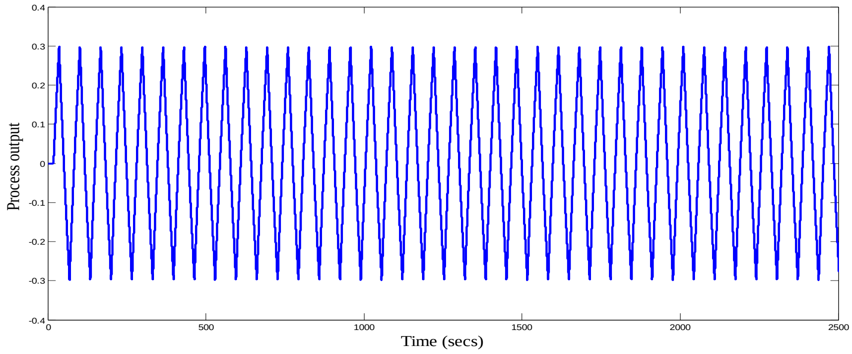

Fig.8 Output response of the process under relay feedback

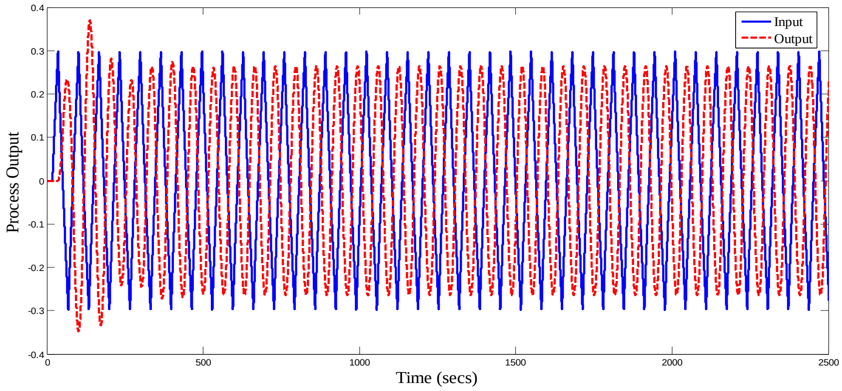

Fig.9 Input and Output of the MRC structure

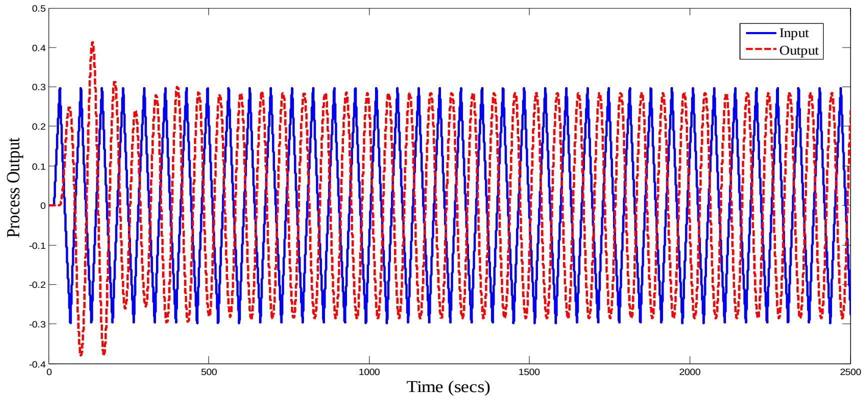

Fig.10 Input and Output of the NMRC structure

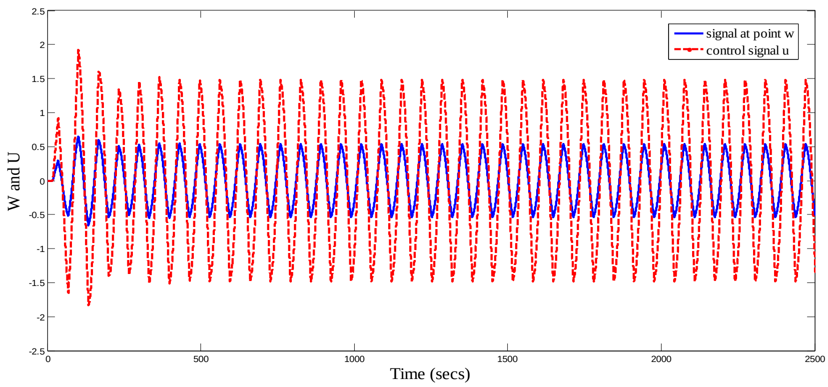

Fig.11 U and W signals based on MRC structure used for identification of controller parameters

