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Regional Rainfall Frequency Analysis By L-Moments Approach For Madina Region, Saudi Arabia

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ABSTRACT:- In arid regions, extreme rainfall event frequency predictions are still a challenging problem, because of the rain gauge stations scarcity and the record length limitation, which are usually short to insure reliable quantile estimates. Regional frequency analysis is one of the popular approaches used to compensate the data limitation. In this paper, regional frequency analysis of maximum daily rainfall is investigated for Madinah province in the Western Kingdom of Saudi Arabia (KSA). The observed maximum daily rainfall records of 20 rainfall stations are selected from 1968 to 2015. The rainfall data is evaluated using four tests, namely, Discordance test (Di), Homogeneity test (H), Goodness of fit test (Zdist) and L-moment ratios diagram (LMRD). The Di of L-moments shows that all the sites belong to one group (Di <3.0). The H test indicated that the region is homogeneous (H<1). Finally, the Zdist is used to evaluate five probability distribution functions (PDFs) including generalized logistic (GLO), generalized extreme value (GEV), generalized normal (GNO), generalized Pareto (GPA), and Pearson Type III (PE3). Zdist and LMRD both showed that PE3 distribution is the best among the other PDFs. The regional parameters of the candidate PDF are computed using L-moments approach and accordingly the regional dimensionless growth curve is developed. The results enhance the accuracy of extreme rainfall prediction at-sites and also they can be used for ungauged catchment in the region.
Keywords:- Regional frequency analysis, L-moments approach, arid regions, Madinah region, Saudi Arabia

I. INTRODUCTION

Magnitude and frequency of extreme rainfall events are usually required for hydraulic structures design in any water resources project. Arid regions have rather short data and limited number of gauge stations. It is a common problem that the observed rainfall data is either not available or not sufficient for reliable designs. In such cases, regional frequency analysis (RFA) is implemented, where the observed rainfall data from several locations are combined for prediction of the frequency of extreme rainfall events in the region. RFA approach can improve the accuracy of the predicted rainfall and allow for more reliable quantile estimates and it also reduces the uncertainties associated with at-site (single) location approach (Buishand, 1991; Smithers and Schulze, 2001; Brath et al. 2003; Gáal et al. 2007; Kysely´ and Picek, 2007). L-moments approach can be considered as the most popular method used in regional frequency analysis (Hosking and Wallis, 1997).

Several studies are available on regional rainfall frequency analysis, mostly in the humid regions. For example, Schaefer (1990) found the EVII as the best annual rainfall distribution for Washington State. In Sicily, Italy, Cannarozzo et al. (1995) preformed two-component extreme value distribution for annual maximum rainfall. Adamowski et al. (1996) numerically analyzed 320 stations in Canada and showed that they may be considered as one homogeneous region, and GEV distribution is the regional parent distribution. To estimate short duration design rainfalls in South Africa, Smithers and Schulze (2001) applied the GEV distribution and regional index storm approach based on L-moments using information from 172 rainfall stations. The region was subdivided into 15 relatively homogeneous clusters.

Sveinsson et al. (2002) developed the regional frequency curves of annual maximum precipitation based on index method for Northeastern Colorado State. On the other hand, Zalina et al. (2002) analyzed the hourly rainfall of seventeen automatic stations in Malaysia, where eight distributions were compared using Probability Plot Correlation Coefficient test, root mean squared error, relative root mean squared error and maximum absolute deviation. It was found that GEV distribution is the most appropriate distribution. Furthermore, Lee and Maeng (2003) analyzed the maximum annual daily rainfall in 38 Korean rainfall stations based on the comparison between three distributions using L-moment ratio diagram and Kolmogorov–Smirnov test. It was found that GEV and GLO distributions are appropriate for Korean rainfall data.

