International Journal of Engineering Research and Development
e- ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 21, Issue 10 (October 2025), PP 11-17

Multi-attribute group decision-making algorithm based on Yager norms for intuitionistic fuzzy soft numbers

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Abstract: *Intuitionistic fuzzy soft set (IFSS) theory offers an effective and comprehensive algorithm to handle uncertainty by incorporating parameterized elements which makes it a strong technique for decision-making (DM). For the purpose to aggregate IFS numbers (IFSNs), we propose new operation rules for IFSNs. Then, by utilizing the proposed operations, we propose intuitionistic fuzzy soft Yager weighted averaging (IFSYWA) and geometric (IFSYWG) aggregation operator (AO). Further, we thoroughly examine the mathematical characteristics of the proposed IFSYWA AO and IFSYWG AO such as idempotency and monotonicity. By using the proposed IFSYWA and IFSYWG AO, we develop a multi-attribute group decision-making (MAGDM) algorithm for IFSNs environment. Usefulness of proposed MAGDM algorithm is illustrated by a real-world MAGDM problem focussed on selecting the best renewable energy project for investment.Lastly, the results confirm that the suggested AOs can be used to solve MAGDM difficulties.*
Keywords:*IFSS; Yager t-norm; Aggregation operator; MAGDM.*

Date of Submission: 25-09-2025 Date of acceptance: 05-10-2025

I. Introduction

The main purpose of fuzzy decision-making (DM) is to manage ambiguity and ill-defined data. Fuzzy sets are commonly used to address uncertainties in such problem. In daily human life, there are many problems which an individual faces either related to business, the military, or other fields, but among all the situations, DM matters the most. DM helps in solving problems by selecting the optimal alternative from two or more possible alternatives. To evaluate the optimal alternative, several steps are involved in DM process which must satisfy all the terms and conditions. The first step to solve any DM problem is problem formulation, where the problem is clearly stated along with the relevant parameters. The next step is to select the evaluation criteria and rank the alternatives using appropriate methodology. Finally, a decision is made by selecting the best option.

Molodtsov [1] initially brought forth soft set (SS) concept for the foundational technique, simulating vagueness and ambiguous information. Several techniques have been developed by researchers to manage uncertainty throughout analytical processes like the theory of fuzzy sets (FS), at first proposed by Zadeh [2], inside of which every element is assigned a membership grade (MG). Then FS is extended to intuitionistic Fuzzy Set (IFS) [3], interval-valued intuitionistic fuzzy set (IVIFS) [4], Pythagorean fuzzy sets (PFS) [5]. However, a common limitation among these models is that none of them have a parameterization tool, which restricts their usefulness in real-world situations. SS theory carefully considers and effectively addresses these kinds of circumstances. Later, theories of fuzzy sets and intuitionistic fuzzy sets were merged using an idea of SS, which led to the identification of FSS (fuzzy soft set)[6] and the fuzzy soft set intuitionistic (IFSS) [7]. In meantime, research on Models that are hybrid, like fuzzy soft sets [6], generalized fuzzy soft sets [9,10],intuitionistic fuzzy soft sets [8], fuzzy number intuitionistic fuzzy soft sets, [7,13, 14] generalized intuitionistic fuzzy soft sets [11, 12] etc. has been developed by combining SS with the other models. Additionally, a variety of distance and similarity measures have been proposed by different scholars [15, 16, 17, 18, 19] in FSS or IFSS contexts. Majumdar and Samanta [16] developed a similarity measure for SSs. Khalid and Abbas [19] proposed distance measure for IFSS and IVIFSS. An algorithm to decision-making issues in an FSS context was proposed by Roy and Maji [20]. Thus, the current research will focus on intuitionistic fuzzy soft numbers (IFSNs), considering that the IFSS is a potent instrument to address the data's ambiguity and vagueness. Their advantage is due to their parameterization's property, which makes it easier to describe real-world situations.

