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Garch Models in Value-At-Risk Estimation for REIT

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Abstract:- In this study we investigate volatility forecasting of REIT, from January 03, 2007 to November 18, 2016, using four GARCH models (GARCH, EGARCH, GARCH-GJR and APARCH). We examine the performance of these GARCH-type models respectively and backtesting procedures are also conducted to analyze the model adequacy. The empirical results display that when we take estimation of volatility in REIT into account, the EGARCH model, the GARCH-GJR model, and the APARCH model are adequate. Among all these models, GARCH-GJR model especially outperforms others.

Keywords:- Value-at-Risks, GARCH-type models, normal distribution, Student-tdistribution forecast volatilities, backtesting

I. INTRODUCTION

Real Estate Investment Trust (REIT) is a crucial financial commodity. For many investors, it is possible to acquire ownership in real estate ventures, as well as some cases operate commercial properties like apartment complexes, office buildings, hospitals, and so on. There are three major types of REITs in the US and they are Equity REITs, Mortgage REITs, and Hybrid REITs. Nowadays, REIT is becoming more popular for investors to invest. Hence it is crucial to understand their price movements and calculate the return and volatility structure.

Considering the importance of relatively accurate volatility forecasting, many pieces of literature have emerged to model and predict volatility in financial markets to calculate value-at-risk (VaR), derivatives pricing and make the hedging decision. A lot of papers focus on aspects of REIT volatility. Stevenson (2002)[12], utilized univariate models to analyze the volatility dynamics on monthly REIT returns. Devaney (2001)[13] used a GARCH-M model with respect to monthly REIT data, which examines the relationship between interests rates and REIT volatility primarily. What's more, Winniford (2003)[14] and Najand and Lin (2004)[15] provided further evidence, which suggests that volatility shocks are persistent, concerning the daily volatility dynamics in the REIT sector. For simplicity and conventionality, one usually assumes that asset returns of econometric time series follow a normal distribution. However, Hsu, Miller and Wichern (1974)[16] and Hagerman (1978)[17] showed that the normal distribution does not fit asset returns significantly. Thus non-Gaussian time series have begun to be noticed and development of forecasting methods is on the way gradually. Accurate volatility forecasts have become a crucial issue because of the increasing volatility. Benjamas and Rizz (2009)[18] utilized the GARCH model to estimate the volatility of U.S Equity REIT based on data of U.S Equity REIT from 1993 to 2006. Cotter and Stevenson (2006)[19] adopted a multivariate GARCH based model to analyze the volatility in REIT.

One widely used measurement of the stock risk is the so-called Value-at-Risk, VaR for short. US investment bank J.P. Morgan introduced and incorporated it in their risk management model RiskMetrics. The Value-at-Risk of a stock is mainly known as the maximum loss that may be suffered on that stock in a short period of time. More precisely, a VaR(α) is the α-quantile of the distribution of the maximum loss, typically α is chosen in the range of 0.01 to 0.05. By varying the value of α, one can investigate a whole risk distribution of the maximum loss.

An investor needs to estimate the volatility of REIT for improving the measure for VaR. As have been shown in empirical studies, financial instruments have heteroscedasticity in the variance. The milestones addressing this observation are the ARCH and GARCH models, which were introduced by Engle (1982)[8] and Bollerslev (1986)[1]. Later on, many new generalized varieties of GARCH models have emerged, which according to different factors to capture the changing volatility over time. However, when we forecast the volatility for all kinds of financial data, it is difficult to say which of the models from the GARCH family is the best. The examined models need to be refined to specific data sets since the availability of plethora of different GARCH models. This paper focus on four of the most influential models, including GARCH(1,1), EGARCH(1,1), GARCH-GJR(1,1), APARCH(1,1).

This paper is organized as follows. Section 2 introduces the sample data and the statistical parameters. We review certain four GARCH-type used in this paper in section 3. We introduce two ways of backtesting VaR in Section 4. Section 5 contains the empirical results with respect to REIT daily log return. At last, we give our conclusion in section 6.

II. DATA AND DESCRIPTIVE STATISTICS

Data Description

In this paper, we mainly concentrate on the daily REIT price time series over the ten-year period. There were 2,492 daily data points from Jan. 3, 2007 to Nov. 18, 2016. The collection of REIT stock was from Yahoo Finance. We use the daily closing price to investigate the portfolio's performance.