In the USA, Trefry et al. (2005) updated the rainfall frequency curve of 76 recording rainfall stations and 152 daily stations in Michigan State using generalized Pareto distribution fit to partial duration series data and generalized extreme value distribution fit to annual maximum series data. Seventy eight daily rainfall stations in Czech are processed by Kysely and Picek (2007) to improve the extreme rainfall predictions, where the study area was subdivided into four homogeneous regions using cluster analysis. According to L-moment ratio diagram and goodness of fit tests, GEV distribution was selected as the most suitable one. Ngongondo et al. (2011) analyzed 23 selected rainfall stations in Southern Malawi. They suggested the k-means cluster analysis and Ward's classification with three homogeneous rainfall regions, and concluded that the regional quantiles had smaller uncertainty as compared to at-site estimates.

Recently, Shahzadi et al. (2013) estimated the regional rainfall quantiles of 23 sites in Pakistan using L-moments approach; different test were applied including independency, stationarity, discordancy, homogeneity tests and the area was subdivided into three regions. In their study L-moment ratio diagram was developed to select the best distributions and the regional quantiles estimations using GNO distribution for large return periods and GEV for small return periods. Three methods of forming regions for use in estimating design rainfalls of 134 rainfall stations in south-east of Queensland and northern New South Wales in Australia were compared by Haddad et al. (2015) using Bayesian Generalized Least Squares Regression. Finally, it was found that all three methods provided good estimates of the L-moment statistics and the rainfall quantiles. Núñez et al. (2016) proposed an integral procedure for the application of L-moments based regional frequency analysis in large network and high as well as complex spatial scale conditions in Latin America. Most recently, monthly precipitation were processed at 17 stations in Northern Pakistan by Khan, et al. (2017) and the L-moments and partial L-moments are computed for four distributions. The results showed than partial L-moments were superior to L-moments for large return estimations.

Evaluation of these various approaches in arid regions has not yet been widely investigated, thus the main objective of the present study is to develop the regional growth curves of the maximum daily rainfall in Madinah region, Western Kingdom of Saudi Arabia (KSA) by using l-moments approach.

II. STUDY AREA AND DATA SET

A. *Study Area*

Madinah region with 150,000 km2 area is located in the Western part of KSA and it is bounded by latitudes 23o and 27oN and longitude 37o and 42oE (Fig. 1). The region can be divided into three topographical areas, which are; the Red Sea coastal plain, the hills and the mountains, where the elevation ranges from 0 up to 2300 meter above the mean sea level. The study area is predominately arid with hot summer and cooler winter seasons. The climate pattern over the study area is characterized by various air masses that affect the rainfall distribution (Alyamani and Şen, 1993). It is a combination of Mediterranean and monsoonal weathered pattern modified by Hijaz Escarpment. Maritime polar air mas and continental tropical air mass are the main two fronts of air moisture flowing into the region (Şen, 1983). The mean temperature ranges from 24°C to 40°C in the summer and from 15°C to 25°C in the winter. Rainfall in the study area is sporadic, characterized by moderate to high variations in space and time (Subyani and Alahmadi, 2011). The rainy season, which is from October to April, produces mean annual rainfall of 40 mm in the coastal plain to around 100 mm in the mountainous areas (Şen, 1983). Extreme rainfall events are caused by a combination of disturbances from the winter Mediterranean and the Sudan trough, which usually generates extreme convective rainfall events over the study area (Alahmadi et al., 2014).

B. *Data Set*

Hydrological network in the KSA has been established since 1960s by Ministry of Environment, Water, and Agriculture (formerly Ministry of Agriculture and Water). There are 20 daily rainfall stations in the study area with their locations in Figure 1. The data of these rain gauges span the period from 1968 to 2015. Table 1 presents the general information about these stations. The station elevations ranges from 119 m up to 1084 m above the mean sea level and the mean annual rainfall (MAR) varies from 24.6 mm up to 102.0 mm. The record lengths span over 20 to 50 years with an average about 42 years.

III. METHODOLOGY

In this study, L-moments approach is implemented starting with computation of l-moment ratios and two data screening approaches including discordancy measure (D_i), and Homogeneity test (H), and subsequently two further approaches are implemented to select the best fitted PDF which are Goodness of fit test (Z_{dist}) and L-moments ratio diagram (LMRD). Finally, the extreme rainfall quantiles are computed and the regional growth curve is developed using the best fit distribution.