An aggregation operator (AO) is essential for ranking all possible alternatives by combining the decision-maker's preferences into a single value, thereby utilizing the rating to determine which option is best. To ensure accurate DM and address uncertainties, many researchers have developed various DM frameworks [21, 22, 23, 24]. Garg and Arora [22] proposed interactive aggregation operations for IFNs. Arora and Garg [25] developed AO and MADM algorithm under IFSS environment. In this article to address decision-making concerns we first propose new operational rules for IFSNs based on Yager's norms [28]. Next, by using the proposed operations, we propose intuitionistic fuzzy soft Yager weighted averaging (IFSYWA) and geometric (IFSYWG) AO concerning the purpose for aggregating IFSNs. Furthermore, we create a multi-attribute group decision-making (MAGDM) algorithm of IFSNs framework. Consequently, to demonstrate the practical efficacy among the proposed MAGDM technique, which we use to a case report about choosing which renewable energy project is the greatest to invest in. The suggested MAGDM algorithm works incredibly well for resolving challenging DM issues in real life.

The remainder of the paper is below: The basic principle is explained in Section 2 relevant to this article. Section 3, introduces new operational rules for IFSNs based on Yager's norms and also, their characteristics are thoroughly described. In Section 4 we propose IFSYWA AO and IFSYWG AO on the basis of proposed operations. In section 5, we develop a novel MAGDM algorithm for IFSNs framework. Section 6 presents a case study involving the choice of the best renewable energy initiative for investment. Finally, Section 7 concludes the article.

II. Preliminaries

***Definition 2.1*[26].**Let E be a set of parameters and K^X indicates the set of all fuzzy subsets of a universal set X. A fuzzy soft set over X is defined as a pair (F, E), where $F: E \to K^X$. If F_{e_j} is a crisp subset of X, then FSS reduces to a SS.

***Definition 2.2*[27].**A pair (F, E) is referred to as IFSS over a universe of discourse X iff $F: E \to IFS(X)$, where IFS(X) indicates the collection of all intuitionistic fuzzy sets of X. For any parameter $e_j \in E$, the mapping is stated as $F_{e_j}(x_i) = \{\langle x_i, \varphi_j(x_i), \psi_j(x_i)\rangle | x_i \in X\}$, where $\varphi_j(x_i)$ and $\psi_j(x_i)$ are MG and non-membership grade (NMG), respectively, with the conditions $0 \leq \varphi_j(x_i), \psi_j(x_i) \leq 1$ and $\varphi_j(x_i) + \psi_j(x_i) \leq 1$.

Definition 2.3[25]. Let $\alpha_{ij} = \langle \varphi_{ij}, \psi_{ij}\rangle$ be any IFSN. Then the score value $S(\alpha_{ij})$ of IFSN α_{ij} is given by:

$$S(\alpha_{ij}) = \varphi_{ij} - \psi_{ij} \tag{1}$$

where $S(\alpha_{ij}) \in [0,1]$. It is clear from this definition that an IFSN with the greatest score is considered the most preferred or highest ranked.

***Definition 2.4*[28].** *Let Φ and Ψ be two real numbers, and $\varrho > 0$. Then Yager's t-norm Y_t and t-conorm Y_c are expressed as:*

$$Y_t(\Phi, \Psi) = 1 - \min(1, ((1-\Phi)^\varrho + (1-\Psi)^\varrho)^{\frac{1}{\varrho}}),$$

$$Y_c(\Phi, \Psi) = \min(1, (\Phi^\varrho + \Psi^\varrho)^{\frac{1}{\varrho}}).$$

III. Proposed operations for IFSNs based on Yager's norm

Definition 3.1.Let $\alpha = \langle \varphi, \psi\rangle, \alpha_{11} = \langle \varphi_{11}, \psi_{11}\rangle, \alpha_{12} = \langle \varphi_{12}, \psi_{12}\rangle$ be three IFSNs, and $\lambda > 0$. Then, the proposed operations of IFSNs based on Yager's norm, described in ***Definition 2.4***, are given as follows:

