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Qualitative Analysis of Prey Predator System With Immigrant Prey

Habtay Ghebrewold and Goteti V. Sarma
Department of Mathematics, Eritrea Institute of Technology Mainefhi, Asmara Eritrea

ABSTRACT: The predator prey system with immigrant prey is introduced and studied through a suitable mathematical model. Existence conditions for interior equilibrium point and their stability is studied under suitable ecological restrictions. Global stability of the system around equilibrium point is also discussed.

I. INTRODUCTION

Interaction between the predators and prey is always influenced by various ecological factors like climatic conditions, distribution of prey, predators ability to detect the prey etc. These interactions caught the attention of several authors and well studied by constructing suitable mathematical models. Some authors studied the affect of providing additional food to the predator, the effect of securing prey in refugee camps in achieving the ecological equilibrium. The element of immigration in any ecological model exerts a lot influence on the existing system which can make or mar the equilibrium in the existing system. By studying systematically the role of immigration in any prey predator system one can understand the inherent inter dependency of the system which enables ecological planners to set rules to permit immigration in to the system.

This article is organized as follows. In section 2 a mathematical model is constructed which incorporates the immigrant prey in the existing predator prey models. Section 3 studies the existence of equilibrium points and establishes the boundedness of the modeal. Section 4 elucidates the dynamical behavior of the system around equilibrium points is analyzed. In section 5 global behavior of the system is discussed. In section 6 scope for further research is mentioned.

II. MATHEMATICAL MODEL FORMULATION:

Let *N* and *T* represent the amount of biomass of prey and predator. Let *A* be the amount of immigrant prey entering into the system. Let *r* represents the natural growth rate of prey and *k* is the maximum sustainable prey populations without predator and immigration. We assume that the predation is according to Holings type II functional response with *c* as the prey to predator conversion coefficient, e_1 as predation ability of the predator and h_1 being the ability of predator to detect immigrant prey.

With the above ecological parameters we propose the following basic mathematical model to understand the effect of immigration in prey predator system

$$\left.\begin{aligned} N^{/} &= (1-A)rN\left(1-\frac{N}{K}\right)-\frac{e_1ANP}{1+e_1h_1AN} \\ P^{/} &= P\left(-m+c\,\frac{e_1AN}{1+e_1h_1AN}\right) \end{aligned}\right\} \qquad (1)$$

Without loss of generality we can restrict *A* in [0, 1].

III. EQUILIBRIUM ANALYSIS :

For equilibrium points consider the stable state equations of (1)

$$\begin{aligned} (1-A)rN\left(1-\frac{N}{K}\right)-\frac{e_1ANP}{1+e_1h_1AN} &= 0 \\ P\left(-m+c\,\frac{e_1AN}{1+e_1h_1AN}\right) &= 0 \end{aligned}$$

Clearly the above system has 3 solutions only which gives the equilibrium points of the dynamical system (1) namely

i. $E_0 = (0, 0)$ a trivial equilibrium point
ii. $E_1 = (k, 0)$ an axial equilibrium point and
iii. The equilibrium point of coexistence $E_2 = (N^*, P^*)$ where

$$N^* = \frac{m}{e_1A(c-mh_1)} \; and \; P^* = (1-A)\frac{cr}{m}N^*\left(1-\frac{N^*}{k}\right)$$

For the existence E_2 it is essential that $c > mh_1$ which can be justified in perspective of ecological parameters and also essentially $A > \frac{m}{e_1k(c-mh_1)}$.

It is interesting to observe that the interior equilibrium point is always dependent on the parametric value of biomass of immigrant prey entered in to the system which in turn dependent on other ecological parameters of the system. Therefore we can determine the amount of biomass of immigrant prey based on required stable levels of biomasses of the prey and predator of the system under investigation.

IV. DYNAMICAL BEHAVIOR OF THE SYSTEM

To understand the dynamics and local stability of the equilibrium points we use the variational principle.
The variational matrix for the system (1) is

$$J(N, P) = \begin{bmatrix} (1-A)r\left(1-\frac{2N}{K}\right)-\frac{e_1AP}{(1+e_1h_1AN)^2} & \frac{-e_1AN}{(1+e_1h_1AN)} \\ \frac{ce_1AP}{(1+e_1h_1AN)^2} & -m+\frac{ce_1AN}{(1+e_1h_1AN)} \end{bmatrix}$$

Now let us analyze each of the equilibrium points E_1, E_2 *and* E_3

1. At the equilibrium point E_1 (0, 0) the variational matrix becomes

$J(0, 0) = \begin{bmatrix} (1-A)r & 0 \\ 0 & -m \end{bmatrix}$ whose eigenvalues are (1-A) r and - m which are having opposite signs hence the equilibrium point E_1 (0, 0) is always a unstable saddle point.

2. At the equilibrium point E_2 (k, 0) the variational matrix becomes J (k, 0)

$$= \begin{bmatrix} -(1-A)r & \frac{-e_1Ak}{(1+e_1h_1Ak)} \\ 0 & -m+\frac{ce_1Ak}{(1+e_1h_1Ak)} \end{bmatrix}$$

whose eigenvalues are - (1-A) r and $-m+\frac{ce_1Ak}{(1+e_1h_1Ak)}$ which are having opposite signs under natural assumption $m < \frac{ce_1Ak}{(1+e_1h_1Ak)}$ hence the equilibrium point (k, 0) is also always a saddle point.