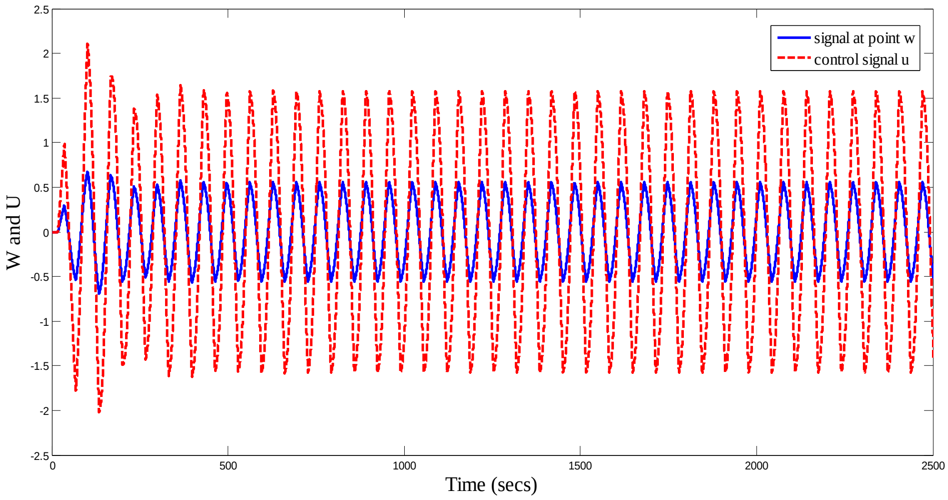

Fig.12 U and W signals based on MRC structure used for identification of controller parameters

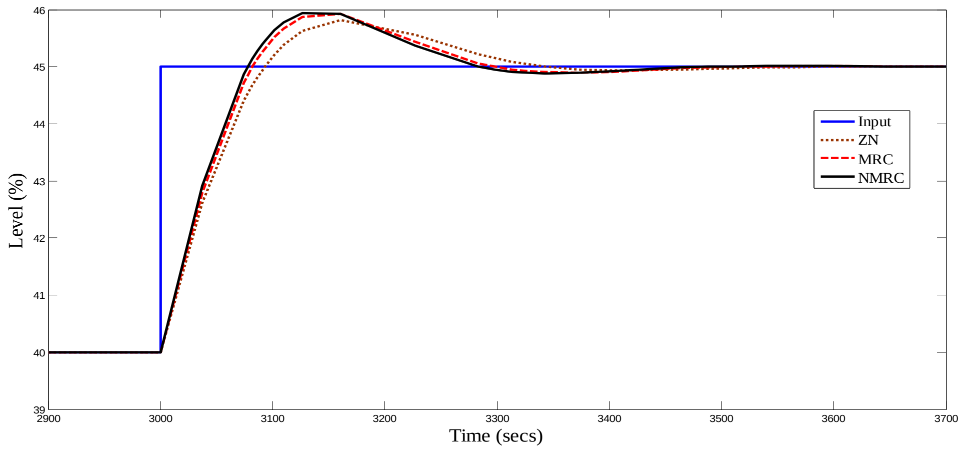

Fig.13 Tracking response of different controllers at operating range of 40 % with 5 % step change

