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A Study on Anti-Picture Fuzzy Subalgebras in BG-algebra

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ABSTRACT: *In this article, the concept of an Anti-Picture Fuzzy BG-subalgebra in a BG-algebra is introduced and illustrated with a suitable example. Several fundamental properties of this structure are investigated. Furthermore, it is proved that the intersection of two Anti-Picture Fuzzy BG-subalgebras is again an Anti-Picture Fuzzy BG-subalgebra.*

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I. INTRODUCTION

Fuzzy set theory was first presented by L. A. Zadeh [12] in 1965 as a tool for managing uncertainty in real-world situations. As an expansion of fuzzy set theory, Atanassov [4] proposed the concept of intuitionistic fuzzy sets in 1986. Cuong [5] extended the concepts of fuzzy sets and intuitionistic fuzzy sets in 2014 with the introduction of picture fuzzy sets. Kim and Kim proposed the idea of BG-algebras [8], which are generalisations of B-algebras. Ahn and Lee [2] later defined fuzzy subalgebras of BG-algebras in 2004. In near-rings, Kim and Yon established the idea of anti-fuzzy ideals [7]. By developing anti-fuzzy ideals of Γ-near-rings [11], Srinivas et al. expanded on this concept. Anti fuzzy BG-ideals in BG-algebras [6] were subsequently introduced by Muhammad Uzair Khan et al. Asif et al. presented picture fuzzy ideals of near-rings [3] in 2020. Zhang et al. proposed picture fuzzy filters of pseudo-BCI algebras [13] in 2017. Picture anti-fuzzy interior ideals of semigroups [1] were recently developed by Adak and Kamal. Furthermore, picture fuzzy subalgebras [9] and interval-valued picture fuzzy subalgebras [10] in BG-algebras were investigated by P. Naga Sriveni et al. In this paper, we present the notion of an anti-picture fuzzy BG-subalgebra and provide an appropriate example. We also discussed some of their properties.

II. MATERIAL AND METHODS

Definition 2.1 [8] A non-empty set $\pounds$ with a constant 0 and a binary operation $*$ is said to be BG-Algebra if it satisfies the following axioms

1. $u * u = 0$
2. $u * 0 = 0$
3. $(u * v) * (0 * v) = u$ for all $u, v \in \pounds$.

Definition 2.2 [8] A non-empty subset $\mathcal{J}$ of a BG-algebra $\pounds$ is called a subalgebra of $\pounds$ if $u * v \in \mathcal{J}$ for all $u, v \in \mathcal{J}$.

Definition 2.3 [13] Let $\pounds$ be the collection of objects. Then a fuzzy set $\mathcal{F}$ in $\pounds$ is defined as

$$\mathcal{F} = \{(u, P_1(u)) \mid u \in \pounds\},$$

where $P_1(u)$ is called the membership degree of u in $\mathcal{F}$ and $0 \leq P_1(u) \leq 1$.

Definition 2.5 [2] A fuzzy set $\mathcal{F}$ is said to be a fuzzy subalgebra of $\pounds$ if

$$P_1(u * v) \geq min\{P_1(u), P_1(v)\} \text{ for all } u, v \in \pounds.$$

Definition 2.6 [6] A fuzzy set $\mathcal{F}$ is said to be an anti-fuzzy subalgebra of £ if

$$P_1(u * v) \le max\{P_1(u), P_1(v)\} \text{ for all } u, v \in £.$$

Definition 2.6 [5] Let £ be a non-empty and finite set. A Picture fuzzy set in £ is defined by

$$P = \{\langle u, P_1(u), P_2(u), P_3(u)\rangle | u \in £\}$$

Where $P_1: £ \to [0,1], P_2: £ \to [0,1]$ and $P_3: £ \to [0,1]$ positive, neutral, and negative membership functions respectively, and $0 \le P_1(u) + P_2(u) + P_3(u) \le 1$. Furthermore, $H(u) = 1 - P_1(u) - P_2(u) - P_3(u)$ is the refusal membership function.