Furthermore, to develop an accurate track record of asset performance, we use Pt to denote the daily closing price of a stock, for integer t $\in$ Z. The stochastic properties of the price time series $\{P_t\}$ are characterized by the relative log returns, which are defined as:

$$r_t = 100(\log P_t - \log P_{t-1})$$

(1)

The daily closing values of REIT and its returns are displayed as following.

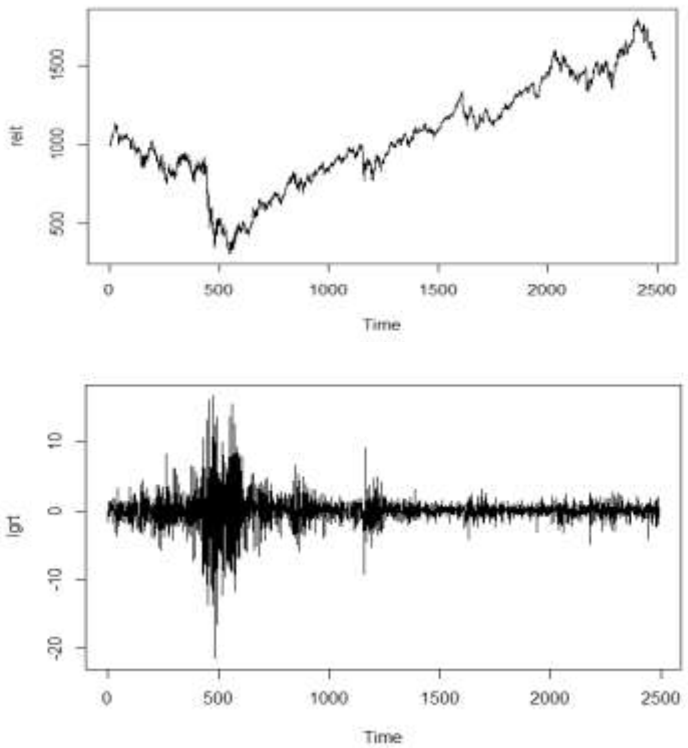

Fig.1: Time plots of REIT stock from 2007-01-03 to 2016-11-18

The upper panel of Figure 1 displays the time plot of daily closing price and the lower panel shows the daily log return. The daily log returns plot shows a recent negative expected return trend. Note that the volatility is relatively stable before 2010, which is followed by more intense turbulence.

Table Ⅰ: Summary Statistics of the REIT Daily Log Returns January 03, 2007-November 18, 2016

Observations	**Kurtosis**	**Skewness**	**Standard Deviation**	**Range**	**Mean**
2491	13.1502	-0.1140	2.3220	(-21.4847, 16.8119)	0.0173

As demonstrated in Table Ⅱ, the mean is low while the corresponding standard deviation is high. Meanwhile, the value of skewness and kurtosis are far away from the standard normal distribution, which implies that the return has a leptokurtic distribution with fat tail. Thereafter, we apply two ways to test normality, the Jarque-Bera test and ShapiroWilk test. Both of them reject the null hypothesis of normal distributed at significance level. Furthermore, we use KPSS test to examine the stationary property of the daily log return, which indicates that the series have weak stationarity. To check the autocorrelation of the returns, we use Ljung-Box test on returns and square returns. The value in the table has shown that all sample returns have long memory.

Test for Normality

While analyzing the time series, one usually assumes that the process follows normal distribution. However, it sometimes contrasts to the truth. Our research on REIT Stock demonstrates that the return is not normally distributed. Here we use three methods to verify it.

At first, we use the Q-Q plot to test the fitting for normal distribution.