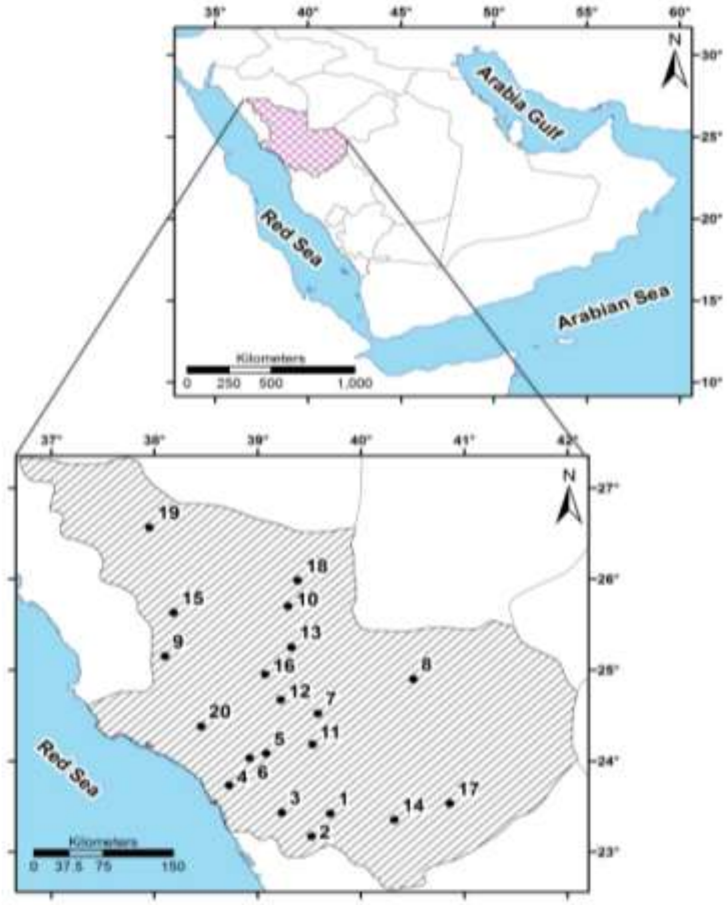

Fig.1: Location of rainfall gauge stations in Madinah Province

Table 1: General information of rainfall stations

ID	Code	Name	Longitude	Latitude	Altitude	MAR	Years
1	J109	Faqair	39.70	23.42	682	89.4	50
2	J110	Malbanah	39.52	23.17	389	63.7	50
3	J112	Umm Albirak	39.23	23.43	210	51.0	40
4	J117	Bader	38.72	23.73	119	42.0	42
5	J118	Mosijid	39.08	24.08	471	74.0	50
6	J133	Umm Dayan	38.92	24.03	310	71.0	44
7	M001	Madina	39.58	24.52	590	59.0	48
8	M004	Hinakyiah	40.51	24.90	865	72.8	45
9	M101	Ise	38.10	25.15	576	51.0	45
10	M102	Khayber	39.29	25.70	774	42.4	48
11	M103	Bir Mashi	39.53	24.18	660	44.0	48
12	M108	Meleileh	39.22	24.67	641	56.0	35
13	M109	Silsilah	39.32	25.25	864	46.4	41
14	M110	Suwayriqiyah	40.32	23.35	870	102	38
15	M111	Dhulaiah	38.18	25.63	295	24.6	37
16	M112	Buwayr	39.07	24.95	681	42.0	45
17	M113	Almahd	40.87	23.50	1084	100	32
18	M116	Oqailah	39.39	25.98	791	24.7	28
19	M117	Al ula	37.95	26.57	663	42.0	38
20	W109	Sowayq	38.45	24.38	177	46.0	38

A. L-moments computation

L-moments approach is developed by Hosking (1990) as modification of Probability Weighted Moments (Greenwood et al. 1979). They are one of the most popular methods for the PDF parameter estimations based on the linear combinations of the order statistics for the sample data that are arranged in ascending order. Let $x_1, x_2, \dots, x_n$ as the sample with size n and $x_{1:n} \le x_{2:n} \le \cdots \le x_{n:n}$ the ordered sample then the following four parameters are defined.

