Model Development for Pressure-Flow Capacity Relations for Gas Pipelines Flow Optimization

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ABSTRACT: - The flow of natural gas in gas pipelines has been attended with the problem of pressure drop along the pipeline. The magnitude of the pressure drop is known to be affected by certain flow variables such as flow velocity or volumetric flow rate, bulk flow temperature, ambient conditions, nominal pipe diameter, pipe roughness (relative or absolute), etc. Other factors that will affect throughput and pressure drop relationship are design and construction of the piping system, piping materials, pipe wall thickness, terrain of a place, enlargement and contraction along the pipes, pipe conditions (vacuum and weight of the overlying materials be it water or earth mass), etc. It is well established that the flow of natural gas in gas pipelines is not essentially single phase flow situation. To tackle flow problems well-known flow equations used in the gas transmission industries are: Weymouth equation, Panhandle A, and Modified Panhandle B. There is no adequate understanding of the flow variables as it affects throughput-pressure drop relationship in a gas flow line. Hence the call for more revelation in this area of knowledge by the development of mathematical models that will optimize flow variables in gas pipelines aroused.

KEYWORDS:- Pressure-Flow Relations; Flow Optimization; Weymouth Equation; Modified Panhandle B Equation; Pipeline Efficiency; Pipeline Efficiency; Compressibility Factor; Average Flow Pressure; Pump Constant, Pump Isentropic Efficiency.

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I INTRODUCTION

Natural gas is the sub-category of petroleum product. It is a naturally occurring substance, a complex mixture of hydrocarbon and traces of inorganic compounds. In terms of the World's energy supply and demand natural gas is next to petroleum (crude oil). One cubic metre of natural gas, when combusted will generates 19 to 43KJ of heat depending on the gas composition [1]. In the global World, natural gas is becoming a competing energy source to petroleum. It is a fuel source for devices like automobiles, aircraft, power plant and domestic appliances. There is every hope and aspiration that with the passage of time it would be a more dominant energy source compared to petroleum or crude oil. Even there is this global anxiety of depletion of crude oil.

Transportation or transmission of crude oil or natural gas is through a piping network. The design, construction, and operation of the piping network rely on adequate knowledge of the flow variables. The metering of the gas is by volume measurement and it is dependent on both pressure and temperature; natural gas being a compressible fluid. These considerations prompt the need for flow variables optimization. Optimization of flow calls for the development of mathematical models that will relate throughput to pressure drop along gas pipelines, more importantly, the main (trunk) line or any category of the pipelines.

II PURPOSE AND SIGNIFICANCE

Attainment of optimal conditions of performance in the cost of investment and operation of gas pipelines assets and facilities; even setting optimal conditions for certain critical flow control variables such as flow capacity and overall line pressure drop. This would go a long way in ascertaining optimum power to conduct a fluid stream through a piping network system.

III MATHEMATICAL MODELS DEVELOPMENT

Unit Consistency

Most of the oil and gas industries use the unconventional system of units like the imperial system of the units, the British system of units, the cgs system of unit (centimetre, gram, second). The approach in this work will be system international (S.I) units.

Weymouth Equation

The constant K, in Weymouth equation shall be converted to the S.I. form.

$$Q = K \left(\frac{T_b}{P_b}\right) \sqrt{\frac{1}{f}} \left[\frac{P_1^2 - P_2^2}{G\overline{T}ZL}\right]^{0.5} D^{2/3}$$
(1)

In the Weymouth equation 1, flow capacity, Q is in m^3/day , pressure, P is in bar, length, L is in Km, temperature, T is in K, diameter, D is in cm, specific gravity, G, friction factor, f, and compressibility factor, Z is dimensionless. The constant, K is obtained in the S.I. by the following

$$\frac{m^3}{3600 \times 24 \times s} = K \frac{{}^{\circ}K}{bar} \left[\frac{bar^2}{{}^{\circ}K \times 1000m} \right]^{1/2} \left(\frac{1}{100m} \right)^{8/3}$$