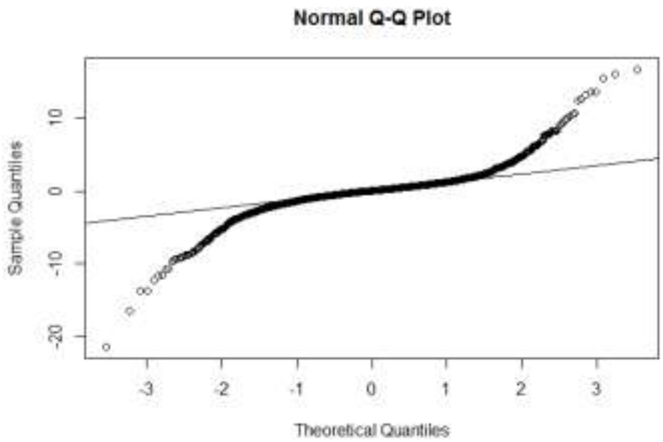

Fig.2: Quantile-Quantile plot of returns against the normal distribution

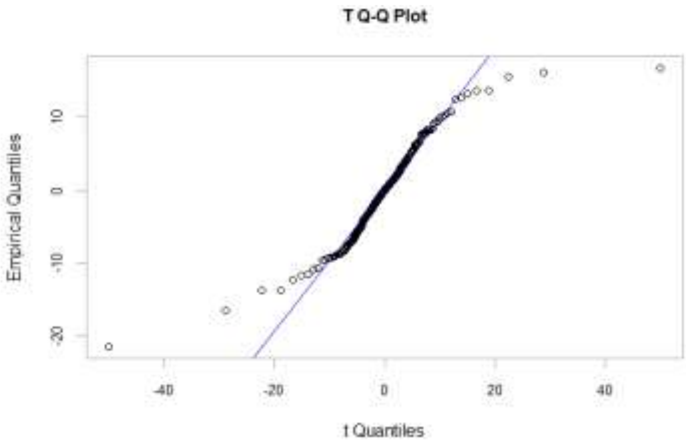

Fig.3: Quantile-Quantile plot of returns against the Student-t distribution

The plots 2 and 3 are the Q-Q plot of the empirical distribution of the daily returns (y-axis) against the normal distribution (x-axis). As shown in the plots, empirical distribution of the daily returns exhibits heavier tails than the normal distribution, which means that normal distribution is unrealistic for the return process. Compare to the normal distribution, the Student-t distribution fits better. So we test all GARCH models with Student-t distribution in our study.

To support our observation, here we use two tests. The first one is the so-called Jarque-Bera test, JB for short, which can be used to test similarity in kurtosis and skewness of the sample data, compare to a normal distribution. The test statistic is defined as:

$$JB = \frac{6}{n}(S^2 + \frac{1}{4}(K-3)^2))$$

(2)

where n is the sample size, S is the sample skewness and K is the sample kurtosis. If the sample data follows normal distribution, the statistic JB should follow asymptotically a Chi-squared distribution with two degrees of freedom. The null hypothesis is that the sample data fit the normal distribution.

The second method named Shapiro-Wilk test, which is considered one of the most powerful tool to test normality. The Shapiro-Wilk test statistics is defined as

$$W = (\sum_{t=1}^{T}(a_t r_t)^2)(\sum_{t=1}^{T}(r_t - \bar{r})^2) \qquad (3)$$

where r_t is the t-th order statistic, $\bar{r}$ is the sample mean, ($a_1, a_2 \cdots a_T$) are the weights. The null hypothesis is W = 1 which indicates the normal distribution. We reject the null hypothesis if p-value is less than the significance level α.

In Table Ⅱ, which shows results of Jarque-Bera test and Shapiro-Wilk test, we can reject the null hypothesis of a normal distribution at all significance levels.

Test for Correlations

To check autocorrelation, we choose the Ljung-Box test by Ljung and Box (1978)[20], which is used to check time serial correlation of returns. The null and alternative hypothesis of the Ljung-Box test is defined respectively as follow:

$$H_0: \hat{\rho}(1) = \hat{\rho}(2) = \cdots = \hat{\rho}(m) = 0 \quad \text{vs} \quad H_1: \hat{\rho}(i) \neq 0 \text{ for some } i \in \{1, 2, \cdots, m\}$$

where n is the sample size, m is the number of lags being tested, and $\hat{\rho}(l)$ is called the lag-l autocorrelation of $\{r_t\}$, i.e. the correlation between $\{r_t\}$ and $\{r_{t-l}\}$.

The Ljung-Box Q test statistic is

$$Q(m) = n(n+2)\sum_{l=1}^{m} \frac{\hat{\rho}(l)^2}{n-l} \tag{4}$$

As Ljung and Box proposed, if we assume that $\{r_t\}_{t=1}^{n}$ are independent, identically distributed, the approximate distribution of Q(m) should be Chi-squared with m degrees of freedom. Here we reject the null hypothesis if Q is too large or the p-value of Q(m) is less than or equal to the significance level of α. The graph of returns and square returns is shown in Figure 4.