Definition 2.7. [9] A Picture fuzzy set $P = (P_1, P_2, P_3)$ in £ is called a Picture Fuzzy BG-Subalgebra if it satisfies the following conditions
(PFBG-SA 1) $P_1(u * v) \ge min\{P_1(u), P_1(v)\}$
(PFBG-SA 2) $P_2(u * v) \ge min\{P_2(u), P_2(v)\}$
(PFBG-SA 3) $P_3(u * v) \le max\{P_3(u), P_3(v)\}$ for all $u, v \in £$.

Definition 2.8. [10] An interval valued picture fuzzy set $\overline{\overline{P}} = ({P_1}^I, {P_2}^I, {P_3}^I)$ in £ is called an interval valued picture fuzzy BG-Subalgebra if it satisfies
(IvPFBG-SA 1) ${P_1}^I(u * v) \gg rmin\{{P_1}^I(u), {P_1}^I(v)\}$
(IvPFBG-SA 2) ${P_2}^I(u * v) \gg rmin\{{P_2}^I(u), {P_2}^I(v)\}$
(IvPFBG-SA 3) ${P_3}^I(u * v) \ll rmax\{{P_3}^I(u), {P_3}^I(v)\}$ for all $u, v \in £$.

III. ANTI PICTURE FUZZY BG-SUBALGEBRA

Definition 3.1. A Picture fuzzy set $P = (P_1, P_2, P_3)$ in £ is called an Anti-Picture Fuzzy BG-Subalgebra if it satisfies the following conditions
(APFBG-SA I) $P_1(u * v) \le max\{P_1(u), P_1(v)\}$
(APFBG-SA II) $P_2(u * v) \le max\{P_2(u), P_2(v)\}$
(APFBG-SA II) $P_3(u * v) \ge min\{P_3(u), P_3(v)\}$ for all $u, v \in £$.

Definition 3.2. Consider a set $£ = \{0, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5\}$ with the binary operation '$*$' which is given in table.1

Table:1

$*$	0	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5
0	0	ϵ_5	ϵ_4	ϵ_3	ϵ_2	ϵ_1
ϵ_1	ϵ_1	0	ϵ_5	ϵ_4	ϵ_3	ϵ_2
ϵ_2	ϵ_2	ϵ_1	0	ϵ_5	ϵ_4	ϵ_3
ϵ_3	ϵ_3	ϵ_2	ϵ_1	0	ϵ_5	ϵ_4
ϵ_4	ϵ_4	ϵ_3	ϵ_2	ϵ_1	0	ϵ_5
ϵ_5	ϵ_5	ϵ_4	ϵ_3	ϵ_2	ϵ_1	0

Then $(£; * ,0)$ is a BG-algebra. Let $P = (P_1, P_2, P_3)$ be a Picture fuzzy set in £ defined by table.2

Table:2

£	0	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5
$P_1(u)$	0.21	0.35	0.21	0.35	0.21	0.35
$P_2(u)$	0.13	0.52	0.13	0.52	0.13	0.52
$P_3(u)$	0.71	0.45	0.71	0.45	0.71	0.45

It is routine to verify that $P = (P_1, P_2, P_3)$ is an Anti-Picture Fuzzy BG-Subalgebra of £.

Proposition 3.3. If $P = (P_1, P_2, P_3)$ in £ is an Anti-Picture Fuzzy BG-Subalgebra in £, then for all $u \in £$ $P_1(0) \le P_1(u), P_2(0) \le P_2(u)$ and $P_3(0) \ge P_3(u)$.

Proof: Let $u \in £$. Then
$P_1(0) = P_1(u * u) \le max\{P_1(u), P_1(u)\} = P_1(u)$

$\Rightarrow P_1(0) \leq P_1(u)$,
$P_2(0) = P_2(u * u) \leq max\{P_2(u), P_2(u)\} = P_2(u)$
$\Rightarrow P_2(0) \leq P_2(u)$,
$P_3(0) = P_3(u * u) \geq max\{P_3(u), P_3(u)\} = P_3(u)$
$\Rightarrow P_3(0) \geq P_3(u)$.

Theorem 3.4. Let $P = (P_1, P_2, P_3)$ be a Anti Picture Fuzzy BG-Subalgebra of £. If there exists a sequence $\{u_n\}$ in £ such that $\lim_{n\to\infty} P_1(u_n) = 1$, $\lim_{n\to\infty} P_2(u_n) = 1$ and $\lim_{n\to\infty} P_3(u_n) = 0$, then $P_1(0) = 0$, $P_2(0) = 0$ and $P_3(0) = 1$.