(i) $\alpha_{11} \oplus \alpha_{12} = \langle \min(1, (\varphi_{11}^\Delta + \varphi_{12}^\Delta)^{1/\Delta}, 1 - \min(1, (1-\psi_{11})^\Delta + (1-\psi_{12})^\Delta)^{1/\Delta}\rangle$;
(ii) $\alpha_{11} \otimes \alpha_{12}$=$\langle 1 - \min(1, ((1-\varphi_{11})^\Delta + (1-\varphi_{12})^\Delta)^{1/\Delta}), \min(1, (\psi_{11}^\Delta + \psi_{12}^\Delta)^{1/\Delta})\rangle$;
(iii) $\lambda\alpha_{11}$=$\langle \min(1, \lambda\varphi_{11}^\Delta)^{1/\Delta}, 1 - \min(1, (\lambda(1-\psi_{11}^\Delta)^{1/\Delta}\rangle$;
(iv) α^λ=$\langle 1 - \min(1, (\lambda(1-\varphi)^\Delta)^{1/\Delta}), \min(1, (\lambda\psi^\Delta)^{1/\Delta})\rangle$.

Theorem 1(Commutative law).Let$\alpha_{ij} = \langle \varphi_{ij}, \psi_{ij}\rangle (i = 1,2)$be two IFSNs, then:
(i) $\alpha_{1j} \oplus \alpha_{2j} = \alpha_{2j} \oplus \alpha_{1j}$;
(ii) $\alpha_{1j} \otimes \alpha_{2j} = \alpha_{2j} \otimes \alpha_{1j}$.
Since this theorem's proof is simple, we omit it out here.

Theorem 2 (Associative law).Let$\alpha_{ij} = \langle \varphi_{ij}, \psi_{ij}\rangle (i = 1,2,3)$be three IFSNs, then:
(i) $(\alpha_{1j} \oplus \alpha_{2j}) \oplus \alpha_{3j} = \alpha_{1j} \oplus (\alpha_{2j} \oplus \alpha_{3j})$

(ii) $(\alpha_{1j} \otimes \alpha_{2j}) \otimes \alpha_{3j} = \alpha_{1j} \otimes (\alpha_{2j} \otimes \alpha_{3j})$

Since this theorem's proof is simple, we omit it out here.

Theorem 3.Let$\alpha_{1j} = \langle \varphi_{1j}, \psi_{1j} \rangle$and$\alpha_{2j} = \langle \varphi_{2j}, \psi_{2j} \rangle$be two IFSNs and $\lambda > 0$, then we have:

(i) $\lambda(\alpha_{1j} \oplus \alpha_{2j}) = \lambda\alpha_{1j} \oplus \lambda\alpha_{2j}$.

(ii) $(\lambda_1 \oplus \lambda_2)\alpha = \lambda_1\alpha \oplus \lambda_2\alpha$.

(iii) $(\alpha_{1j} \otimes \alpha_{2j})^\lambda = \alpha_{1j}^\lambda \otimes \alpha_{2j}^\lambda$.

(iv) $\alpha^{\lambda_1} \otimes \alpha^{\lambda_2} = \alpha^{\lambda_1 + \lambda_2}$.

Proof: We have

(i)Let $\alpha_{1j} = \langle \varphi_{1j}, \psi_{1j} \rangle$ and $\alpha_{2j} = \langle \varphi_{2j}, \psi_{2j} \rangle$ be two IFSNs and $\lambda > 0$ be an accurate figure, then we have,

$$\alpha_{1j} \oplus \alpha_{2j} = \langle \min(1, (\varphi_{1j}^\Delta + \varphi_{2j}^\Delta))^{1/\Delta}, 1 - \min(1, ((1 - \psi_{1j})^\Delta + (1 - \psi_{2j})^\Delta)^{1/\Delta}) \rangle$$

$$\lambda(\alpha_{1j} \oplus \alpha_{2j}) = \langle \min(1, \lambda(\varphi_{1j}^\Delta + \varphi_{2j}^\Delta))^{1/\Delta}, 1 - \min(1, (\lambda((1 - \psi_{1j})^\Delta + (1 - \psi_{2j})^\Delta))^{1/\Delta}) \rangle$$

$$= \lambda\alpha_{1j} \oplus \lambda\alpha_{2j}.$$

Hence, the result.