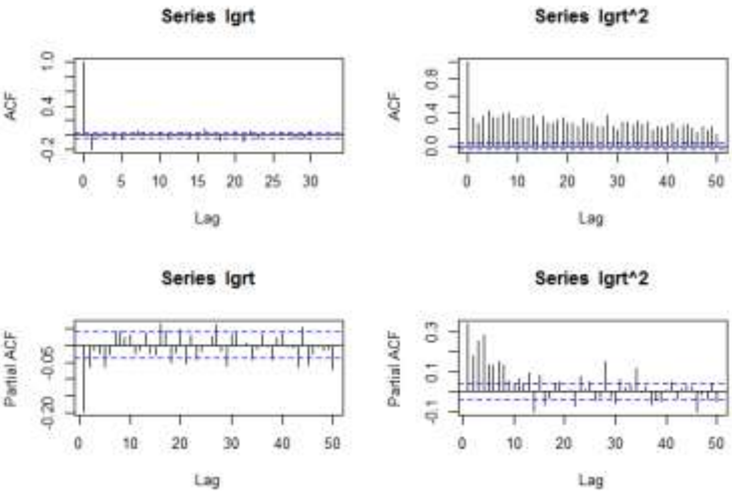

Fig.4: Sample autocorrelation coefficients and partial autocorrelation coefficients for REIT daily log returns and square returns

Descriptive statistics and hypothesis test results for REIT returns are as follows.

Table Ⅱ: Tests for the REIT Daily Log Returns January 03,2007-November 18,2016

p-value	**Statistic**	
0.1	0.08	KPSS test
2.2e-16	0.82	Shapiro-Wilk test
2.2e-16	17989.2	Jarque-Bera test
2.2e-16	108.24	LB-Q(5)
2.2e-16	144.24	LB-Q(16)
2.2e-16	1547.3	LB-Qs(5)
2.2e-16	4704.7	LB-Qs(16)
2.2e-16	251.53	ARCH test Qs(5)
2.2e-16	380.6	ARCH test Qs(16)

As demonstrated in Table Ⅱ, the null hypothesis of weak stationarity fails to be rejected at the 5% significant level. The Ljung-Box Q statistics on the 5th and 16th lags of the REIT returns are significant.

Meanwhile, the Ljung-Box test results for square returns confirm that ARCH effect presents and return series have long memory.

Based on the analysis above, we can conclude that the daily log return are stationary, nonnormal distributed and have long memory. All of these test results show that the REIT return series have rather complicated statistics properties. To overcome these difficulties, we use GARCH type models to estimate the volatility and ARMA model to estimate the mean.

Methodology

Defining Value-at-Risk

VaR is such a quantity that might be lost in a portfolio of assets over a specific time period T with a specified small failure probability α. Here we set this time period as one day. Suppose a random variable X, which denotes the distribution of daily return in some financial asset, the α-quantile of the portfolio is defined to be the VaR_α:

$$P(X \leq VaR_\alpha) = \alpha$$

(5)

The VaR_α is the largest value for X such that the probability of a loss over the time horizon T is less than α. Although we can choose the parameter α arbitrarily, it is normal to choose $\alpha \in \{0.005, 0.01, 0.05\}$. Therefore the crux to estimate VaR accurately is in estimating the cut off return of VaR_α.

To estimate VaR accurately, it is essential to process accurate volatility estimates. In this context, we develop different ways to estimate volatility. When we find models to fit REIT return, we need to take the volatility clustering phenomenon in account. Bollerslevn (1986) [1] generalized ARCH model to GARCH model, which is able to capture the time-varying volatility. This GARCH model uses a linear function of the squared historical innovations to approximate conditional variance. But we cannot forget to mention drawbacks of this model, since it overlooks the leverage effect in REIT return's volatility. The EGARCH, GARCH-GJR and APARCH models are applied here to show the conditional asymmetry properties. In this paper, we are focusing upon the use of these GARCH-type models to estimate and forecast daily VaR of the Real Estate Investment Trust (REIT) stock in fixed period time.