Proof: Using the proposition 3.3, we know that $P_1(0) \leq P_1(u_n)$ $P_2(0) \leq P_2(u_n)$ and $P_3(0) \geq P_3(u_n)$ for every positive integer n. Note that

$$0 \leq P_1(0) \leq \lim_{n\to\infty} P_1(u_n) = 0$$
$$0 \leq P_2(0) \leq \lim_{n\to\infty} P_2(u_n) = 0$$
$$1 \geq P_3(0) \geq \lim_{n\to\infty} P_3(u_n) = 1$$

Therefore $P_1(0) = 0$, $P_2(0) = 0$ and $P_3(0) = 1$.

Proposition 3.5. If a Picture fuzzy set $P = (P_1, P_2, P_3)$ in £ be a Anti Picture Fuzzy BG-Subalgebra, then for all $u \in$ £ $P_1(0 * u) \leq P_1(u)$, $P_2(0 * u) \leq P_2(u)$ and $P_3(0 * u) \geq P_3(u)$.

Proof: For all $u \in$ £, we have

$$P_1(0 * u) \leq max\{P_1(0), P_1(u)\} = max\{P_1(u * u), P_1(u)\}$$
$$\leq max\{max\{P_1(u), P_1(u)\}, P_1(u)\} = P_1(u)$$
$$\Rightarrow P_1(0 * u) \leq P_1(u),$$

$$P_2(0 * u) \leq max\{P_2(0), P_2(u)\} = max\{P_2(u * u), P_2(u)\}$$
$$\leq max\{max\{P_2(u), P_2(u)\}, P_2(u)\} = P_2(u)$$
$$\Rightarrow P_2(0 * u) \leq P_2(u),$$

$$P_3(0 * u) \geq min\{P_3(0), P_3(u)\} = min\{P_3(u * u), P_3(u)\}$$
$$\geq min\{min\{P_3(u), P_3(u)\}, P_3(u)\} = P_3(u)$$
$$\Rightarrow P_3(0 * u) \geq P_3(u).$$

Definition 3.6. Let $P_1 = (P_{1_1}, P_{2_1}, P_{3_1})$ *and* $P_2 = (P_{1_2}, P_{2_2}, P_{3_2})$ be two Picture Fuzzy Sets, then the intersection is defined as

$$P_1 \cap P_2 = \left\{\left(u, max\left(P_{1_1}(u), P_{1_2}(u)\right), max\left(P_{2_1}(u), P_{2_2}(u)\right), min\left(P_{3_1}(u), P_{3_2}(u)\right)\right) : u \in £\right\}$$

Theorem 3.7. Let P_1 and P_2 be two Anti Picture Fuzzy BG-Subalgebras of £, then $P_1 \cap P_2$ is a Anti Picture Fuzzy BG-Subalgebra of £.

Proof: Let $u, v \in P_1 \cap P_2$, then $u, v \in P_1$ and $u, v \in P_2$.

$$P_{1_{P_1 \cap P_2}}(u * v) = max\left\{P_{1_{P_1}}(u * v), P_{1_{P_2}}(u * v)\right\}$$
$$\leq max\left\{max\left\{P_{1_{P_1}}(u), P_{1_{P_1}}(v)\right\}, max\left\{P_{1_{P_2}}(u), P_{1_{P_2}}(v)\right\}\right\}$$
$$= max\left\{max\left\{P_{1_{P_1}}(u), P_{1_{P_2}}(u)\right\}, max\left\{P_{1_{P_1}}(v), P_{1_{P_2}}(v)\right\}\right\}$$
$$= max\left\{P_{1_{P_1 \cap P_2}}(u), P_{1_{P_1 \cap P_2}}(v)\right\}$$

$$P_{2_{P_1 \cap P_2}}(u * v) = max\left\{P_{2_{P_1}}(u * v), P_{2_{P_2}}(u * v)\right\}$$
$$\leq max\left\{max\left\{P_{2_{P_1}}(u), P_{2_{P_1}}(v)\right\}, max\left\{P_{2_{P_2}}(u), P_{2_{P_2}}(v)\right\}\right\}$$

