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The two Channel Wavelet Filter Bank Design

Swati D.Botla[1], Shirish S. Kulkarni[2]
[1]*M.E Student, Department of Instrumentation, RamraoAdik Institute of Technology, Mumbai*
[2]*Asst. Professor, Department of Instrumentation, RamraoAdik Institute of Technology, Mumbai*

Abstract:-Wavelets, filter banks and multi-resolution analysis have been used independently in the fields of applied mathematics, signal processing and image processing. It is shown that the wavelets are closely related to filter banks and there is a direct analogy between multi-resolution analysis in continuous time and a filter bank in discrete time. A two-channel filter bank has a low-pass and a high-pass filter in the decomposition phase and another low-pass and a high-pass filter in the reconstruction phase.

We have designed a two channel wavelet FIR filter bank by factorization of half-band polynomial with all the degrees of freedom allocated to the number of vanishing moments. In this paper, orthogonal wavelet filter bank is constructed and their filters are derived. The construction begins with choosing the product filter of degree 4p-2 and factorizing it into minimum phase and maximum phase low-pass filters by spectral factorization. The construction for degree 14 filter is examined and the location of zeros of the filters associated with it are shown.

Keywords:-wavelets, wavelet filter bank, half band filter, spectral factorization, orthogonal wavelets.

I. INTRODUCTION

Wavelets and filter banks have been used independently in the fields of applied mathematics, computer vision, signal processing and image processing. It is interesting to note that they performed similar functions in different fields. The fundamental idea was the same in all fields i.e. breaking down of the signal or functions into components signals at different resolutions. Filter banks are basically a bank of low-pass and high-pass filters linked by sampling operators and sometimes by delays [1]. The study of literature reveals a close relationship between the DWT and digital filter banks [2]. It turns out that a tree of digital filter banks without computing mother wavelets can simply achieve the wavelet transform. Hence, the filter banks have been playing a central role in the area of wavelet analysis. Perfect reconstruction filter banks were initially developed in the 1980's independently of wavelet theory [1]. They have become an immensely popular signal processing tool. The theories of wavelets and filter banks were unified within the framework of Meyer and Mallat's Multi-resolution Analysis. By multi-resolution, we mean a decomposition in sub signals with different resolution leading to a perfect reconstruction filter bank but the converse is not always true unless the filter bank satisfies an extra property which is referred to as "zeros at π" for the conventional two-channel filter banks.

Filter banks used for subband decomposition of images has been known since the early 1980's [1], [2]. The first perfect reconstruction filter banks were found by Smith and Barnwell and Mintzer. The initial solutions were for two channel systems with finite impulse response filters, although M-channel solutions quickly followed. The QMF (Quadrature Mirror Filter Bank) was first presented by Croisier, Esteban and Galand where they choosed alternating signs for the low-pass and high-pass filters. These filter banks have been shown to be closely related to wavelets [1] [2]. The simplest approach of design is the spectral factorization of a low pass half band filter with some regularity constraints (vanishing moments) [10]. The Daubechies wavelets construction requires the finding of a scaling function and a wavelet function [1]. This construction is best described via a two-channel perfect reconstruction filter bank and depends on the distribution of the zeros of some polynomials in the plane. The literature provides many theorems describing geometric locations of the roots of certain polynomials. Orthogonal and Bi-orthogonal Daubechies families of wavelets are considered and their filters are derived [4].

In this paper, we have shown the construction of orthogonal wavelets via the two channel filter bank. The construction is done through spectral factorization of halfband polynomial and their filter coefficients are derived. We also examined the locations of these zeros of these filters.

II. THE TWO CHANNEL FILTER BANK

A two-channel filter bank is shown in Fig.1. Let a discrete signal x(n) be the input signal. The objective in designing the filter bank is to find filters H0(z) and F0(z) such that the output is a delay version of input. In other words, a two channel filter bank gives perfect reconstruction when its filters satisfy these conditions:

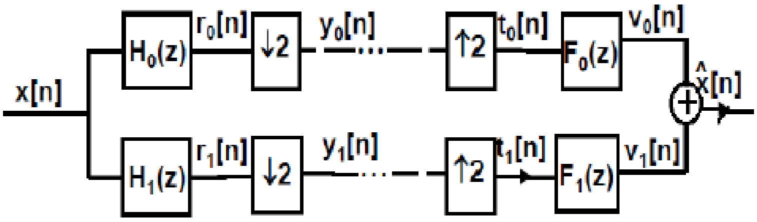

Fig. 1: Block diagram of Two Channel Filter Bank [1]

For Alias cancellation : F0(z)H0(z)+F1(z)H1(z)= $2z^{-l}$…….(1)
For No distortion :F0(z)H0(-z)+ F1(z)H1(-z) = 0 …….(2)

