International Journal of Engineering Research and Development
e- ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 22, Issue 4 (April 2026), PP 66-70

Anisotropic Vacuum Cosmology in Bimetric Gravity: Effects of Cosmic Strings and Electromagnetic Fields

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ABSTRACT:
The aim of our research is to study the Bianchi type-I cosmological model as a coupling between a cosmic string and an electromagnetic field in Bimetric theory. This led to the existence of the "False Vacuum Model" of the universe $(\rho = \lambda = 0)$ *being filled with "Vacuum energy". The constant value of the Hubble parameter*(H) *indicates the expansion of the universe at a constant rate. i.e.* $70kms^{-1}/Mpc = H_0$*which is a result of the constant value of scalar expansion*(θ)*. The expansion is found to be uneven from the constant value of the anisotropy parameter*(A)*.*
KEYWORDS: *Electromagnetic Field, Cosmic string, False Vacuum Model, Vacuum energy, Cosmic Parameter, Gravitational Field.*

Date of Submission: 20-03-2026 Date of acceptance: 03-04-2026

I. INTRODUCTION:

The standard cosmological model based on Einstein's general theory of relativity has beenremarkably successful in explaining a wide range of large-scale observational phenomena.Nevertheless, several fundamental issues, such as the nature of vacuum energy, the originof cosmic anisotropies, and the role of topological defects in the early universe, have motivated the study of alternative theories of gravitation and anisotropic cosmological models.Among these alternatives, Rosen's bimetric theory of gravitation offers a natural frameworkin which both curved and flat background space-times coexist within a unified gravitationaldescription [1, 2].

Rosen introduced bimetric theory by employing two metric tensors: the physical Riemannian metric tensor gij, which governs gravitational interactions, and a background flat metric tensor γij, representing an empty universe satisfying the cosmological principle [1].This formulation has been widely applied in cosmology to investigate vacuum-dominateduniverses, anisotropic models, and non-standard matter distributions beyond the scope ofgeneral relativity.

Anisotropic cosmological models are particularly relevant for describing the early evolutionary stages of the universe before establishing large-scale isotropy. The Bianchi type-space-time, being the most straightforward anisotropic generalization of the Friedmann–Lemaitre–Robertson–Walker (FLRW) model, allows independent expansion along the threespatial directions and provides a practical framework for studying anisotropic expansion,shear, and directional effects in cosmology [3].

Topological defects such as cosmic strings are predicted to form during symmetry-breakingphase transitions in the early universe [4, 5]. These one-dimensional defects have significant energy density and tension, which can affect the geometry of space-time and cosmicdynamics. Recent studies have renewed interest in cosmic strings because of their potential observational signatures, including gravitational waves and vacuum-related effects [6, 7].The anisotropic stresses associated with string networks naturally motivate their inclusionin anisotropic cosmological models.

Electromagnetic fields are also expected to play an essential role in the anisotropic evolution of cosmology. Observations of large-scale magnetic fields in galaxies and clusters suggesta primordial origin [8,9]. The coupling of electromagnetic fields with anisotropic space-timescan significantly modify the expansion dynamics and contribute to the effective energy density of the universe. Various isotropic and anisotropic Bianchi type-I cosmological modelshave been extensively studied within the framework of bimetric theory, considering perfectfluids, cosmic strings, and electromagnetic fields as possible sources of gravitation [10].

Several authors have extended the study of anisotropic Bianchi type-I cosmological models within the framework of Rosen's bimetric theory by considering magnetized dark energy,stiff matter, and cosmological constant scenarios. These investigations highlight the role ofanisotropy, electromagnetic fields, and accelerated expansion in bimetric gravity [11–14].

Recent investigations have further highlighted the importance of vacuum polarizationand the induced vacuum energy effects in the presence of cosmic strings and electromagneticbackgrounds. Quantum field-theoretic investigations have shown that topological defectscan generate nontrivial vacuum expectation values of the energy-momentum tensor, leading to practical vacuum energy contributions [15–17]. Recent studies have demonstratedthat vacuum polarization effects persist in curved backgrounds, such as de Sitter spacetime,further strengthening the connection between cosmic strings and vacuum-driven acceleratedexpansion [18].

