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Specially Structured Two Stage Flow Shop Scheduling To Minimize the Rental Cost, Processing Time, Set up Time Are Associated With Their Probabilities Including Job Block Criteria And Job Weightage

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Abstract*—The present paper is attempt to develop a new heuristic algorithm, an alternative to the traditional algorithm proposed by Johnson's (1954) to find the optimal sequence to minimize the utilization time of the machines and hence their rental cost for two stage specially structured flow shop scheduling under specified rental policy in which processing times and set up time are associated with their respective probabilities, including job block criteria. Further jobs are attached with weights to indicate their relative importance. The proposed method is very simple and easy to understand and also provide an important tool for the decision maker. Algorithm is justified by numerical illustration.

Keywords*—Specially structured flow shop scheduling. Rental policy, Processing time, weightage of jobs, Set up, Job block.

I. INTRODUCTION

Scheduling theory deals with formulation and study of various scheduling models. Some widely studied classical models comprise single machine, parallel machine, flow shop scheduling, job shop scheduling, open shop scheduling etc. The objective of flow shop scheduling problem is to find a permutation schedule that minimizes the maximum completion time of a sequence. Scheduling has become a major field with in operation research with several hundred publications appearing each year. The majority of scheduling research assumes set up as negligible or part of processing time. While this assumption adversely effects solution quality for many application which require explicit treatment of setup. Johnson [9] first of all gave a method to minimize the make span for n-jobs, two machine scheduling problems. Gupta J.N.D. [7] gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Gupta [5] studied specially structured flow shop problem to minimize the rental cost of the machine under predefined rental policy in which the probabilities have been associated with processing time with weightage of jobs and job block criterion. Yoshida and Hitomi [17] further considered the problem with set up time. The basic concept of equivalent job for a job block has been introduced by Maggu & Das [10]. Singh T.P. and Gupta Deepak [13] studied the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. The work was developed, Chander Shekheran [3], Bagga & P.C. [2] and Gupta Deepak et. al [14] by considering various parameters.

This paper is an attempt to extend the study made by Gupta & Singla [5] by introducing set up time separated from processing time. Thus the problem discussed in this paper become wider and very close to practical situation in manufacturing/ process industry. We have obtained an algorithm which gives minimum possible rental cost while minimizing total utilization time.

II. PRACTICAL SITUATION

The practical situation of specially structured flow shop scheduling occur in our day to day working, in banking, offices, educational institutions, factories and industrial concern e.g In a readymade garment manufacturing plant which has mainly two machines. viz, cutting machine and sewing machine , in which the processing time of jobs on 2^{nd} machine(sewing machine) will always be greater than the processing time of jobs on first machine(cutting machine). Moreover different quality of garment are to be produced with relative importance i.e. weight of jobs become significant. Various practical situations occur in real life when one has got the assignment but does not have one's own machine or does not have enough money to purchase machine. Under such circumstances the machine has to be taken on rent in order to complete the assignment. Rental of various equipments is an affordable and quick solution for a businessman, a manufacturer or a company, which presently constrained by the availability of limited funds due to recent global economic recession. Renting enables saving working capital, gives option for having the equipment and allows up-gradation to new technology. Further the priority of one job over the other may be significant due to some urgency or demand of one particular type of job over other. Hence the job block criteria become important.

III. NOTATIONS

S : Sequence of jobs 1, 2, 3,….,n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------ r.
M_j : Machine j, j= 1,2.
a_{ij} : Processing time of i^{th} job on machine M_j
s_{ij} : Set up time of i^{th} job on machine M_j
p_{ij} :Probability associated to the processing time a_{ij}
q_{ij} : Probability associated to the processing time s_{ij}
A_{ij} : Expected processing time of i^{th} job on machine M_j
S_{ij} : Expected set up time of i^{th} job on machine M_j