Estimating μ_{t+1} and σ_{t+1} Using ARMA-GRACH-type Model

Let $\{r_t\}$ be the daily log return of REIT. Let F_t be the historical information about the return process available up to time t. Since the volatility and leptokurtosis exists, we make an assumption that the conditional mean of r_t fits an autoregressive average model AR(1) and the conditional volatility follows an univariate GARCH-type model. We give the representation of r_t as follow.

$$\begin{cases} r_t = \mu_t + \sigma_t \varepsilon_t \\ \mu_t = \phi_0 + \phi_1 r_{t-1} \\ \sigma_t^2 \text{ follows a GARCH-type model} \end{cases}$$

Where the innovation $\{\varepsilon_t\}$ are white noise process with zero mean and unit variance; the conditional mean is defined as $\mu_t = E(r_t \mid F_{t-1})$ and the conditional volatility is $\sigma_t^2 = \mathrm{var}(r_t \mid F_{t-1})$. In this paper, we assume $\{\varepsilon_t\}$ follows normal and Student's t-distribution respectively.

***Grach Model*:** The Generalized ARCH (GARCH) model of Bollerslevn (1986) [1] is based on an infinite ARCH specification and it allows to impose nonlinear restrictions on parameters to reduce the number of them. The GARCH(p, q) model is given by:

$$\begin{cases} r_t = \mu_t + \sigma_t \varepsilon_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \eta_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \end{cases}$$

where p is the order of GARCH and q is the order of ARCH process, α_i and β_j are parameters and we expect the sum of them is less than 1.

French, Schwert and Stambaugh (1987) [2]; Pagan and Schwert (1990) [3]; Franses and Van Dijk (1996) [4] show that the basic GARCH(1, 1) model suits well in most financial time series. Furthermore, according to Brooks (2008), it is sufficient to capture all the volatility clustering in the data if we just set the lag order (1, 1). The GARCH(1, 1) model is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1\eta_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

(6)

The GARCH-type models described above follows that positive and negative error terms have equal contribution to the volatility. However, we all know that the volatility tends to increase dramatically following bad news, according to Angabini and Wasiuzzaman (2011) [23]. Thereafter, the Exponential GARCH (EGARCH), GARCH-GJR and Asymmetric Power ARCH (APARCH) models are applied to capture the asymmetry in return volatility, i.e. leverage effect.

EGRACH Model: The Exponential GARCH model, introduced by Nelson (1991) [6] originally. For p, q > 0, the EGARCH (p, q) model is given by:

$$\log\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}[\alpha_i\eta_{t-i} + \gamma_i(|\eta_i| - E|\eta_{t-i}|)] + \sum_{j=1}^{q}\beta_j \log\sigma_{t-j}^2$$

(7)

The logarithmic form in Nelson's EGARCH model makes it possible to relax the parameters. And the conditional variance is always positive even if the coefficients are negative.

GRACH-GJR Model: Glosten, Jagannatahan and Runkle (1993) [21] developed the GARCH-GJR model, which is another kind of asymmetric GARCH models. It is given by:

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{p}\alpha_i\eta_{t-i} + \sum_{i=1}^{q}\beta_i \log\sigma_{t-i}^2 + \gamma_i I_{t-i}\eta_{t-i}^2$$

(8)

where α and β are constants, and I is an indicator function when η_{t-i} is negative.

Aparch Model

Asymmetric power ARCH (APARCH) is another asymmetric model, which was introduced by Ding Engle and Granger (1993) [22] and can be written as:

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{p}(\alpha_i|\eta_{t-i}| - \gamma_i|\eta_{t-i}|)^{\delta} + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2)^{\delta} \quad (9)$$

This model captures the leverage effect by changing the error term into a more flexible varying exponent.

We make an estimation of the following symmetric GARCH models: the GARCH(1, 1) model with normal distribution and Student's t-distribution as well as the following asymmetric GARCH models like Egarch(1, 1) with normal distribution and Student's t-distribution, GARCH-GJR model and APARCH model. The estimated results are shown as follows.