In general relativity, vacuum Bianchi type-I space-times reduce to the well-known Kasnersolution [19], in which the directional scale factors evolve as power laws of cosmic time.These solutions are non-accelerating and are dominated by shear, providing a useful buthighly idealized description of anisotropic vacuum cosmologies. Such Kasner-type behaviorhas been extensively discussed in the context of spatially homogeneous cosmologies [20].Unlike the Kasner vacuum solution of general relativity, which exhibits power-law expansionand no acceleration, the present bimetric model admits an exponentially expanding vacuumsolution with a constant Hubble parameter and persistent anisotropy.

Motivated by these developments, the present work investigates a Bianchi type-I cosmological model within Rosen's bimetric theory, accounting for the combined effects of cosmic strings and electromagnetic fields. By imposing the false vacuum conditionρ = λ = 0, exact solutions of the bimetric field equations are obtained. The resulting modelexhibits a constant Hubble parameter, accelerated expansion, and persistent anisotropy,providing a consistent description of vacuum-dominated anisotropic cosmological evolution.

The paper is organized as follows: Section 2 presents the metric and field equations inRosen's bimetric theory. Section 3 discusses the model's physical and geometrical properties. Finally, Section 4 summarizes the main conclusions and cosmological implications ofthe study.

II. METRIC AND FIELD EQUATIONIN ROSEN'S BIMETRIC THEORY:

Rosen proposed the bimetric theory of gravitation in 1973,which involves two metric tensors i.e. the Riemannian metric tensor g_{ij} describing the geometry of curved space-time and the background metric γ_{ij} of flat space-time. γ_{ij} is the metrictensor of empty Universe where the cosmological principle holds good.The line elements are expressed as

$$ds^2 = g_{ij}dx^i dx^j \quad (1)$$

of curved space-time and

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \quad (2)$$

of flat space-time

Hereds is the interval between two neighbouring events as measured by a clock and a measuring rod. $d\sigma$ is an abstract or a geometric quantity that is not directly measurable.

The field equations of the theory of gravitation proposed by Rosen [1] are

$$N_j^i - \frac{1}{2}Ng_j^i = -8\pi kT_j^i \quad (3)$$

Where $N_j^i = \frac{1}{2}\gamma^{ab}\left(g^{hi}g_{hj;a}\right)_{;b}$ (4)

Here $(;)$ denotes the covariant differentiation with respect tog_{ij}.

Again, $N = \sum N_j^i = N_1^1 + N_2^2 + N_3^3 + N_4^4$ (5)

And $g = det(g_{ij})$, $\gamma = det(\gamma_{ij})$, $k = \left(\frac{g}{r}\right)^{\frac{1}{2}}$

Here, T_{ij} is the energy-momentum tensor of the matter.

We considered the Bianchi type-I metric of curved space-time

$$ds^2 = A^2dx^2 + B^2dy^2 + C^2dz^2 - dt^2 \quad (6)$$

Where A, B and C are functions of 't' and the background metric of flat space-time.

$$d\sigma^2 = dx^2 + dy^2 + dz^2 - dt^2 \quad (7)$$

The energy-momentumtensor for a cosmic string associated with an electromagnetic field is written as

$$T_j^i = T_{j(string)}^i + T_{j(mag)}^i \quad (8)$$

$$T_{j(string)}^i = \rho u^i u_j - \lambda x^i x_j \quad (9)$$

Where$\rho$and $\lambda$are the energy density and tension density of strings. u^i is four-velocity vector is aligned with the time direction. x^ispace-like vector pointing along the string direction. The four-velocity vector describes the motion of the string cloud in space. It is a time-like vector and satisfies the normalisation condition$u^i u_j = 1$.

Hence $(u_1, u_2, u_3, u_4) = (0,0,0,1)$ and $(u^1, u^2, u^3, u^4) = (0,0,0,-1)$ as $u^4 = g^{44}u_4 = -1$

The space-like vector defines the orientation of the string, which is orthogonal to the four-velocity vector, satisfying the condition $x^i x_j = -1$ and $u^i x_j = 0$.