(ii)Proof is same as part (i).

(iii)Let $\alpha_{1j} = \langle \varphi_{1j}, \psi_{1j} \rangle$ and $\alpha_{2j} = \langle \varphi_{2j}, \psi_{2j} \rangle (i = 1,2)$ be two IFSNs real number $\lambda > 0$, we have $\alpha_{1j} \otimes \alpha_{2j} = \langle 1 - \min(1, ((1 - \varphi_{1j})^\Delta + (1 - \varphi_{2j})^\Delta)^{1/\Delta}), \min(1, (\psi_{1j}^\Delta + \psi_{2j}^\Delta)^{1/\Delta}) \rangle$ and $\alpha^\lambda = \langle 1 - \min(1, (\lambda(1 - \varphi)^\Delta)^{1/\Delta}), \min(1, (\lambda\psi^\Delta)^{1/\Delta}) \rangle$. Then

$$(\alpha_{1j} \otimes \alpha_{2j})^\lambda = \langle 1 - \min(1, (\lambda((1 - \varphi_{1j})^\Delta + (1 - \varphi_{2j})^\Delta))^{1/\Delta}), \min(1, (\lambda(\psi_{1j}^\Delta + \psi_{2j}^\Delta))^{1/\Delta}) \rangle$$

$$= \langle 1 - \min(1, (\lambda(1 - \varphi_{1j})^\Delta + \lambda(1 - \varphi_{2j})^\Delta))^{1/\Delta}, \min(1, (\lambda\psi_{1j}^\Delta + \lambda\psi_{2j}^\Delta))^{1/\Delta} \rangle$$

$$= \alpha_{1j}^\lambda \otimes \alpha_{2j}^\lambda.$$

Hence, the result.

(iv)The proof is same as part (iii).

IV.Proposed intuitionistic fuzzy soft Yager Aggregation operators of IFSNs

We propose new aggregation operators (AOs) on the basis of proposed operations of IFSNs defined in ***Definition 3.1***, namely the intuitionistic fuzzy soft Yager weighted averaging (IFSYWA) AO and geometric (IFSYWG) AO.

4.1 Intuitionistic fuzzy soft Yager weighted averaging (IFSYWA) AO

Definition 4.1.Let$\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm}$be IFSNs. Then, we have:

$$IFSYWA(\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm}) = \overset{m}{\underset{j=1}{\oplus}} \zeta_j \left(\overset{n}{\underset{i=1}{\oplus}} \eta_i \alpha_{ij} \right)$$

$$= \left\langle \min\left(1, \left(\sum_{j=1}^m \zeta_j \sum_{i=1}^n \eta_i \varphi_{ij}^\Delta\right)^{\frac{1}{\Delta}}\right), 1 - \min\left(1, \left(\sum_{j=1}^m \zeta_j \sum_{i=1}^n \eta_i (1 - \psi_{ij})^\Delta\right)^{\frac{1}{\Delta}}\right) \right\rangle \quad (2)$$

where $\zeta_1, \zeta_2, \dots, \zeta_m$ are the weights for parameters and $\eta_1, \eta_2, \dots, \eta_n$ are weight for the experts x_i, respectively alongwith the condition $\zeta_j, \eta_i > 0$, $\sum_{j=1}^m \zeta_j = 1$ and $\sum_{i=1}^n \eta_i = 1$.

Theorem 4.The overall value by utilizing IFSYWA AO is an IFSN defined as follows:

$$IFSYWA(\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm}) = \left\langle \min\left(1, \left(\sum_{j=1}^m \zeta_j \sum_{i=1}^n \eta_i \varphi_{ij}^\Delta\right)^{\frac{1}{\Delta}}\right), 1 - \min\left(1, \left(\sum_{j=1}^m \zeta_j \sum_{i=1}^n \eta_i (1 - \psi_{ij})^\Delta\right)^{\frac{1}{\Delta}}\right) \right\rangle$$