Operation:

$$K = \frac{1000^{1/2} \times 100^{8/3} m^3 \times bar \times^{\circ} K \times m^{1/2} \times m^{8/3}}{3600 \times 24 \times bar \times s}$$
$$= 78.85 m^{8/6} {}^{\circ} K^{-1/2} s^{-1}$$

A. Panhandle A. Equation

$$Q = K \left(\frac{T_b}{P_b}\right)^{1.788} \left(\frac{1}{f}\right)^{0.5394} \left[\frac{P_1^2 - P_2^2}{\overline{T}ZL}\right]^{0.5394} \left(\frac{1}{G}\right)^{0.4606} D^{2.6182}.$$
(2)

In the Panhandle A. equation 2, flow capacity, Q is in m^3/day , pressure, P is in bar, length, L is in Km, temperature, T is in K, diameter, D is in cm, specific gravity, G, friction factor, f, and compressibility factor, Z is dimensionless. The constant, K is obtained in the S.I. by the following

$$\frac{m^3}{3600 \times 24s} = K \left(\frac{K}{bar}\right) \left[\frac{bar^2}{K \times 1000m}\right]^{0.5394} \left(\frac{1}{100}\right)^{2.6182}$$

Operation:

$$K = \frac{m^3 \times bar^{1.0788} \times^{\circ} K \times m^{0.5394} \times 1000^{0.5394} \times 100^{2.6182}}{{}^{\circ}K^{1.0788} \times bar^{1.0788} \times 3600 \times 24 \times s \times m^{2.6182}}$$
$$= 82.81m^{0.9} {}^{\circ}K^{-0.54} s^{-1}$$

B. Modified Panhandle B. Equation

$$Q = K \left(\frac{T_b}{P_b}\right)^{1.02} \left(\frac{1}{f}\right)^{0.51} \left[\frac{P_1^2 - P_2^2}{\overline{T}ZLG^{0.961}}\right]^{0.51} D^{2.53}$$
(3)

In the Modified Panhandle B equation 3, flow capacity, Q is in m^3/day , pressure, P is in bar, length, L is in Km, temperature, T is in K, diameter, D is in cm, specific gravity, G, friction factor, f and compressibility factor, Z are dimensionless. The constant, K is obtained in the S.I. by the following operation:

$$\frac{m^{3}}{3600 \times 24 \times s} = K \left(\frac{K}{bar}\right)^{1.02} \left[\frac{bar^{2}}{m^{\circ}K}\right]^{0.51} \left(\frac{m}{100}\right)^{2.53}$$
$$K = 45.03m^{-0.04} \otimes K^{-0.49}$$

C. Model Development for the Pump Constant

Model development for the pump constant, k_p , is as follows:

Let Q be the average flow capacity in m^3/s and P the average stream pressure in bar (N/m^2) . The required power to conduct flow streams through the conduit is expressed as;

$$P_W = FV$$
(4)
Where F is the driving force and V is the flow velocity in m/s.

$$P = F / A; \quad \therefore \quad F = PA \tag{5}$$

Combining equations 4 and 5;

$$P_{W} = PAV = PQ \tag{6}$$

The isentropic efficiency of the pump or compressor be ni is given as:

$$\eta_{i} = \frac{\text{Re al Power}}{\text{Ideal Power}} = \frac{P_{R}}{P_{W}} = \frac{P_{R}}{PQ}$$

$$P_{P} = \eta_{i} P Q$$
(7)

 $P_{R} = \eta_{i} P Q$

The losses in power due to irreversibility inherent in the pump/compressor could be expressed as; $P_{L} = P_{W} - P_{R} = PQ - \eta_{i}PQ = PQ(1 - \eta_{i}) = h_{l}\rho gQ(1 - \eta_{i})$ (8)