$$= max\left\{max\left\{P_{2_{P_1}}(u), P_{2_{P_2}}(u)\right\}, max\left\{P_{2_{P_1}}(v), P_{2_{P_2}}(v)\right\}\right\}$$
$$= max\left\{P_{2_{P_1 \cap P_2}}(u), P_{2_{P_1 \cap P_2}}(v)\right\},$$

$$P_{3_{P_1 \cap P_2}}(u * v) = min\left\{P_{3_{P_1}}(u * v), P_{3_{P_2}}(u * v)\right\}$$
$$\geq min\left\{min\left\{P_{3_{P_1}}(u), P_{3_{P_1}}(v)\right\}, min\left\{P_{3_{P_2}}(u), P_{3_{P_2}}(v)\right\}\right\}$$
$$= min\left\{min\left\{P_{3_{P_1}}(u), P_{3_{P_2}}(u)\right\}, min\left\{P_{3_{P_1}}(v), P_{3_{P_2}}(v)\right\}\right\}$$
$$= min\left\{P_{3_{P_1 \cap P_2}}(u), P_{3_{P_1 \cap P_2}}(v)\right\}.$$

Hence $P_1 \cap P_2$ is a Anti Picture Fuzzy BG-Subalgebra of £. Theorem 3.7. can be generalizes as follows.

Theorem 3.8. Let $\{P_i: i = 1,2,3 \dots \dots\}$ be a family of Anti Picture Fuzzy BG-Subalgebra of £. Then $\bigcap P_i$ is also an Anti Picture Fuzzy BG-Subalgebra of £.

Theorem 3.9. A Picture Fuzzy Set $P = (P_1, P_2, P_3)$ is a Anti Picture Fuzzy BG-Subalgebra of £ if and only if the fuzzy sets P_1, P_2 and ${P_3}^c$ are anti fuzzy subalgebra of £.

Proof: Let $P = (P_1, P_2, P_3)$ be a Anti Picture Fuzzy BG-Subalgebra of £ then we have
$P_1(u * v) \leq max\{P_1(u), P_1(v)\}$,
$P_2(u * v) \leq max\{P_2(u), P_2(v)\}$ and
$P_3(u * v) \geq min\{P_3(u), P_3(v)\}$ for all $u, v \in$ £.

Clearly P_1, P_2 are anti fuzzy subalgebra of £. Now

$$1 - P_3(u * v) \leq 1 - min\{P_3(u), P_3(v)\}$$
$$\Rightarrow {P_3}^c(u * v) \leq max\{1 - P_3(u), 1 - P_3(v)\} = max\{{P_3}^c(u), {P_3}^c(v)\}.$$

Hence ${P_3}^c$ is an anti-fuzzy subalgebra of £.

Conversely, assume that P_1, P_2 and ${P_3}^c$ are anti fuzzy subalgebra of £.
For every $u, v \in$ £ we have

$P_1(u * v) \leq max\{P_1(u), P_1(v)\}$,
$P_2(u * v) \leq max\{P_2(u), P_2(v)\}$
${P_3}^c(u * v) \leq max\{{P_3}^c(u), {P_3}^c(v)\}$

$\Rightarrow 1 - {P_3}^c(u * v) \geq 1 - max\{{P_3}^c(u), {P_3}^c(v)\} = min\{1 - {P_3}^c(u), 1 - {P_3}^c(v)\}$
$\Rightarrow P_3(u * v) \geq min\{P_3(u), P_3(v)\}$.
Hence $P = (P_1, P_2, P_3)$ is an anti picture fuzzy BG-Subalgebra of £.

Definition 3.10. Let $P = (P_1, P_2, P_3)$ is a Picture fuzzy set defined on £.
The operator $\sqcap P$ *and* $\sqcup P$ are defined as $\sqcap P = (P_1, P_2, {P_1}^c)$ and $\sqcup P = ({P_3}^c, P_2, P_3)$ respectively.

Theorem 3.11. If $P = (P_1, P_2, P_3)$ be an Anti-Picture Fuzzy BG-Subalgebra of £ then
$(i) \sqcap P$
$(ii) \sqcup P$ both are Anti-Picture Fuzzy BG-Subalgebras.