Proof. If $\alpha_{ij} = \langle \varphi_{ij}, \psi_{ij} \rangle$ where $i = 1,2, \dots, n$, and $j = 1,2, \dots, m$, then

$$\eta_i \alpha_{ij} = \left\langle \min\left(1, \left(\eta_i \varphi_{ij}^\Delta\right)^{\frac{1}{\Delta}}\right), 1 - \min\left(1, \left(\eta_i (1 - \psi_{ij})^\Delta\right)^{\frac{1}{\Delta}}\right) \right\rangle$$

$$\overset{n}{\underset{i=1}{\oplus}} \eta_i \alpha_{ij} = \left\langle \min\left(1, \left(\sum_{i=1}^n \eta_i \varphi_{ij}^\Delta\right)^{\frac{1}{\Delta}}\right), 1 - \min\left(1, \left(\sum_{i=1}^n \eta_i (1 - \psi_{ij})^\Delta\right)^{\frac{1}{\Delta}}\right) \right\rangle$$

Also,

$$\zeta_j\left(\bigoplus_{i=1}^{n}\eta_i\alpha_{ij}\right)=\left\langle \min\left(1,\left(\zeta_j\sum_{i=1}^{n}\eta_i\varphi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right),1-\min\left(1,\left(\zeta_j\sum_{i=1}^{n}\eta_i(1-\psi_{ij})^{\Delta}\right)^{\frac{1}{\Delta}}\right)\right\rangle$$

Hence, we have

$$\bigoplus_{j=1}^{m}\zeta_j\left(\bigoplus_{i=1}^{n}\eta_i\alpha_{ij}\right)=\left\langle \min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\varphi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right),1-\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i(1-\psi_{ij})^{\Delta}\right)^{\frac{1}{\Delta}}\right)\right\rangle.$$

Example 4.1.Let $\alpha_{11}=\langle\varphi_{11},\psi_{11}\rangle=\langle 0.2,0.6\rangle,\alpha_{12}=\langle\varphi_{12},\psi_{12}\rangle=\langle 0.4,0.2\rangle,\alpha_{21}=\langle\varphi_{21},\psi_{21}\rangle=\langle 0.1,0.3\rangle$,and$\alpha_{22}=\langle\varphi_{22},\psi_{22}\rangle=\langle 0.8,0.2\rangle$be four IFSNs where$\langle\zeta_1,\zeta_2\rangle=\langle 0.4,0.6\rangle$are weights of the parameters and $\langle\eta_1,\eta_2\rangle=\langle 0.23,0.77\rangle$ are weights of the experts and let $\Delta=2$. Using IFSYWA Operator,we have

$$\bigoplus_{j=1}^{2}\zeta_j\left(\bigoplus_{i=1}^{2}\eta_i\alpha_{ij}\right)=\left\langle \min\left(1,\left(\sum_{j=1}^{2}\zeta_j\sum_{i=1}^{2}\eta_i\varphi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right),1-\min\left(1,\left(\sum_{j=1}^{2}\zeta_j\sum_{i=1}^{2}\eta_i(1-\psi_{ij})^{\Delta}\right)^{\frac{1}{\Delta}}\right)\right\rangle,$$

$$=\langle\min(1,[(0.4(0.23(0.2)^2+0.77(0.1)^2))+(0.6(0.23(0.4)^2+0.77(0.8)^2))]^{1/2}),1-\min(1,[(0.4(0.23(1-0.6)^2+0.77(1-0.3)^2))+(0.6(0.23(1-.2)^2+0.77(1-0.2)^2))]^{1/2})\rangle$$

$$=\langle\min(1,0.56966),1-\min(1,0.7413)\rangle$$

$$=\langle 0.5697,0.2587\rangle.$$

Theorem 5 (Idempotency).If every IFSNs is the same *i.e* $\alpha_{ij}=\alpha$afterwards, $IFSYWA(\alpha_{11},\alpha_{12},\dots,\alpha_{mn})=\alpha$.