$$\beta_0 = n^{-1} \sum_{j=1}^{n} x_{j:n}$$

$$\beta_1 = n^{-1} \sum_{j=2}^{n} \frac{j-1}{n-1} x_{j:n}$$

$$\beta_2 = n^{-1} \sum_{j=3}^{n} \frac{(j-1)(j-2)}{(n-1)(n-2)} x_{j:n}$$

and

$$\beta_3 = n^{-1} \sum_{j=4}^{n} \frac{(j-1)(j-2)(j-3)}{(n-1)(n-2)(n-3)} x_{j:n}$$

Using the PWMs, L-moments (λ_r) can be computed by,

$$\lambda_1 = \beta_0$$
$$\lambda_2 = 2\beta_1 - \beta_0$$
$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$$
$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0$$

where $\lambda_1$is the mean of the distribution (or L-location), while $\lambda_2$is the measure of dispersion or scale (L=scale) of the distribution. The L-moment ratios (τ_r) are then expressed as,

$$\tau_r = {}^{\lambda_r}/_{\lambda_2} \qquad r = 3, 4, \dots$$
$$L - C_V = \tau = {}^{\lambda_2}/_{\lambda_1}$$
$$L - C_s = \tau_3 = {}^{\lambda_3}/_{\lambda_2}$$
$$L - C_k = \tau_4 = {}^{\lambda_4}/_{\lambda_2}$$

where L-Cv (τ) is the coefficient of L-variation, L-Cs (τ_3) is the L-skewness and L-Ck (τ_4) is the L-kurtosis of the distribution.

Regional frequency analysis based on L-moments approach as by Hosking and Wallis (1997) supposes that the data are available at N sites, with site i having sample size ni and observed data xij , j =1, 2, …, ni.

Four evaluation tests are used in this study, which are the discordancy measure (Di), Homogeneity test (H) goodness of fit test (Zdist), and L-moment ratios diagram (LMRD).

B. Discordancy measure (Di)

Discordancy measure (Di) identifies the unusual sites in the region or those sites that do not appear to belong to the cloud of sample L-moments (L-Cv, L-Cs, L-Ck). Let $u_i = [\tau_i, \tau_{3i}, \tau_{4i}]^T$ be a vector containing the L-moment ratios for site i then the group average of this vector can expressed as,

$$\bar{U} = N^{-1} \sum_{i=1}^{N} u_i$$

The sample covariance matrix of sum of squares and cross- products can be defined as follows.

$$S = (N-1)^{-1} \sum_{i=1}^{N} (u_i - \bar{U})(u_i - \bar{U})^T$$

The Discordancy measure can be computed using the as,

$$D_i = \frac{1}{3} N (u_i - \bar{U})^T S^{-1} (u_i - \bar{U})$$

Hosking and Wallis (1993) suggested that sites with Di ≥ 3 can be considered as unusual (discordant) with the group as a whole.

C. Homogeneity test (H)

It is used to estimate the degree of homogeneity in a group of sites and to assess whether the sites might reasonably be treated as a homogeneous region. *H* estimated by using the regional average of L-moment ratios (L-C_v, L-C_s, L-C_k), which are represented as follows:

$$t^R = \sum_{i=1}^{N} n_i\, t^{(i)} \Big/ \sum_{i=1}^{N} n_i$$

$$t_3^R = \sum_{i=1}^{N} n_i\, t_3{}^{(i)} \Big/ \sum_{i=1}^{N} n_i$$

$$t_4^R = \sum_{i=1}^{N} n_i\, t_4{}^{(i)} \Big/ \sum_{i=1}^{N} n_i$$

The weighted standard deviation of the at-site sample of L-moments is calculated as,

$$V = \left\{ \sum_{i=1}^{N} n_i \left(t^{(i)} - t^R\right)^2 \Big/ \sum_{i=1}^{N} n_i \right\}^{1/2}$$