Where $u^i = g^{ij}u_j$ and $x^i = g^{ij}x_j$

Hence $(x_1, x_2, x_3, x_4) = (A^2, 0,0,0)$ And $(x^1, x^2, x^3, x^4) = (1,0,0,0)$

Using the above conditions in (3), the existing components are

$T_1^1 = -\lambda A^2, T_2^2 = T_3^3 = 0$ and $T_4^4 = \rho$ (10)

The electromagnetic energy tensor is

$E_j^i = -F_{jn}F^{in} + \frac{1}{4}F_{ab}F^{ab}g_j^i$ (11)

Maxwell's equations for the electromagnetic field is

$F_{(ij,k]} = F_{ij,k} + F_{jk,i} + F_{ki,j} = 0$ (12)

The non-vanishing component of F_{ij} is F_{23}which leads to the result

$$F_{23} = constant = m\ (say)$$

And

$$F^{23} = g^{22}g^{33}F_{23} = MB^{-2}C^{-2}$$

Using the above values, the existing components of the electromagnetic field are

$E_1^1 = \frac{M^2B^{-2}C^{-2}}{2}, E_2^2 = E_3^3 = E_4^4 = -\frac{M^2B^{-2}C^{-2}}{2}$ (13)

Using (10) and (13)in (8), the existing components of the energy-momentumtensor of the cosmic string associated with the electromagnetic field are

$$T_1^1 = -\lambda A^2 + \frac{m^2}{2B^2C^2}$$
$$T_2^2 = T_3^3 = -\frac{m^2}{2B^2C^2}$$
$$T_4^4 = -\left(\rho + \frac{m^2}{2B^2C^2}\right)$$

As $T^{ij} = g^{ij}T_j^i$

$$T^{11} = \frac{1}{A^2}\left(-\lambda A^2 + \frac{m^2}{2B^2C^2}\right)$$
$$T^{22} = T^{33} = -\frac{m^2}{2B^2C^2}$$

$T^{44} = \rho + \frac{m^2}{2B^2C^2}$ (14)

The Rosen field eq (1) for the metric (6) and (7) using eq(14) becomes

$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} = 16\pi k\left(-\lambda A^2 + \frac{m^2}{2B^2C^2}\right)$ (15)

$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = \frac{8\pi k m^2}{B^2C^2}$ (16)

$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} = \frac{8\pi k m^2}{B^2C^2}$ (17)

$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} = 16\pi k\left(\rho + \frac{m^2}{2B^2C^2}\right)$ (18)

"Vacuum model" refers to a theoretical framework that describes the universe as being filled with "vacuum energy," essentially energy inherent to empty space, arising from Quantum field fluctuations and considered a potential explanation for the observed acceleration and expansion of the universe.

Using the condition for the vacuum model$\rho = \lambda = 0$ in the above equations and solving we got the values of the cosmic parameters as

$A = e^{k_1t+k_2}$, $B = e^{k_3t+k_4}$, $C = e^{k_5t+k_6}$

Where k_1, k_2, k_3, k_4, $k_5$and $k_6$are constants.