***Proof*.** Since for the benefit of each *i,j,* α_{ij} are equal, i.e. $\alpha_{ij}=\alpha$, then from Eq.(2), we have

$$IFSYWA(\alpha_{11},\alpha_{12},\dots,\alpha_{nm})$$

$$=\left\langle \min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\varphi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right),1-\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i(1-\psi_{ij})^{\Delta}\right)^{\frac{1}{\Delta}}\right)\right\rangle$$

$$=\left\langle \min\left(1,\left(\varphi^{\Delta}\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\right)^{\frac{1}{\Delta}}\right),1-\min\left(1,\left((1-\psi)^{\Delta}\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\right)^{\frac{1}{\Delta}}\right)\right\rangle$$

$$=\langle\min(1,\varphi),1-\min(1,(1-\psi))\rangle$$

$$=\langle\varphi,1-(1-\psi)\rangle$$

$$=\langle\varphi,\psi\rangle.$$

Theorem 6 (Monotonicity).Let $\alpha_{ij}^{*}=\alpha_{11}^{*},\alpha_{12}^{*},\dots,\alpha_{nm}^{*}$ *and* $\alpha_{ij}=\alpha_{11},\alpha_{12},\dots,\alpha_{nm}$ be two collection of IFSNs. If$\varphi_{ij}^{*}\le\varphi_{ij}and\psi_{ij}^{*}\ge\psi_{ij}$then $IFSYWA(\alpha_{11}^{*},\alpha_{12}^{*},\dots,\alpha_{nm}^{*})\le IFSYWA(\alpha_{11},\alpha_{12},\dots,\alpha_{nm})$.

Proof. Let IFSYWA$(\alpha_{11}^{*},\alpha_{12}^{*},\dots,\alpha_{nm}^{*})=(G^{*},H^{*})$ and $IFSYWA(\alpha_{11},\alpha_{12},\dots,\alpha_{nm})=(G,H)$. We need to prove that $G^{*}\le G$ and $H^{*}\ge H$. Since, it is given that $\varphi_{ij}^{*}\le\varphi_{ij}$ and $\psi_{ij}^{*}\ge\psi_{ij}$, then

$$\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i{\varphi_{ij}^{*}}^{\Delta}\right)^{\frac{1}{\Delta}}\le\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\varphi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}$$

$$\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i{\varphi_{ij}^{*}}^{\Delta}\right)^{\frac{1}{\Delta}}\right)\le\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\varphi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right)$$

Therefore, $G^{*}\le G$. Also, $\psi_{ij}^{*}\ge\psi_{ij}$

$$\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i{\psi_{ij}^{*}}^{\Delta}\right)^{\frac{1}{\Delta}}\le\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\psi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}$$

$$\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i{\psi_{ij}^{*}}^{\Delta}\right)^{\frac{1}{\Delta}}\right)\le\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\psi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right)$$

$$1-\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i{\psi_{ij}^{*}}^{\Delta}\right)^{\frac{1}{\Delta}}\right)\ge1-\min\left(1,\left(\sum_{j=1}^{m}\zeta_j\sum_{i=1}^{n}\eta_i\psi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right)$$

Therefore, $H^* \geq H$.

Hence, IFSYWA$(\alpha_{11}^*, \alpha_{12}^*, \dots, \alpha_{nm}^*) \leq IFSYWA(\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm})$.

4.2 Intuitionistic fuzzy soft Yager weighted geometric (IFSYWG) AO

Definition 4.2 Let$\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm}$be IFSNs. Then, we have

$$IFSYWG(\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm}) = \underset{j=1}{\overset{m}{\otimes}} \left(\underset{i=1}{\overset{n}{\otimes}} \alpha_{ij}^{\eta_i} \right)^{\zeta_j},$$

$$= \left\langle 1 - \min\left(1, \left(\sum_{j=1}^{m} \zeta_j \sum_{i=1}^{n} \eta_i (1-\varphi_{ij})^{\Delta}\right)^{\frac{1}{\Delta}}\right), \min\left(1, \left(\sum_{j=1}^{m} \zeta_j \sum_{i=1}^{n} \eta_i \psi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right)\right\rangle, \quad (3)$$

where $\zeta_1, \zeta_2, \dots, \zeta_m$ are the weights for parameters and $\eta_1, \eta_2, \dots, \eta_n$ are weight for the experts x_i, respectively alongwith the condition $\zeta_j, \eta_i > 0$, $\sum_{j=1}^{m} \zeta_j = 1$ and $\sum_{i=1}^{n} \eta_i = 1$.