The analysis section of filter bank consists of a lowpass filter H0(z) and highpass filter H1(z). The convolved output of H0(z) followed by downsampler is

$$Y0(z) = \frac{1}{2}[(H0(z^{1/2})X(z^{1/2}) + H0(-z^{1/2})X(-z^{1/2})]$$

Similarly the output of H1(z) followed by downsampler

$$Y1(z) = \frac{1}{2}[(H1(z^{1/2})X(z^{1/2}) + H1(-z^{1/2})X(-z^{1/2})]$$

After the upsampling and the synthesis low-pass filter F0 and high-pass filter F1, the output of the filters are

$$V0(z) = \frac{1}{2}\,F0(z)[\,H0(z)X(z)+ H1(-z)X(-z)]$$
$$V1(z) = \frac{1}{2}\,F1(z)[\,H0(z)X(z)+ H1(-z)X(-z)]$$

The outputs are combined so that the processed output $\widehat{X}(z)$ is

$$\widehat{X}(z) = \frac{1}{2}[F0(z)H0(z)+F1(z)H1(z)]\,X(z) + \frac{1}{2}[F0(z)H0(-z)+ F1(z)H1(-z)]\,X(-z)$$

The second term of the equation contains the alias term. For perfect reconstruction, we may choose filters F0 and F1 to eliminate it.

$$F0(z) = H1(-z) \text{ and } F1(z) = -H0(-z)$$

This choice automatically satisfies the above mentionedconditions of equation (1) and equation (2) and the output of filter bank becomes,

$$\widehat{X}(z) = \frac{1}{2}\,X(z)[H0(z)H1(-z) + H1(z)H0(-z)]$$

The perfect reconstruction (PR) condition requires that $\widehat{X}(z)$ can only be a delayed version of input X(z) [i.e. $\widehat{X}(z) = z^{-l}X(z)$ for some delay 'l']
We obtain the following relations

$$\begin{aligned} H0(z)F0(z)+ H1(z)\,F1(z) &= H0(z)\,H1(-z) - H1(z)\,H0(-z) \\ &= H0(z)F0(z)- H1(z)\,F1(z) \\ &= 2z^{-l} \end{aligned}$$

To simplify the analysis, let us also define composite filters P0(z) and P1(z) as product filters for the two filtering paths.

$$\begin{aligned} P0(z) &= H0(z)\,F0(z) = -\,H0(z)\,H1(-z) \\ P1(z) &= H1(z)\,F1(z) = -\,H1(z)\,H0(-z) \\ &= -\,F0(-z)\,H0(-z) \\ &= -P0(-z) \end{aligned}$$

where we have made use of the aliasing free condition. In terms of composite filters, the PR condition becomes

$$P0(z) - P0(-z) = 2z^{-l} \quad ………..(3)$$

If we design the product filter P0(z) that satisfies the above condition, then the filters H0(z) and F0(z) can be obtained through spectral factorization [15]. In the next section, we will discuss the design of the filter bank using half band filter approach.

III. DESIGN PROCEDURE

There have been many filter design methods developed in the filter bank literature that allow us to construct new wavelet filters based on a variety of design criteria [1]. In general, these design methods revolve around the design of FIR PR-QMF filters and can be divided into three groups [15]. The first group is based on the design of half-band filters followed by spectral factorization [4], [7]. The second group is based on the design using lattice structures that are associated with efficient implementations and the third group is based on the formulation of the problem in the time-domain and solving it using an optimization algorithm [6], [8].

The design of two channel filter bank is reduced to two steps [1].

1. Design a product lowpass filter P0(z) satisfying P0(z) - P0(-z) = $2z^{-l}$.

2. Factorize P0(z) into H0(z) and F0(z), then find F0(z) and F1(z).

The length of P0 determines the sum of the lengths of H0 and F0 and there are many ways to design P0 and many ways to factor it. The reconstruction equation (3) can be made a little more convienent since the left side is an odd function, then l is odd.

To center, normalize P0(z) by z^l so the normalized product filter is P(z) = z^l P0(z).

Substituting P(-z) =z^{-l} P0(-z) The reconstruction equationtakesa simple form when both sides multiplied by z^l.Hence the perfect reconstruction condition becomes

$$P(z) + P(-z) = 2 \qquad \ldots\ldots\ldots\ldots(4)$$

This implies that P(z) is a halfband filter with all its coefficients zero except the constant term 1. Furthermore, the odd powers cancel when we add P(z) and P(-z). Therefore, the coefficientsof odd powers in P(z) are design variables in two channel filter bank.

The design of lowpass and highpass filter for orthogonal filter bank considers the two main properties. The wavelet filters must be orthogonal and must have maximum flatness at ω = 0 and at ω = π in their frequency response. This filter bank is orthogonal and the product filters has length 4p- 1. The low pass filters will have p = 1,2,3,4 zeros at 'π'. They have 2p = 2, 4, 6, 8, coefficients so that their length is 2p. The coefficients of highpass filter H1 are obtained by alternating flip construction [1].P0(z) can be factorized as

$$P0(z) = \left(\frac{1+z^{-1}}{2}\right)^{2p} Q2p - 2(z)$$

where polynomial Q2p-2(z) of degree 2p- 2 is chosen to satisfy equation (3). P0(z)will be ahalf band by special choice of Q(z).For p=4, db4 wavelet is obtained. To carry out factorization, P0(z) has 8 roots at z = -1 and other roots occur in pairs(z and 1/z). These eight roots are equally divided between the two filters. All the zeros of H0(z) are inside (and on the unit circle). It is therefore minimum phase factorization. Similarly, all zeros of F0(z) are outside and on the unit circle. It is therefore a maximum phase factorization. The fig.2.shows the pole zero plot of P0(z) and low-pass filters H0 and F0 and fig.3.shows the frequency response of P0(z). The frequency response of all the four filters is shown in fig.4. The scaling and wavelet functions shown in fig.5.are then derived from the coefficients of these filters completing the construction. Table 1. shows the designed coefficients of H0 and H1 and fig.6.shows the impulse response of all four filters.