Kappa distribution is then fitted to the regional L-moment ratios $(1, t^R, t_3^R, t_4^R)$. A large number (*Nsim*) of regional realization with N sites and the same record lengths as their real world counterparts, is simulated, for each region, where *V* is calculated. The mean (μ_V) and standard deviation (σ_V) of the *Nsim* are computed from these simulations. The homogeneity measure then can be computed as,

$$H = \frac{(V - \mu_V)}{\sigma_V}$$

If *H* < 1 then the region is “acceptably homogeneous”; if 1≤ *H* < 2 then the region is “possibly heterogeneous” and finally, if *H* ≥ 2 then the region is “definitely heterogeneous”.

D. Goodness of fit test (Z_{dist})

This test is used to select from a number of distributions, the one that fits the data closely by how well the L-skewness and L-kurtosis of the fitted distribution match the regional average L-skewness and L-kurtosis of the recorded data. In this test, the three parameter distributions are usually used such as the generalized logistic (GLO), generalized extreme value (GEV), generalized Normal (GNO), generalized Pareto (GPA) and Pearson type III (PE3). Goodness of fit test (Z_{dist}) can be computed as follows.

$$Z^{Dist} = \left(\tau_4^{Dist} - \bar{t}_4\right) / \sigma_4$$

Herein, Z^{Dist} is the goodness fit value of the three parameter distribution, τ_4^{Dist} is the L-kurtosis of the fitted distribution, Dist can be any of GLO, GEV, GNO, GPA, or PE3. $\bar{t}_4$ is the regional average L-kurtosis and σ_4 is the standard deviation of $\bar{t}_4$, which is computed by repeated simulation of kappa distribution in the same manner as used in the calculation of the homogeneity test described above. The distribution is considered as a good fitting to the regional data provided that Z^{Dist} value is sufficiently close to zero, a reasonable criterion being $|Z^{Dist}| \leq 1.64$.

E. L-moment Ratio diagram (LMRD)

L-moments ration diagram developed by Hosking (1990) is a graphical plot between L-skewness and L-kurtosis by comparing visually sample L-moment ratios to theoretical values. LMRD can be used as a guide tool in selecting an appropriate distribution (Vogel and Wilson, 1996; Peel et. al., 2001). The distribution with theoretical value visually close to sample values can be considered as the most suitable PDF that can represent the sample data well. This evaluation test is used as a supportive visual evaluation to ensure that the selected overall best distribution fits the observed data well.

F. Regional growth curve development

The regional average L-moment ratios (t^R, t_3^R, t_4^R) are used to compute the parameters of best fit (candidate) distribution. These parameters are then used to calculate the standardized quantiles for the region at specific recurrence interval (probability of non-exceedance). Finally, the regional growth curve of the candidate distribution is developed.

IV. RESULTS AND DESCUSIONS

A. *L-moments computation*

L-moments statistics are computed for each site using observed rainfall data through the use of lmomco R package developed by Asquith (2011, 2017).Table 2 shows the first four l-moment ratios, which are mean L-Cv, L-Cs, and L-Ck. The mean ranges from 13.42 to 31.12 mm, whereas the L-CV variation domain is from 0.29 to 0.44, which implies high to very high variability. On the other hand, the L-Cs has values in the range from 0.118 to 0.376, which implies high to very high skewness. Finally, L-Ck ranges from 0.033 to 0.338.

Table 2: Computed L-moment statistics

ID	Mean	L-C_v	L-Cs	L-C_k	ID	Mean	L-C_v	L-Cs	L-C_k
1	29.49	0.3433	0.1182	0.0481	11	22.75	0.3330	0.2572	0.2027
2	31.12	0.4127	0.1728	0.0330	12	21.15	0.4180	0.1717	0.0662
3	25.19	0.3616	0.2484	0.1345	13	20.38	0.3979	0.2554	0.1544
4	20.31	0.4321	0.2740	0.1425	14	26.20	0.2943	0.2264	0.2341
5	27.95	0.4381	0.3352	0.2848	15	14.50	0.3664	0.1308	0.1496
6	26.85	0.3530	0.2382	0.0505	16	15.52	0.4100	0.2643	0.1616
7	22.20	0.4120	0.3021	0.2575	17	20.88	0.3180	0.2159	0.3380
8	20.92	0.3506	0.3014	0.2106	18	13.42	0.4090	0.2625	0.1294
9	20.61	0.3704	0.3137	0.1444	19	19.41	0.4037	0.3755	0.2473
10	29.53	0.3364	0.1401	0.0568	20	18.26	0.4415	0.1974	0.0396