Theorem 7.The overall value by using IFSYWG AO is an IFSN defined as follows*:*

$$IFSYWG(\alpha_{11}, \alpha_{12}, \dots, \alpha_{nm}) = \underset{j=1}{\overset{m}{\otimes}} \left(\underset{i=1}{\overset{n}{\otimes}} \alpha_{ij}^{\eta_i} \right)^{\zeta_j}$$

$$= \left\langle 1 - \min\left(1, \left(\sum_{j=1}^{m} \zeta_j \sum_{i=1}^{n} \eta_i (1-\varphi_{ij})^{\Delta}\right)^{\frac{1}{\Delta}}\right), \min\left(1, \left(\sum_{j=1}^{m} \zeta_j \sum_{i=1}^{n} \eta_i \psi_{ij}^{\Delta}\right)^{\frac{1}{\Delta}}\right)\right\rangle \quad (4)$$

Proof. We may demonstrate this result using the same reasoning as in ***Theorem 4***.

Aggregation operator IFYWG furthermore fulfills the requirement of idemotency and monotonicity.

V.Novel MAGDM algorithm

This section provides a methodology for resolving DM issues using IFSYWA AO and IFSYWG AO within IFSS context.

Examine a list of options, indicated as $\{A_1, A_2, \dots, A_t\}$, which are evaluated by a team of of professionals $X = \{x_1, x_2, \dots, x_n\}$ based on parametersations as $E = \{e_1, e_2, \dots, e_m\}$. Evaluations provided by expert $u_1, u_2, \dots, u_n$ for parameters $e_1, e_2, \dots, e_m$ represented as IFSNs and are expressed in the form of $\alpha_{ij} = (\zeta_{ij}, \psi_{ij})$, ensuring that $0 \leq \zeta_{ij}, \psi_{ij} \leq 1$ and $\zeta_{ij} + \psi_{ij} \leq 1$.

The following steps outline the process to determine the most suitable alternatives using the identified operations:

(Step 1:)Information Collection: Gather information associated with each other alternative $A_1, A_2, \dots, A_t$ in form of of IFSNs $\alpha_{ij}^1, \alpha_{ij}^2, \dots, \alpha_{ij}^t$ and compile it into a matrix $(\alpha_{ij}^d)_{n*m}$.

(Step 2:)Aggregation: Determine the outcome α_{ij}^d of each alternative A_d into a single aggregated value α^d use the operator IFSWYA.

(Step 3:) Score Calculation: Determine the α^d score for every alternative $(d = 1,2, \dots, t)$.

(Step 4:) Ranking: Arrange the score values $S(\alpha^d)$ in decreasing order to rank the alternatives $A_d (d = 1,2, \dots, t)$. The alternatives with higher scores are considered better.

(Step 5:) Conclusion: Conclude the selection process based on the ranking.

VI.Illustrative Example

A case study involving the government agency which needs to select the best renewable energy project for investment has been used to illustrate the DM process. A team of three experts$\{x_1, x_2, x_3\}$ is involved in evaluating four candidate projects A_1, A_2, A_3, A_4 based on four parameters e_1, e_2, e_3, e_4, which are mentioned below:

Table1: Intuitionistic fuzzy soft matrix (IFSmx) given by the experts for A_1.

	e_1	e_2	e_3	e_4
x_1	⟨0.3,0.5⟩	⟨0.1,0.3⟩	⟨0.5,0.2⟩	⟨0.6,0.2⟩
x_2	⟨0.1,0.4⟩	⟨0.6,0.4⟩	⟨0.4,0.3⟩	⟨0.7,0.1⟩
x_3	⟨0.2,0.7⟩	⟨0.5,0.5⟩	⟨0.3,0.2⟩	⟨0.8,0.1⟩