Table Ⅲ: Estimation Results of Different Volatility Models for REIT

APARCH	**GARCH-GJR**	**GARCH-GJR**	**Egarch**	**Egarch**	**Garch**	**GARCH Type**
normal	Std	normal	Std	normal	Std	distribution
0.0396	0.0700	0.0436	0.0678	0.364	0.0882	ϕ_0
-0.0363	-0.0426	-0.0375	-0.0431	-0.0415	-0.0457	ϕ_1
0.0183	0.0120	0.0191	0.0023	0.0125	0.0141	α_0
0.0926	0.0502	0.0524	-0.0626	-0.0643	0.1050	α_1
0.9109	0.9076	0.8980	0.9935	0.9892	0.8940	β_1
0.3439	0.0794	0.0862	0.0119	0.1789		γ_1
1.3989						δ
-4352.581	-4320.949	-4355.499	-4324.104	-4362.463	-4328.335	log(L)
3.5003	3.4749	3.5018	3.4774	3.5074	3.4800	AIC
3.5166	3.4912	3.5158	3.4938	3.5214	3.4940	BIC

Table Ⅲ demonstrates the results of all GARCH-type models. The log likelihood and AIC statistics shows the above specified GARCH models adequately capture the serial correlation in conditional means and variances. The nonlinear asymmetric models EGARCH, GARCHGJR and APARCH are used to capture the leverage effect. The coefficient γ_1 in these models is statistically significant at 5% significant level, which implies the existence of asymmetry. Meanwhile, the positive value means the leverage effect exists.

Table Ⅳ: Value-at-Risk in Different Models for REIT

Aparch	Garch-Gjr	Garch-Gjr	Egarch	Egarch	Garch	Garch Type
Normal	Std	Normal	Std	Normal	Std	Distribution
-1.48	-1.46	-1.43	-1.49	-1.46	-1.45	$Var_{0.01}$
-1.03	-0.93	-0.996	-0.947	-1.022	-0.911	$Var_{0.05}$

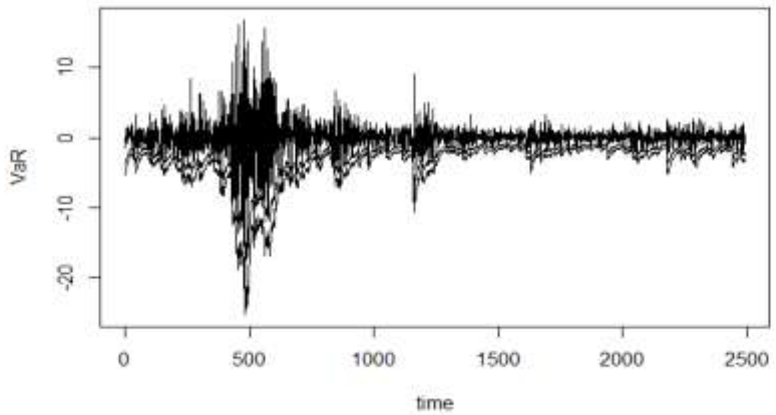

Fig.5: One-day-ahead VaR forecasts of REIT based on the GARCH-GJR model at quantile 1% and 5%

We have forecasted the volatility for one-day-ahead on the basis of estimation of parameters in all models. The estimation of VaR of REIT at quantile 1% and 5% are shown in Figure 5. And the forecasted values of the VaR at quantile 1% and 5% are shown in Table Ⅳ.

III. BACKTESTING

To gauge a model's accuracy and effectiveness, we choose to use backtest, which is a technique for approximating a model on historical data. In value at risk, backtesting is used to compare the predicted losses from the calculated value at risk with the actual losses realized at the end of the specified time horizon. This comparison identifies the periods where the value at risk is underestimated, i.e. where the original expected value at risk are less than the portfolio losses. The most popular two ways to backtest VaR are introduced by Kupiec (1995) [24] and Christoffersen (1998) [25].

UnConditional Coverage

To count the number of VaR exception is the most common test of a VaR model. We can imply that the system overestimates risk if the number of exceptions is less than the selected confidence level. Denote x as the number of exceptions and T as the number of the observations, the failure rate is defined by x/T. If we fix α be the confidence level and let $p = 1 - \alpha$, then the null hypothesis is that the expected proportion of exception is equal to α, which means that H_0: $x/T = \alpha$.

Under the null hypothesis, the statistic function is given by:

$$LR_{uc} = 2\ln((1-\frac{x}{T})^{T-x}(\frac{x}{T})^{x}) - 2\ln((1-\alpha)^{T-x}\alpha^{x}),$$

(10) which is a Chi-square distribution with one degree of freedom. Therefore, we can reject H_0 if the value of LR is greater than the critical value or the p-value is less than the significance level.