Denoting the pressure drop in the pomp or compressor due to irreversibility by ΔP , the power losses in the compressor is,

$$P_L = \Delta P_P Q \tag{9}$$

Relating equation 8 to 9,

$$\Delta P_{p}O = PO(1-\eta_{i})$$

$$\Delta P_P = P(1 - \eta_i) \tag{10}$$

$$P = \frac{F}{A} = \frac{\dot{m}V}{A} = \frac{\rho Q V}{A} = \frac{\rho Q^2}{A^2}$$
(11)

Applying equation 10 in 11,

$$\Delta P_{p} = \frac{16\rho(1-\eta_{i})}{\pi^{2}d^{4}}Q^{2}$$

= $K_{p}Q^{2}$ (12)

where
$$K_P = \frac{16\rho(1-\eta_i)}{\pi^2 d^4}$$

K_p—Pump constant

The isentropic efficiency, $\eta_i = 85\%$ to 97.5% for most pumps and compressors [5].

D. Development of Mathematical Models for Change In Flow Capacity Per Unit Change In Pressure Drop

The design, construction and sizing the inline facilities on gas pipelines require accurate values of the flow capacity and overall pressure drop along the line. That in itself is not complete without a mathematical model to monitor how changes in flow capacity affect overall pressure drop along a line. A mathematical model is

developed to determine change in flow capacity per unit change in the overall pressure drop.

The Weymouth equation is expressed as [2]:

$$Q = k \left(\frac{T_b}{P_b} \right) \left[\frac{(P_1 + P_2)(P_1 - P_2)}{Gf\overline{TZL}} \right]^{0.5} D^{8/3}$$

$$\Delta P = P_1 - P_2$$

$$Q = k_1 \Delta P^{0.5}$$

$$k_1 = k \left(\frac{T_b}{P_b} \right) \left[\frac{P_1 + P_2}{Gf\overline{TZL}} \right]^{0.5} D^{8/3}$$

$$\frac{\partial Q}{\partial (\Delta P)} = \frac{0.5k_1 \partial (\Delta P)}{\Delta P^{0.5}}$$
(14)

With respect to equation 14 the following expressions are derived:

(i) Change in flow rate per unit change in frictional pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{f})} = \frac{0.5k_{1}(1 + \Delta P_{e} + \Delta P_{a} + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_{v} + \Delta P_{ft} + \Delta P_{p})}{\Delta P^{0.5}}$$

(ii) Change in flow rate per unit change in elevation pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_e)} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p)}{\Delta P^{0.5}}$$

(iii) Change in flow rate per unit change in acceleration pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_a)} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_e + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_f + \Delta P_p)}{\Delta P^{0.5}}$$

(iv) Change in flow rate per unit change in entrance pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ent})} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_f + \Delta P_p)}{\Delta P^{0.5}}$$

(v) Change in flow rate per unit change in exit pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{exit})} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ext} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p)}{\Delta P^{0.5}}$$

(vi) Change in flow-rate per unit change in enlargement and contraction pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ec})} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_v + \Delta P_{ft} + \Delta P_p)}{\Delta P^{0.5}}$$

(vii) Change in flow rate per unit change in valve pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{v})} = \frac{0.5k_{1}\left(1 + \Delta P_{f} + \Delta P_{e} + \Delta P_{a} + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_{ft} + \Delta P_{p}\right)}{\Delta P^{0.5}}$$

(viii) Change in flow-rate per unit change in fittings pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ft})} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_p)}{\Delta P^{0.5}}$$

(ix) Change in flow rate per unit change in pump pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_p)} = \frac{0.5k_1(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft})}{\Delta P^{0.5}}$$

E. Development of Mathematical Model for Change In Flow Capacity Per Unit Change In Pressure Drop Using Panhandle A. Equation