Table2: IFSmx given by the experts for A_2.

	e_1	e_2	e_3	e_4
x_1	⟨0.6,0.4⟩	⟨0.3,0.6⟩	⟨0.9,0.1⟩	⟨0.4,0.3⟩
x_2	⟨0.4,0.5⟩	⟨0.4,0.5⟩	⟨0.8,0.1⟩	⟨0.5,0.2⟩
x_3	⟨0.5,0.5⟩	⟨0.2,0.1⟩	⟨0.7,0.1⟩	⟨0.3,0.2⟩

Table 3: IFSmx given by the experts for A_3.

	e_1	e_2	e_3	e_4
x_1	⟨0.1,0.4⟩	⟨0.2,0.4⟩	⟨0.9,0.1⟩	⟨0.2,0.5⟩
x_2	⟨0.2,0.7⟩	⟨0.2,0.6⟩	⟨0.7,0.2⟩	⟨0.4,0.1⟩
x_3	⟨0.3,0.5⟩	⟨0.4,0.3⟩	⟨0.6,0.1⟩	⟨0.6,0.2⟩

Table 4: IFSmx given by the experts for A_4.

	e_1	e_2	e_3	e_4
x_1	⟨0.6,0.3⟩	⟨0.7,0.2⟩	⟨0.7,0.3⟩	⟨0.4,0.1⟩
x_2	⟨0.4,0.2⟩	⟨0.6,0.1⟩	⟨0.8,0.2⟩	⟨0.6,0.2⟩
x_3	⟨0.7,0.1⟩	⟨0.5,0.4⟩	⟨0.6,0.3⟩	⟨0.4,0.3⟩

- The Cost Efficiency (e_1) : Measures the cost-effectiveness of the project.
- Environmental Impact (e_2): Evaluates the project's sustainability and ecological effects.
- Energy Output (e_3): Represents the expected energy generation capacity.
- Scalability (e_4): Assesses how well the project can be expanded in the future.

The expert weight vectors and the parameters weight vectors are $(0.25,0.35,0.4)$and $(0.2,0.3,0.4,0.1)$ respectively. The procedure listed below are used to choose the most promising renewable energy project for funding.

(Step 1:) For each candidate, each alternatives rates as part of IFSNs, which compiled in Tables 1, 2, 3, and 4.

(Step 2:) Using the IFSWYA operator, the total values of all the options are :

$\alpha^1 = \langle 0.4365,0.3209\rangle, \alpha^2 = \langle 0.5856,0.2416\rangle, \alpha^3 = \langle 0.4473,0.2878\rangle, \alpha^4 = \langle 0.6262,0.2321\rangle$.

(Step 3:) Using score value defined through Eq.(1) ,score values for each alternative is:

$S(\alpha^1) = 0.1156, S(\alpha^2) = 0.3440, S(\alpha^3) = 0.1596, S(\alpha^4) = 0.3941$.

(Step 4:) IFSWYA operator ranks options as $A_4 \succ A_2 \succ A_3 \succ A_1$. Consequently, the most desirable candidate is A_4.

VII.Conclusion

In addressing MAGDM problems, AO plays a significant part in aggregating the data given which are evaluated by a team of experts into collective data. In the following paragraphs, we have proposed fresh operation rules for IFSNs such as addition, multiplication, scalar multiplication, and scalar power operation based on Yager's norms. Then, by utilizing the proposed operations, we have introduced intuitionistic fuzzy soft Yager weighted averaging (IFSYWA) and geometric (IFSYWG) aggregation operator (AO). Next, we have examined the mathematical properties of the proposed IFSYWA AO and IFSYWG AO. Furthermore, by utilizing the proposed IFSYWA AO and IFSYWG AO we have developed an MAGDM algorithm and also demonstrated its effectiveness and applicability by solving a case study which is focussed on selecting the most suitable renewable energy project for investment. Finally, the results clearly validate that the proposed MAGDM algorithm is capable in managing circumstances involving complex and ambiguous decision-making.

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