Proof: (i) It is sufficient to show that ${P_1}^c$ satisfies the condition (APFBG-SA III).
Let $u, v \in$ £ then we have

$P_1(u * v) \leq max\{P_1(u), P_1(v)\}$
$\Rightarrow 1 - P_1(u * v) \geq 1 - max\{P_1(u), P_1(v)\}$
$\Rightarrow {P_1}^c(u * v) \geq min\{1 - P_1(u), 1 - P_1(v)\}$
$= min\{{P_1}^c(u), {P_1}^c(v)\}$.

Hence $\sqcap P = (P_1, P_2, {P_1}^c)$ is an Anti-Picture Fuzzy BG-Subalgebra of £.

(ii) It is sufficient to show that ${P_3}^c$ satisfies the condition (APFBG-SA I).
Let $u, v \in$ £ then we have
$P_3(u * v) \geq min\{P_3(u), P_3(v)\}$
$\Rightarrow 1 - P_3(u * v) \leq 1 - \min\{P_3(u), P_3(v)\}$
$\Rightarrow {P_3}^c(u * v) \leq \max\{1 - P_3(u), 1 - P_3(v)\}$
$\Rightarrow {P_3}^c(u * v) \leq max\{{P_3}^c(u), {P_3}^c(v)\}$.

REFERENCES

[1]. Adak, A.K. and Kamal, N., 2025. Picture Anti-Fuzzy Interior Ideals of Semi-groups. Journal of Fuzzy Extension and Applications. J. Fuzzy. Ext. Appl.,1:1, 50–58.
[2]. Ahn, S. S. and Lee, H. D., (2004), Fuzzy subalgebras of BG-algebras, Commun. Korean Math. Soc. 19(2), 243-251.
[3]. Asif, A., Aydi, H., Arshad, M., Rehman, A. and Tariq, U., 2020. Picture Fuzzy Ideals of Near-Rings. Journal of Mathematics, 1, p.8857459.
[4]. Atanassov, K. T., (1986), Intuitionistic fuzzy sets, Fuzzy Sets Syst., 20(1), 87–96, doi: https://doi.org/10.1016/S0165-0114(86)80034-3.
[5]. Cuong, B., 2013, Picture fuzzy sets-first results. Part 1. In Proceedings of the Third World Congress on Information and Communication WICT'2013, Hanoi, Vietnam, 1–6.
[6]. Khan, M.U., Khan, R., Shah, S.I.A. and Luqman, M., 2019. Anti Fuzzy BG-ideals in BG-algebra. Journal of New Theory, 27, pp.1-10.
[7]. Kim K .H and Yon Y. H., 2005 Oct 21, On anti fuzzy ideals in near-rings. Iranian Journal of Fuzzy Systems. 2:2, 71-80.
[8]. Kim, C. B. and Kim, H. S., 2008, On BG-algebras, Demonstratio Mathematica, 41, 497-505.
[9]. Naga Sriveni, P. and Parameswari, Parameswari, K. L., 2024, "Picture Fuzzy BG-Subalgebra in BG-algebra," International Journal of Advance Research and Innovative Ideas in Education, 10:5, 250-255, https://ijariie.com/AdminUploadPdf/Picture_Fuzzy_BG_Subalgebra_in_BG_algebra_ijariie24932.pdf?srsltid=AfmBOoqi9Kf4yLAd9e5MV76ppq_0Rty-G4zxslhkhizKzlJyzaEG8iSa
[10]. Parameswari, K. L., Anusha, D., and Naga Sriveni, P., 2025, Interval-Valued Picture Fuzzy BG-Subalgebra in BG-algebra, International journal of Multidisciplinary research in Science, Engineering and Technology, 8(8), August 2025 11914—11918 https://www.ijmrset.com/upload/12_Interval-Valued%20Picture%20Fuzzy%20BG-Subalgebra%20in%20BG-algebraC.pdf
[11]. Srinivas, T., Nagaiah, T. and Swamy, P.N., 2012. Anti fuzzy ideals of Γ-near-rings. Ann. Fuzzy Math. Inform, 3(2), pp.255-266.
[12]. Zadeh, L. A., (1965), Fuzzy sets, Inform. and Control, 8, 338-353.
[13]. Zhang, X., Bo, C. and Park, C., 2017. Picture fuzzy filters of pseudo-BCI algebras. In Fuzzy Systems and Data Mining III (pp. 254-260). IOS Press.