$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
w_i : weight of i^{th} job.
β : Equivalent job for job-block(k, *m)*
G_i : weighted flow time of i^{th} job on machine M_1.
H_i : weighted flow time of i^{th} job on machine M_2.
$U_j(S_k)$: Utilization time for which machine M_j is required.
Cj : Renal cost per unit time of j^{th} machine.
$R(S_k)$: Total rental cost for the sequence S_k of all machine

IV. DEFINITION

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = max\ (t_{i-1,,j},+ S_{i-1,,j,}\ t_{i,,j-1}) + A_{ij}\ ;\ \ j \geq 2.$
where Aij=*Expected* processing time of i^{th} job on j^{th} machine. S_{ij}= Expected set up time of i^{th} job on j^{th} machine.

V. RENTAL POLICY (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

VI. PROBLEM FORMULATION

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let A_{ij} & S_{ij} respectively be the expected processing time and set up time of i^{th} job on j^{th} machine. Let w_i be weight of the i^{th} job. β = (k, m) be equivalent job for job block (k,m).Our aim is to find the sequence $\{S_k\}$ of jobs which minimize the rental cost of the machines while minimizing the utilization time of machines.The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1				Machine M_2				Weight of jobs
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	wi
1	a_{11}	p_{11}	s_{11}	q_{11}	a_{12}	p_{12}	s_{12}	q_{12}	w_1
2	a_{21}	p_{21}	s_{21}	q_{21}	a_{22}	p_{22}	s_{22}	q_{22}	w_2
3	a_{31}	p_{31}	s_{31}	q_{31}	a_{32}	p_{32}	s_{32}	q_{32}	w_3
-	-	-	-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	a_{n2}	p_{n2}	S_{n2}	q_{n2}	w_n

Table -1

Mathematically, the problem is stated as:
Minimize $U_2(S_k)$ and hence

Minimize $R\left(S_k\right)=\sum_{i=1}^{n} A_{i1}\times C_1+U_j\left(S_k\right)\times C_2$

Subject to constraint: Rental Policy (P).
i.e. our objective is to minimize utilization time of machine and hence rental cost of machines.

VII. THEOREM

If $A_{i1} \leq A_{j2}$ *for all i, j, i ≠ j,* then $k_1, k_2 \ldots\ldots k_n$ is a monotonically decreasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $A_{i1} \leq A_{j2}$ for all $i, j\ ,\ i \neq j$
i.e., max $A_{i1} \leq$ min A_{j2} for all $i, j\ ,\ i \neq j$

Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Therefore, we have $k_1 = A_{11}$

Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \leq A_{11}$ ($\because A_{21} \leq A_{12}$)

$\therefore k_1 \leq k_2$

Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$

$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \leq k_2$ ($\because A_{31} \leq A_{22}$)

Therefore, $k_3 \leq k_2 \leq k_1$ or $k_1 \geq k_2 \geq k_3$.

Continuing in this way, we can have $k_1 \geq k_2 \geq k_3 \geq \ldots\ldots \geq k_n$, a monotonically decreasing sequence.

Corollary 1: The total rental cost of machines is same for all the sequences, if

$A_{i1} \leq A_{i2}$, for all i, j, $i \neq j$.

Proof: The total elapsed time $T(S) = \sum_{i=1}^{n} A_{i2} + k_1 = \sum_{i=1}^{n} A_{i2} + A_{11}$.

It implies that under rental policy P the total elapsed time on machine M_2 is same for all the sequences thereby the rental cost of machines is same for all the sequences.

VIII. THEOREM

If $A_{i1} \geq A_{j2}$ for all $i, j, i \neq j$, then $K_1, K_2 \ldots\ldots K_n$ is a monotonically increasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \geq A_{j2}$ for all $i, j, i \neq j$ i.e., min $A_{i1} \geq$ max A_{j2} for all $i, j, i \neq j$

Here $k_1 = A_{11}$

$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \geq k_1$ ($\because A_{21} \geq A_{j2}$)

Therefore, $k_2 \geq k_1$.