B. *Discordancy measure (D_i)*

Discordancy measure (D_i) is also computed for each site assuming that the 20 sites constitute a single region. The critical value of D_i for all sites is 3. It can be noticed from Table 3 that Di ranges from 0.06 to 2.9 and all 20 sites have D_i values less than 3.0, which implies that they come from one region.

Table 3: Discordancy measure (***D_i***) for each site

ID	D_i	ID	D_i
1	1.18	11	0.55
2	0.84	12	0.86
3	0.25	13	0.06
4	0.52	14	1.51
5	1.73	15	1.50
6	1.51	16	0.19
7	0.83	17	2.90
8	0.66	18	0.21
9	1.14	19	1.35
10	1.02	20	1.19

C. *Homogeneity test (H)*

The computed L-moment ratios in Table 2 are used to calculate the regional average L-moment ratios (t^R, t_3^R, t_4^R), which are 0.38, 0.24, and 0.15, respectively. These regional average L-moment ratios are then used in the simulation of the kappa distribution and the homogeneity test (H) is 0.647, which implies that the region is "acceptably homogeneous".

D. *Goodness of fit test (Zdist)*

In this test, five 3-parameter distributions, generalized logistic (GLO), generalized extreme value (GEV), generalized Normal (GNO), generalized Pareto (GPA) and Pearson type III (PE3) are evaluated. . Table 4 has the computed $|Z^{Dist}|$ values for each distribution, which varies from 0.78 to 4.1 and two distributions with $|Z^{Dist}| \leq 1.64$, which are GNO and PE3, and the distribution with lowest $|Z^{Dist}|$ value (PE3) will be selected as the best fit distribution.

E. *L-moment Ratio diagram (LMRD)*

In Figure 2, L-moments ratio diagram (LMRD) is given, where the curves show the theoretical relationships between L-skewness and L-kurtosis of various candidate distributions. The points show the regional weighted average and at- sites values of L-skewness (L-C_s) and L-kurtosis (L-C_k) are already presented in Table.2. It can be seen that the regional weighted average of L-C_s and L-C_k value lies between PE3 and GNO distributions and closer to PE3, which is the same conclusion for the Z_{dist}, and finally, PE3 distribution is selected as a candidate distribution for the rainfall in Madinah region.