Now we are going to discuss the nature of the interior equilibrium point $E_2 = (N^*, P^*)$ where $N^* = \frac{m}{e_1A(c-mh_1)}$ *and* $P^* = (1-A)\frac{cr}{m}N^*\left(1-\frac{N^*}{k}\right)$ and $c > mh_1$ and $A > \frac{m}{e_1k(c-mh_1)}$

Theorem 4.1: The interior equilibrium point $E_2 = (N^*, P^*)$ is locally asymptotically stable if $A < \frac{m}{e_1k(c-mh_1)}\left[\frac{2-m(c-mh_1)}{1-m(c-mh_1)}\right]$

Proof: The variational matrix at the interior equilibrium point is

$V(N^*, P^*) = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$

Where $v_{11} = (1-A)r\left[1-\frac{2N^*}{k}-\frac{m\left(1-\frac{N^*}{k}\right)}{1+e_1h_1AN^*}\right]$, $v_{12} = -\frac{m}{c}$,

$v_{21} = \frac{P^*}{N^*}\left(\frac{m}{1+e_1h_1AN^*}\right)$ and $v_{22} = 0$

For asymptotical stability of E_2, by Routh Hurwitz criteria

$v_{11}+v_{22} < 0$ and $v_{12}+v_{21} > 0$

For this $1-\frac{2N^*}{k}-\frac{m\left(1-\frac{N^*}{k}\right)}{1+e_1h_1AN^*} < 0$

$$\Rightarrow \frac{2N^*}{k}+\frac{m\left(1-\frac{N^*}{k}\right)}{1+e_1h_1AN^*} > 1$$

$$\Rightarrow \frac{m\left(1-\frac{N^*}{k}\right)}{1+e_1h_1AN^*} > 1-\frac{2N^*}{k}$$

On substituting N^* and rearranging gives the restriction

$$A < \frac{m}{e_1k(c-mh_1)}\left[\frac{2-m(c-mh_1)}{1-m(c-mh_1)}\right].$$

Surprisingly when this restriction holds, it satisfies $v_{12}+v_{21} > 0$ also.

Hence the proof completed.
Now we are going to establish the global stability of the system through the following theorem:
Theorem 4.2 : For each A, $0 \leq A \leq 1$, any solution of the ecological system with positive initial conditions enters the bounded region R in finite time and remains there thereafter.
Proof: Take the curve $mP = c(1-A)r\,N\left(1-\frac{N}{K}\right)$
Then clearly $mP \leq F(N)$ where $F(N) = crN\left(1-\frac{N}{k}\right)$
Here F(N) is a parabola with downward opening which has maximum at $N = \frac{K}{2}$ and its maximum value is $\frac{crk}{4}$.
Also $F(N) = 0$ has a unique positive solution at $N = k$
Now define $F^*(N) = crN\left(1-\frac{N}{k}\right) + \varepsilon$ where ε can be choosen so that F^*(N) = 0 has a positive root.
Now choose l^* such that the line $cN + P = l^*$ either tangent to F^*(N) or passes through the positive root of $F^*(N) = 0$.
Then for (N, P) on F^*(N), $c(1-A)r\,N\left(1-\frac{N}{K}\right) - mP \leq -\varepsilon$
Now consider $I = cN + P$
Then $I' = cN' + P'$

$= c(1-A)r\,N\left(1-\frac{N}{K}\right) - mP \leq -\varepsilon$ for every $I > l^*$

Thus for any initial conditions in the first quadrant the solution of the ecological system reaches to the line $cN + P = l^*$ in finite time.
Hence the solution enters the region R bounded by the lines $N \geq 0, P \geq 0, cN + P \leq l^*$ and never leaves it.

V. GLOBAL STABILITY ANALYSIS:

In this section we will study the global stability of the system around the interior equilibrium point $E_2 = (N^*, P^*)$.
First let us construct a suitable Lyapunov function as
$L(N,P) = c_1(N - N^* lnN) + c_2(P - P^* lnP)$ where c_1, c_2 are positive constants.
Then clearly $L(N,P) > 0$ i.e L is a positive definite function
Now

$$L^{/}(N,P) = c_1\frac{N^{/}}{N}(N - N^*) + c_2\frac{P^{/}}{P}(P - P^*)$$
$$= c_1\left[(1-A)r\left(1-\frac{N}{K}\right) - \frac{e_1 AP}{1+e_1 h_1 AN}\right](N - N^*) + c_2\left(-m + c\frac{e_1 AN}{1+e_1 h_1 AN}\right)(P - P^*)$$

Now it can be easily shown that $L^{/}(N,P) \leq 0$ for a proper choice of the constants c_1, c_2 which depends on the parameters A, K, m and e_1.

VI. CONCLUSIONS AND SCOPE FOR FURTHER INVESTIGATIONS

In this research article the effect of immigrant prey to the existing prey predator model is studied. It is observed that the global stability of the model can be achieved through the careful allowance of amount of immigrant prey in to the system. This model also offers to study in detail for periodic solutions, occurrences of Hopf bifurcation and optimal analysis for further investigations. Further study is also possible on replacing the immigration prey quantity with a variable.

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