Also, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22})$

$= k_2 + (A_{31} - A_{22}) \geq k_2$ ($\because A_{31} \geq A_{22}$)

Hence, $k_3 \geq k_2 \geq k_1$.

Continuing in this way, we can have $k_1 \leq k_2 \leq k_3 \ldots\ldots \leq k_n$, a monotonically increasing sequence.

Corollary 2: The total elapsed time of machines is same for all the possible sequences, if $A_{i1} \geq A_{j2}$ for all i, j, $i \neq j$.

Proof: The total elapsed time

$$T(S) = \sum_{i=1}^{n} A_{i2} + k_n = \sum_{i=1}^{n} A_{i2} + \left(\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + \left(\sum_{i=1}^{n} A_{i2} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + A_{n2}$$

Therefore total elapsed time of machines is same for all the sequences.

IX. ASSUMPTIONS

1. Jobs are independent to each other. Let n jobs be processed thorough two machines M_1 and M_2 in order M_1M_2
2. Machine breakdown is not considered.
3. Pre-emption is not allowed.
4. Weighted flow time must has the following structural relation

   i.e. Either $G_i \geq H_i$

   or $G_i \leq H_i$ for all i

X. ALGORITHM

Step 1: Calculate the expected processing times, $A_{ij} = a_{ij} \times p_{ij}$; $S_{ij} = s_{ij} \times q_{ij}$

Step 2: Compute $A'_{i1} = A_{i1} - S_{i2}$

$A'_{i2} = A_{i2} - S_{i1}$

Step 3: Calculate weighted flow shop time G_i & H_i as follow

If min $(A'_{i1}, A'_{i2}) = A'_{i1}$

Then $G_i = \dfrac{(A'_{i1} + w_i)}{w_i}$ $\quad H_i = \dfrac{A'_{i2}}{w_i}$

And

If min $(A'_{i1}, A'_{i2}) = A'_{i2}$

Then $G_i = \dfrac{A'_{i1}}{w_i}$ $\quad H_i = \dfrac{(A'_{i2} + w_i)}{w_i}$

Step 4: Take equivalent job β = (k,m) and calculate processing time G_β and H_β on the guide lines of Maggu & Dass (1977) as follows:
$G_\beta = G_k + G_m - min\ (G_m,H_k)$
$H_\beta = H_k + H_m - min\ (G_m,H_k)$

Step 5: Define a new reduced problem with processing time G_i & H_i as defined in Step 3 and jobs (k,m) are replaced by single equivalent job β with processing time G_β & H_β as defined in step 4.

Step 6: Check the structural relationship
Either $G_i \geq H_i$
or $G_i \leq H_i$, for all i
if the structural relation hold good go to Step 6 other wise modified the problem..
Step 7: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position and go to step 10 otherwise go to step 8.
Step 8: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1 call this difference as G_i. also take the difference of processing time of job J_n on M_2 from job J_{n-1} (say) having next minimum processing time on M_2 . Call the difference as G_2.
Step 9: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.
Step 10: Arrange the remaining (n-2) jobs between 1st job & last job in any order, thereby we get the sequences $S_1, S_2 \ldots S_r$.
Step 11: Compute in - out table for any one (say S_1) of the sequence $S_1, S_2, \ldots S_n$.
Step 12: Compute the total completion time $CT(S_1)$.

Step 13: Calculate utilization time U_2 of 2nd machine where
$U_2(S_1) = CT(S_1) - A_{i1}(S_1)$;
Step 14: Find rental cost

$$R(S_1) = \sum_{i=1}^{n} A_{i1}(S_1) \times C_1 + U_2(S_1) \times C_2$$

where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

XI. NUMERICAL ILLUSTRATION

Consider 5 jobs, 2 machines problem to minimize the rental cost. The processing times set up times and weight in jobs are given in the following table. Let β = (2,4) as equivalent job block. The rental cost per unit time for machines M_1 and M_2 are 10 units and 7 units respectively.