Conditional Coverage

The unconditional coverage tests only focus on the number of exception, whereas our expectation in theory is those exceptions can be spread evenly. Since occurrence of large losses is more likely to lead to disastrous events, VaR users want to detect the clustering behavior of exceptions. Christoffersen (1998) [25] generalizes the Kupiec test by including a separate statistic for independence of exceptions.

The test is proposed first by defining an indicator I_t satisfies: equals to 1 if VaR is exceeded and equals to 0 if VaR is not exceeded. Then define n_{ij} as the number of days when j occurred under the assumption that i occurred on the previous day. What's more, the probability of observation of an exception on condition i is denoted by π_i:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}, \quad \pi_1 = \frac{n_{11}}{n_{10} + n_{11}}, \quad \pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}$$

(11)From the definition, we imply the model is accurate if $\pi_0 = \pi_1$. The test statistic is given by:

$$LR_{ind} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right)$$

(12)We obtain a joint test which examines both properties of a great VaR model by combining LR_{uc} and LR_{ind}, i.e. conditional coverage:

$$LR_{CC} = LR_{uc} + LR_{ind}$$

(13)where LR_{cc} follows Chi-squared distribution with two degree of freedom. We reject the test if LR_{cc} is greater than the critical value of χ^2 distribution.

Table Ⅴ: Backtesting Results of VaR for GARCH-type Models

Aparch	**Garch-Gjr**	**Garch-Gjr**	**Egarch**	**Egarch**	**Garch**	**Garch Type**
Normal	Std	Normal	Std	Normal	Std	Distribution
25	25	25	25	25	25	Ee(A=0.05)
32	30	30	33	32	35	Ae(A=0.05)
1.903	0.992	0.992	2.459	1.903	2.613	Lr_{uc}(A=0.05)
5.533	3.334	3.334	7.981	5.533	4.379	Lr_{cc}(A=0.05)
5	5	5	5	5	5	Ee(A=0.01)
11	10	11	10	11	9	Ae(A=0.01)
5.419	3.914	5.419	3.914	5.419	3.756	Lr_{uc}(A=0.01)
6.848	5.665	6.848	5.665	6.848	10.725	Lr_{cc}(A=0.01)

According to the results in Table Ⅴ for REIT, all GARCH-type models pass both LR_{uc} and LR_{cc} tests. Furthermore, with minimum value of LR_{uc} and LR_{cc}, we can conclude that GARCH-GJR model has the best performance than others.

IV. EMPIRICAL RESULTS AND DISCUSSIONS

We used the autoregressive model to filter out the autocorrelation of the REIT in this paper. According to the graphs of ACF and PACF, we finger out that AR(1) model to calculate the mean of the time series. Following the minimum AIC value and description of the volatility clustering and asymmetry, EGARCH with Student t distribution outperforms other models. The volatility will decrease when the value rises since or all the coefficient γ is greater than 0 for all models. Meanwhile, for EGARCH with normal distribution, EGARCH with Student t distribution and APARCH model, the coefficient of σ_{t-1}^2 is greater than 0.9, which means the probability of current variance shock can still be captured in the future is over 90 percent. Table Ⅱ tells us that the daily log return does not follow normal distribution. As shown in Table Ⅴ, where the VaR's level is 0.05, reveals that all GARCH-type models perform well since they all pass the LR_{uc} and LR_{cc} test. With a minimum value for LR_{uc} and LR_{cc}, we can reach the conclusion that GARCH-GJR model outperforms others. What we find is that we can get accurate estimation of VaR if we take some specialized facts such as fat tail, leptokurtosis, volatility clustering and asymmetry in consideration.

V. CONCLUSIONS

In this paper we investigate some GARCH-type model for data set of Real Estate Investment Trust (REIT). Besides GARCH-GJR and APARCH model, we focus on GARCH and EGARCH with both normal and Student t distribution. Our findings reveal that the real estate daily log return is characterized by fat tail, volatility clustering and asymmetry. By using the backtesting of VaR, we find that GARCH-GJR(1, 1) model has the best performance.

VI. ACKNOWLEDGMENT

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