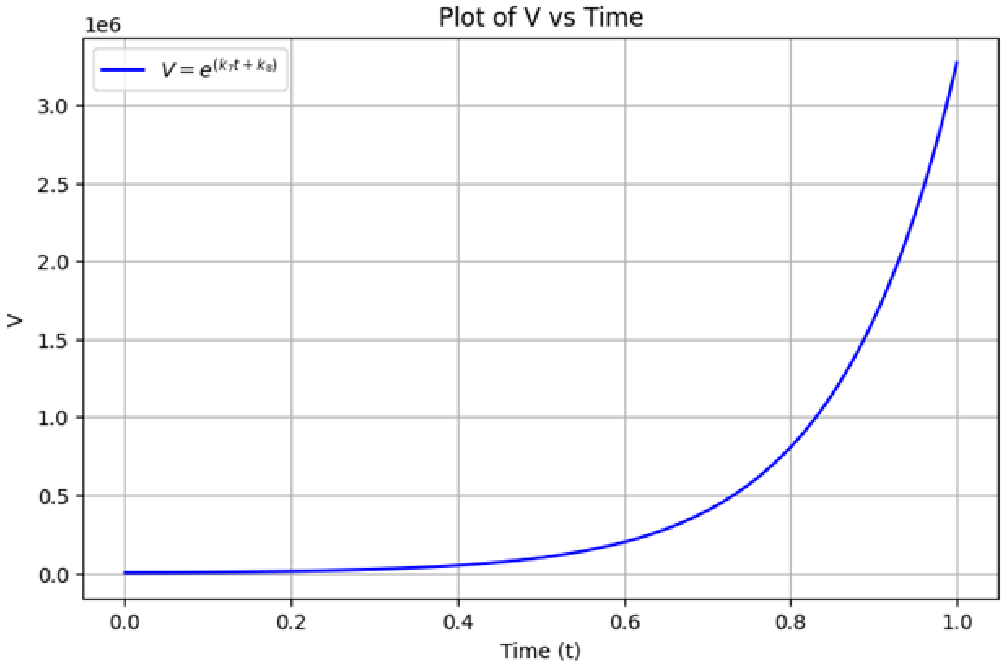
Hence the false vacuum model of our metric as a coupling effect of cosmic string and electromagnetic field takes the form

$ds^2 = e^{2(k_1t+k_2)}dx^2 + e^{2(k_3t+k_4)}dy^2 + e^{2(k_5t+k_6)}dz^2 - dt^2$ (19)

III. PHYSICAL PROPERTIES:

- The proper volume is found to be$V = \sqrt{-g} = e^{k_7t+k_8}$

Where $k_7 = k_1 + k_3 + k_5$ and $k_8 = k_2 + k_4 + k_6$



The variation of volume with respect to cosmic time has been shown in the $V \sim t$ graph.

- Deceleration parameter(q) is calculated as $q = -1 - \frac{\dot{H}}{H^2} = -\frac{V\ddot{V}}{\dot{V}^2} = -1$, as $\dot{H} = 0$

i.e., the model is accelerating. This verifies the expansion of the universe.

- Hubble Parameter in the direction x, y, z are

$H_x = \frac{\dot{A}}{A} = k_1$, $H_y = \frac{\dot{B}}{B} = k_5$, $H_z = \frac{\dot{C}}{C} = k_3$ and $H_t = \dot{H} = 0$

$\therefore H = \frac{1}{3}(H_x + H_y + H_z) = \frac{k_9}{3} = k_{10} = constant$, where $k_9 = k_1 + k_3 + k_5$

The constant value of the Hubble parameter indicates that the universe is expanding at a consistent rate.

- The Anisotropy Parameter(A) is

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 9k_{11} = constant$$

The “constantanisotropy parameter” in cosmology describe the degree of unevenness or directional difference in the expansion of the universe, where this degree remains unchanging throughout the cosmic time which shows the non-uniformity in the expansion across different directions.

- The expansion scalar(θ) is proportional to H. i.e.$\theta = 3H = 3k_{12} = k_{13} = constant$

The constant value of expansion scalar refers to the Hubble constant(H_0), which represents the current rate of expansion of the universe and although it can change over cosmic time scale. The current measured value of H_0 is $70\text{km}s^{-1}/\text{Mpc}$. i.e. (Kilometre per second per mega parsec)

IV. CONCLUSION:

In this paper, a four-dimensionalBianchi type-I cosmological model has been considered as a coupling between a cosmic string and an electromagnetic field in a bimetric theory. We found a solution using the condition of “False Vacuum Model,” i.e. $(\rho = \lambda = 0)$whereλ and ρ are the tension density and energy density of the strings, respectively. Themodel is found to be expanding at a constant rate, which is verified by the constant value of scalar expansion(θ) i.e.$H_0 = 70\text{km}s^{-1}/\text{Mpc}$ as $\theta \propto H$. The model is found to be anisotropic as the anisotropy parameter(A) is constant. This suggests that expansion is uneven and will remain unchanged throughout cosmic time.

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