Jobs	Machine M_1				Machine M_2				Weight of jobs
I	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	Wi
1	140	0.2	3	0.3	90	0.2	2	0.3	1
2	160	0.3	4	0.2	110	0.1	3	0.1	2
3	130	0.2	2	0.1	70	0.2	1	0.2	3
4	180	0.2	6	0.2	80	0.2	5	0.2	1
5	220	0.1	5	0.2	50	0.3	4	0.2	2

Table :2

Solution : As per step 1: The expected processing time & expected set up times for machines M_1 and M_2 are as follow

Jobs	Machine M_1		Machine M_2		Weight of jobs
i	A_{i1}	S_{i1}	A_{i2}	S_{i2}	Wi
1	28.0	0.9	18.0	0.6	1
2	48.0	0.8	11.0	0.3	2
3	26.0	0.2	14.0	0.2	3
4	36.0	1.2	16.0	1.0	1
5	22.0	1.0	15.0	0.8	2

Table : 3

As per step 2: Expected flow time for two machines M_1 and M_2 as follow :

Jobs	Machine M_1	Machine M_2	Weight
i	A_{i1}'	A_{i2}'	w_i
1	27.4	17.1	1
2	47.7	10.2	2
3	25.8	13.8	3
4	35.0	14.8	1
5	21.2	14.0	2

Table : 4

As per step 3: Weighted flow time for machines M_1 and M_2 as follow :

Jobs	Machine M_1	Machine M_2
I	G_i	H_i
1	27.40	18.1
2	23.85	6.1
3	08.60	5.6
4	35.00	15.8
5	10.60	8.0

Table : 5

As per step 5: the new reduced problem become as under:

Jobs	Machine M_1	Machine M_2
I	G_i	H_i
1	27.40	18.1
B	52.74	15.8
3	08.60	6.0
5	10.60	8.0

Table : 6

Here, $G_i \geq H_i$ for all *i*.
As per step 7 max G_i = 52.74 which is for job 2 i.e. J_1 =2
And min H_i = 5.6 which is for job 3 i.e. J_n = 3.
Since $J_1 \neq J_n$. we put J_1 = 2 on the first position
And J_n = 3 on the last position
Therefore the optimal sequences are S_1= 2 – 1 – 5 – 3 = 2 – 4 – 1 – 5 – 3 .
S_2= 2 – 5 – 1 – 3 = 2 – 4 – 5 – 1 – 3
Due our structural conditions the total elapsed time is same for all these 2 possible sequences S_1, S_2; say for S_1 = 2 – 4 – 1 – 5 – 3 is :

Jobs	Machine M_1	Machine M_2
I	In-Out	In-Out
2	0.0 – 48.0	48.0 – 59.0
4	48.0 – 84.8	84.8 – 100.8
1	86.0 – 114.0	114.0 – 132.0
5	114.9 – 136.9	136.9 – 151.9
3	137.9 – 163.9	163.9 – 177.9

Table : 7

Therefore, the total elapsed time = $CT(S_1)$ = 177.9 units
Utilization time of machine $M_2 = U_2(S_1)$ = 177.9 – 48.0
= 129.9 units

Also $\sum_{i=1}^{n} A_{i1}$ =163.9 units.

Therefore the total rental cost for each of the sequence (S_k); k = 1, 2 is
$R(S_k)$ = 163.9 × 10 + 129.9 × 5
= 1639 + 649.5
= 2288.5 units.

XII. REMARKS

a. If we solve the same problem by Johnson's methods we get the optimal sequence as S= 1 – 2 – 4 – 5 – 3.

The in – out flow table is:

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
3	0 – 28.0	28.0 – 46.0

4	28.9 – 76.9	76.9 – 87.9
2	77.7 – 113.7	113.7 – 129.7
5	114.9 – 136.9	136.9 – 151.9
1	137.9 – 163.9	163.9 – 177.9

Therefore, the total elapsed time = CT(S) = 177.9 units
Utilization time of machine $M_2 = U_2(S)$ = 149.9 units

Also $\sum_{i=1}^{n} A_{i1}$ =163.9 units.