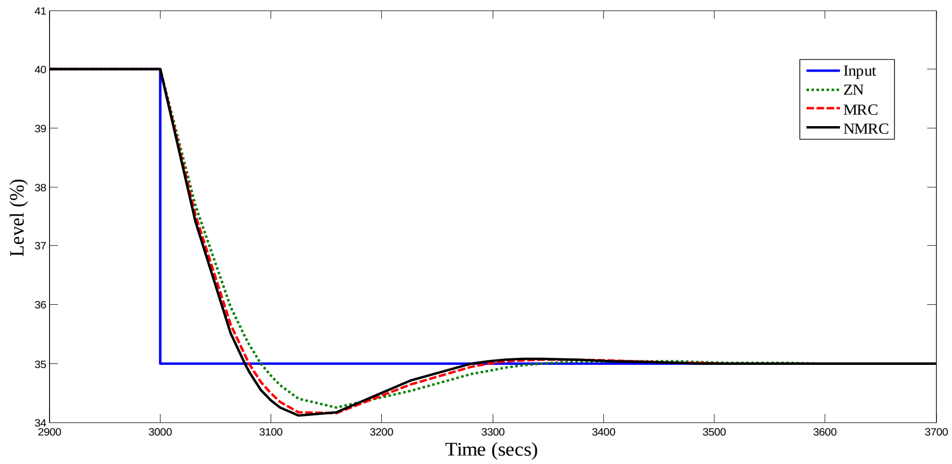

Fig.14 Tracking response of different controllers at operating range of 40 % with -5 % step change

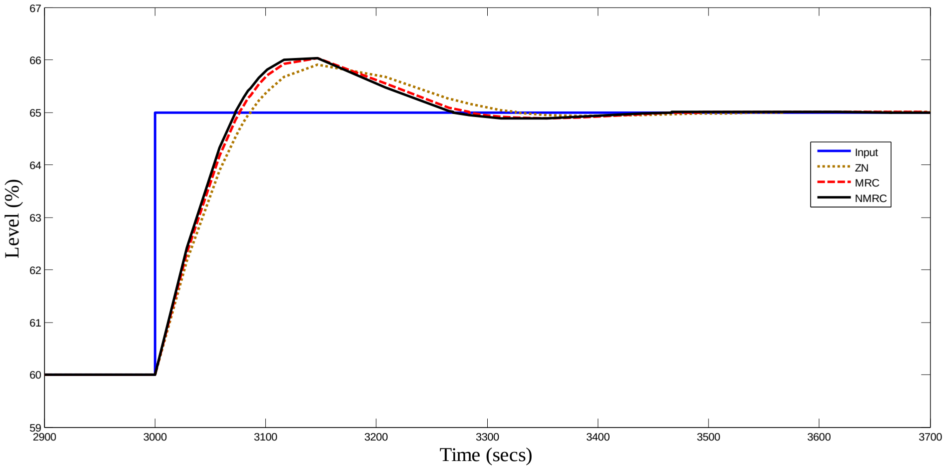

Fig.15 Tracking response of different controllers at operating range of 60 % with 5 % step change

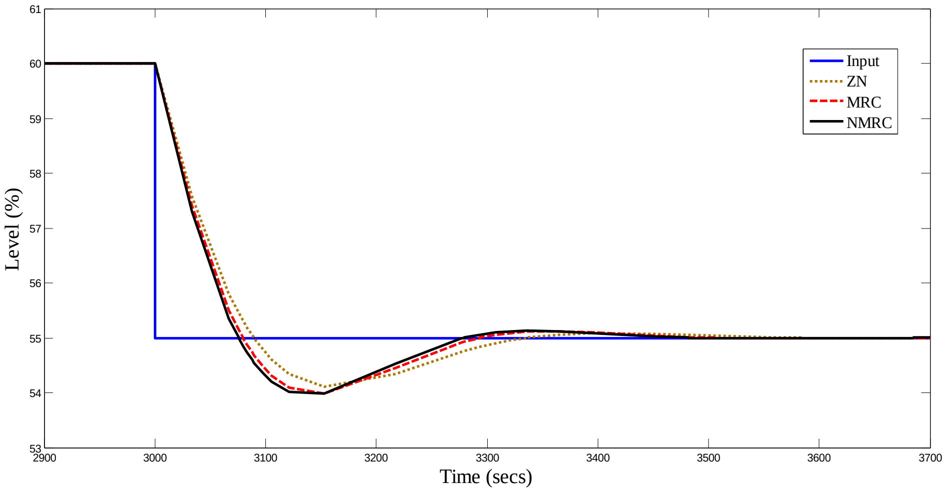

Fig.16 Tracking response of different controllers at operating range of 60 % with -5 % step change