The Panhandle A. equation is given as [3]:

$$Q = k_{p1} \left(\frac{T_{b}}{P_{b}}\right)^{1.0788} \left[\frac{(P_{1} + P_{2})(P_{1} - P_{2})}{f\overline{T}\overline{Z}L}\right]^{0.5394} \left(\frac{1}{G}\right)^{0.4606} D^{2.6182}$$

$$\Delta P = P_{1} - P_{2}$$

$$Q = k_{2} \Delta P^{0.5394}$$

$$k_{2} = k \left(\frac{T_{b}}{P_{b}}\right)^{1.0788} \left[\frac{(P_{1} + P_{2})}{f\overline{T}\overline{Z}L}\right]^{0.5394} \left(\frac{1}{G}\right)^{0.4606} D^{2.6182}$$

$$\frac{\partial Q}{\partial(\Delta P)} = \frac{0.5394 k_{2} \partial(\Delta P)}{\Delta P^{0.4606}}$$
(15)

With respect to equation 15 the following expressions are derived: (i) Change in flow-rate per unit change in frictional pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_f)} = \frac{0.5394k_2(1 + \Delta P_e + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p)}{\Delta P^{0.4606}}$$

(ii) Change in flow-rate per unit change in elevation pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_e)} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p)}{\Delta P^{0.4606}}$$

(iii) Change in flow-rate per unit change in acceleration pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_a)} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p)}{\Delta P^{0.4606}}$$

(iv) Change in flow-rate per unit change in entrance pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ext})} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_e + \Delta P_{ext} + \Delta P_{ext} + \Delta P_e + \Delta P_f + \Delta P_f)}{\Delta P^{0.4606}}$$
(v) Change in flow-rate per unit change in exit pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ext})} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_e + \Delta P_{ext} + \Delta P_{ext} + \Delta P_e + \Delta P_f)}{\Delta P^{0.4606}}$$
(vi) Change in flow-rate per unit change in enlargement and contraction pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ext})} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_e + \Delta P_{ext} + \Delta P_{ext} + \Delta P_v + \Delta P_f)}{\Delta P^{0.4606}}$$
(vii) Change in flow-rate per unit change in valve pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_v)} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_e + \Delta P_{ext} + \Delta P_{ext} + \Delta P_v + \Delta P_f)}{\Delta P^{0.4606}}$$
(viii) Change in flow-rate per unit change in fittings pressure drop.
(viii) Change in flow-rate per unit change in fittings pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_f)} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ext} + \Delta P_{ext} + \Delta P_{ec} + \Delta P_f)}{\Delta P^{0.4606}}$$
(viii) Change in flow-rate per unit change in fittings pressure drop.
(viii) Change in flow-rate per unit change in fittings pressure drop.
(viii) Change in flow-rate per unit change in fittings pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_f)} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ext} + \Delta P_{ext} + \Delta P_{ec} + \Delta P_v + \Delta P_f)}{\Delta P^{0.4606}}$$
(ix) Change in flow-rate per unit change in fittings pressure drop.
(ix) Change in flow-rate per unit change in pump pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_f)} = \frac{0.5394k_2(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ext} + \Delta P_{ext} + \Delta P_{ec} + \Delta P_v + \Delta P_f)}{\Delta P^{0.4606}}$$

F. Development of Mathematical Model For Change In Flow Capacity Per Unit Change In Pressure Drop Using Modified Panhandle B Equation

The modified Panhandle B Equation goes thus [4]:

$$Q = k \left(\frac{T_b}{P_b} \right)^{1.02} \left[\frac{(P_1 + P_2)(P_1 - P_2)}{f\overline{TZ}LG^{0.961}} \right]^{0.51} D^{2.53}$$

$$\Delta P = P_1 - P_2$$

$$Q = k_3 \Delta P^{0.51}$$

$$k_3 = k \left(\frac{T_b}{P_b} \right)^{1.02} \left[\frac{P_1 + P_2}{f\overline{TZ}LG^{0.961}} \right]^{0.51} D^{2.53}$$

$$\frac{\partial Q}{\partial (\Delta P)} = \frac{0.51k_3 \partial (\Delta P)}{\Delta P^{0.49}}$$
(16)