Therefore the total rental cost is
$R(S_k)$ = 163.9 × 10 + 149.9 × 5
= 1639 + 750.5
= 2389.5 units.

b. Equivalent job formation is associative in nature ie block ((k, m)n) = ((k)m, n)
c. The equivalent job formation rule is non commutative i.e. block (k, m) ≠ (m, k) .
d. If set up times of each machine is negligible small, the results are similar as [5].

XIII. CONCLUSION

The algorithm proposed here for specially structured two stage flow shop scheduling problem setup time separated from processing time , with weightage of jobs including job block criterion is more efficient as compared to the algorithm proposed by Johnson(1954) to find an optimal sequence to minimize the utilization time of the machines and hence their rental cost. The study may further be extended by considering various parameters like breakdown effect, transportation time etc.

REFERENCES

[1] . Anup (2002), "On two machine flow shop problem in which processing time assumes probabilities and there exists equivalent for an ordered job block", JISSOR, Vol. XXIII No. 1-4, pp. 41-44.
[2] . Bagga P C (1969), "Sequencing in a rental situation", Journal of Canadian Operation Research Society, Vol.7, pp.152-153.
[3] . Chandrasekharan R (1992), "Two-Stage Flowshop Scheduling Problem with Bicriteria " O.R. Soc. ,Vol. 43, No. 9, pp.871-84.
[4] . Gupta Deepak & Sharma Sameer (2011), "Minimizing rental cost under specified rental policy in two stage flow shop , the processing time associated with probabilities including breakdown interval and Job-block criteria", European Journal of Business and Management, Vol. 2, No. 3, pp.85-103.
[5] . Gupta D., Shashi B and Singla P (2012), "Minimizing rental cost for specially structured two stage flow shop scheduling, processing time associated with probabilities including weightage of jobs and job block criterion", International Journal of Research in Management, Vol. 3, No. 2, pp. 79-92.
[6] . Gupta J N D (1975), "Optimal Schedule for specially structured flow shop", Naval Research Logistic, Vol.22, No.2, pp. 255-269.
[7] . Ignall E and Schrage L (1965), "Application of the branch and bound technique to some flow shop scheduling problems", Operation Research, Vol.13, pp.400-412.
[8] . Johnson S M (1954), "Optimal two and three stage production schedule with set up times included", Naval Research Logistic, Vol.1, No.1, pp. 61-64.
[9] . M. Dell'Amico, Shop problems with two machines and time lags, Oper. Res. 44 (1996) 777–787.
[10] . Maggu P L and Das G (1977), "Equivalent jobs for job block in job scheduling", Opsearch, Vol. 14, No.4, pp. 277-281.
[11] . Narian L Bagga P C (2005), "Two machine flow shop problem with availability constraint on each machine", JISSOR, Vol. XXIV 1-4, pp.17-24.
[12] . Pandian & Rajendran (2010), "Solving Constraint flow shop scheduling problems with three machines", Int. J. Contemp. Math. Sciences, Vol.5, No. 19, pp.921-929.
[13] . Singh T P (1985), "On n×2 shop problem involving job block. Transportation times and Break-down Machine times", PAMS, Vol. XXI , pp.1-2.
[14] . Singh T P, K Rajindra & Gupta Deepak (2005), "Optimal three stage production schedule the processing time and set times associated with probabilities including job block criteria", Proceedings of National Conference FACM-2005,pp. 463-492.
[15] . Singh, T P, Gupta Deepak (2005), "Minimizing rental cost in two stage flow shop , the processing time associated with probabilies including job block", Reflections de ERA, Vol 1, No.2, pp.107-120.
[16] . Szware W (1977), "Special cases of the flow shop problems", *Naval Research Logistic*, Vol.22, No.3, pp. 483-492.
[17] . Yoshida and Hitomi (1979), "Optimal two stage production scheduling with set up times separated", AIIE Transactions, Vol.11, No. 3,pp. 261-269.