Table 4: Goodness of fit test

Dist	$\|Z^{Dist}\|$
GLO	4.10
GEV	1.88
GNO	0.94
PE3	0.78
GPA	3.60

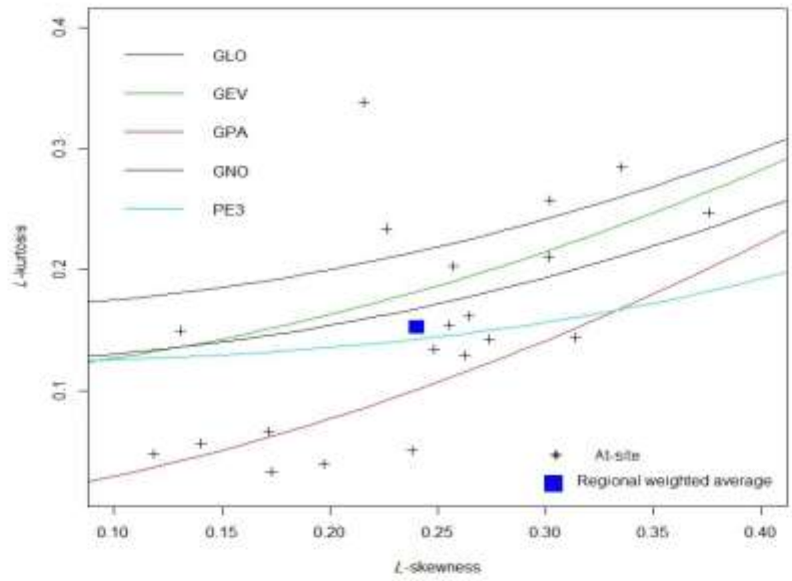

Fig. 2: L-moments ratio diagram of rainfall in Madinah region

F. *Regional growth curve development*

The regional average of L-moment ratios (t^R, t_3^R, t_4^R) are used to compute the three parameters of PE3 distribution (mu, sigma, gamma), which are 1.00, 0.719 and 1.448, respectively. These parameters are then used to calculate the regional quantiles on the basis of the PE3 distribution function. Table 4 presents the regional quantiles, while Figure 2 is for the regional growth curve in the region.

Table 5: Regional quantiles for different probabilities

Probabilities	0.1	0.2	0.5	0.8	0.9	0.95	0.98	0.99	0.999
Regional Quantiles	0.26	0.4	0.83	1.5	1.96	2.4	2.96	3.37	4.71

V. CONCLUSIONS

Arid regions suffer from the scarcity of rain gauges and the record length limitation of observed rainfall data, which are usually short to insure reliable quantile estimates. Regional frequency analysis is one of the most popular approaches used to compensate the limitation of data availability. The applications of these approaches have not yet been widely investigated in arid regions. In this study, 20 daily rainfall stations in Madinah region, Western Kingdom of Saudi Arabia (KSA) are processed to develop the regional growth curve of the maximum daily rainfall using l-moments approach. Computation of l-moments showed high variability and high skewness of the rainfall data in the region (L-CV range from 0.29 to 0.44,

which implies high to very high variability, while the L-Cs ranges from 0.118 to 0.376 also with implication of high to very high skewness). Discordancy measure (Di) for each site was less than 3.0 showing that the 20 sites constitute as one region. Homogeneity test (H) was <1, which meant that the region is “acceptably homogeneous”. Five 3-parameter distributions were evaluated using Goodness of fit test (Zdist) and L-moments ratio diagram (LMRD) showing that generalized Normal (GNO) and Pearson type III (PE3) distributions were the best with superiority of PE3 distribution.

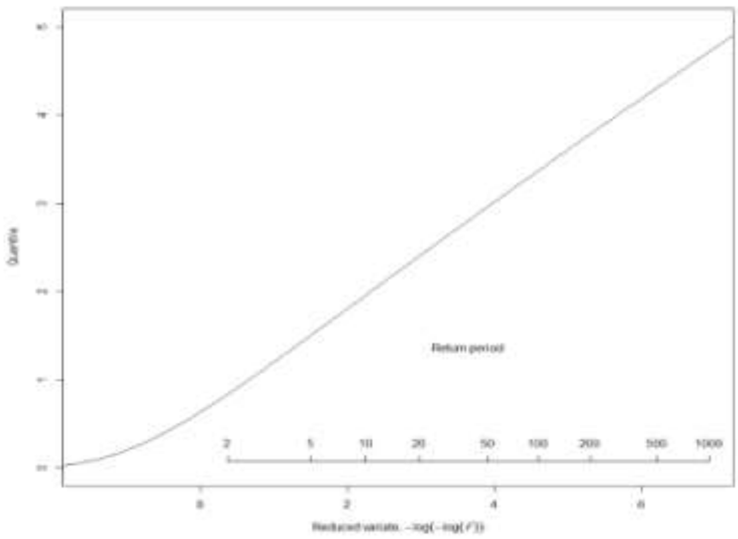

Fig. 3: Regional growth curve for Madinah region using PE3 distribution

Finally, various quantiles at specific recurrence interval (probability of non-exceedance) were computed for the region and the regional growth curve of the candidate distribution was developed, which can be used for ungauged sites or sites with limited rainfall data.

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