With respect to equation 16 the following expressions are derived:(i) Change in flow-rate per unit change in frictional pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{f})} = \frac{0.51k_{3}(1 + \Delta P_{e} + \Delta P_{a} + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_{v} + \Delta P_{f} + \Delta P_{p})}{\Delta P^{0.49}}$$

(ii) Change in flow-rate per unit change in elevation pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_e)} = \frac{0.51k_3 \left(1 + \Delta P_f + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p\right)}{\Delta P^{0.49}}$$

(iii) Change in flow-rate per unit change in acceleration pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_a)} = \frac{0.51k_3 \left(1 + \Delta P_f + \Delta P_e + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_f + \Delta P_p\right)}{\Delta P^{0.49}}$$

(iv) Change in flow-rate per unit change in entrance pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ent})} = \frac{0.5394k_2 \left(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft} + \Delta P_p\right)}{\Delta P^{0.4606}}$$

(vi) Change in flow-rate per unit change in exit pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{exit})} = \frac{0.51k_{3}(1 + \Delta P_{f} + \Delta P_{e} + \Delta P_{a} + \Delta P_{exit} + \Delta P_{exit} + \Delta P_{y} + \Delta P_{ft} + \Delta P_{p})}{\Delta P^{0.49}}$$

(vii) Change in flow-rate per unit change in enlargement and contraction pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ec})} = \frac{0.51k_3 \left(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_v + \Delta P_{ft} + \Delta P_p\right)}{\Delta P^{0.49}}$$

(viii) Change in flow-rate per unit change in valve pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{v})} = \frac{0.51k_{3}(1 + \Delta P_{f} + \Delta P_{e} + \Delta P_{a} + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_{ft} + \Delta P_{p})}{\Delta P^{0.49}}$$

(ix) Change in flow-rate per unit change in fittings pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_{ff})} = \frac{0.51k_{3}(1 + \Delta P_{f} + \Delta P_{e} + \Delta P_{a} + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_{v} + \Delta P_{p})}{\Delta P^{0.49}}$$

(x) Change in flow-rate per unit change in pump pressure drop.

$$\frac{\partial Q}{\partial (\Delta P_p)} = \frac{0.51k_3 \left(1 + \Delta P_f + \Delta P_e + \Delta P_a + \Delta P_{ent} + \Delta P_{exit} + \Delta P_{ec} + \Delta P_v + \Delta P_{ft}\right)}{\Delta P^{0.49}}$$

The flow friction factor for single phase regime is expressed as [5]:

$$f_{0} = \left[0.00140 + \frac{0.125}{\left(\rho_{G} \overline{V}_{G} D / \mu_{G}\right)^{0.32}} \right]$$
(17)

The analysis and deductions so far with respect to equations 13 to 16 will suffice for determination and optimization of flow parameters in a gas flow line when the flow regime is essentially single phase flow situation.

IV MODEL APPLICATIONS

The mathematical models can be applied to any section of a natural gas pipeline network be it the field pipeline or gathering system, the main (trunk) line or transmission system, or the service line or distribution system. This is to determine the effect of variability in overall pressure drop to changes in certain paramount control variables

V CONCLUSIONS

The computer simulation of the deductions from equations 13 to 16 can be of immense value in the analysis of variables governing pressure-flow problems in gas transmission lines. Such analysis will go a long way in the design, construction, operation and investment cost of setting up gas transmission lines as well as the cost of maintenance of the lines.

VI RECOMMENDATION FOR FUTURE RESEARCH

Computer simulation of equations 13 to 16 can be written to determine the various pressure drop components, the overall pressure pressure drop and the change in flow capacity per unit change in the overall pressure drop for different natural gas piping network design.

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