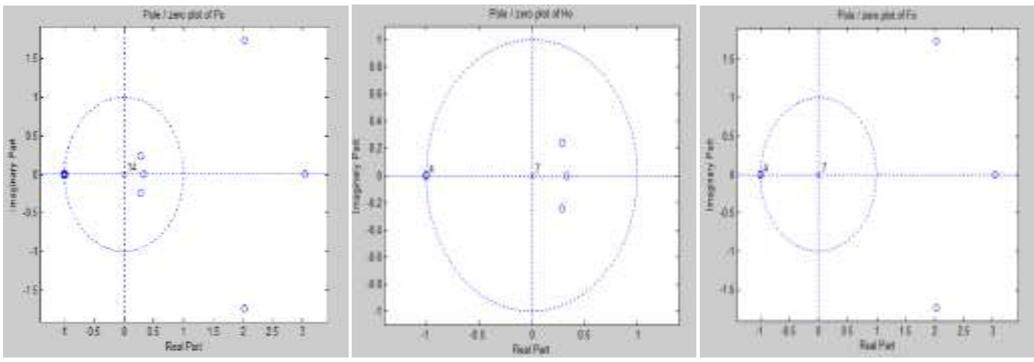

Fig. 2: Zeros of P0(z), H0(z) and F0(z)

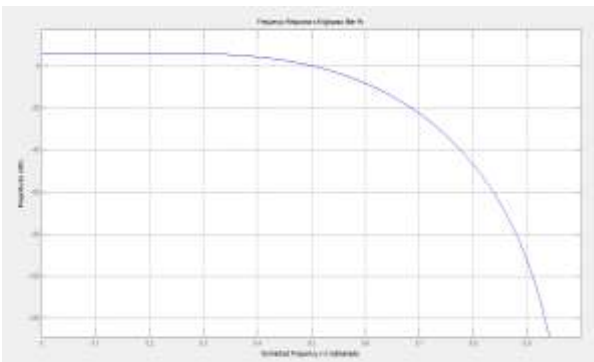

Fig. 3: Frequency response of P0(z).

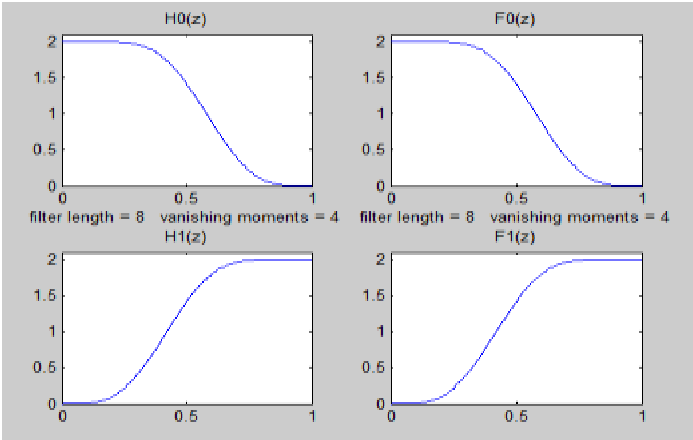

Fig. 4: Frequency response of decomposition and reconstruction filters.

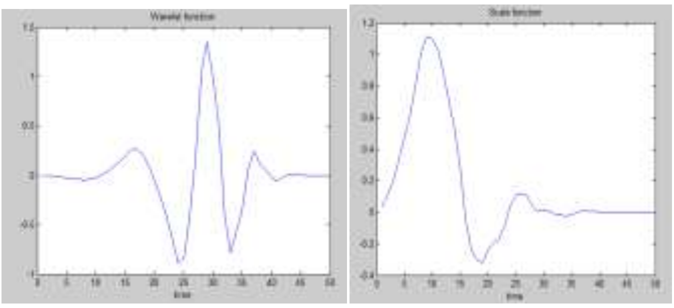

Fig. 5: Scalingand wavelet function.

Table I: The coefficients of lowpass filter H0and highpass filter H1.

H0	**H1**
0.2304	-0.0106
0.7148	-0.0329
0.6309	0.0308
-0.0280	0.1870
-0.1870	-0.0280
0.0308	-0.6309
0.0329	0.7148
-0.0106	-0.2304

It can be noted that these wavelet filters have four vanishing moments which is maximum possible for this particularlength.

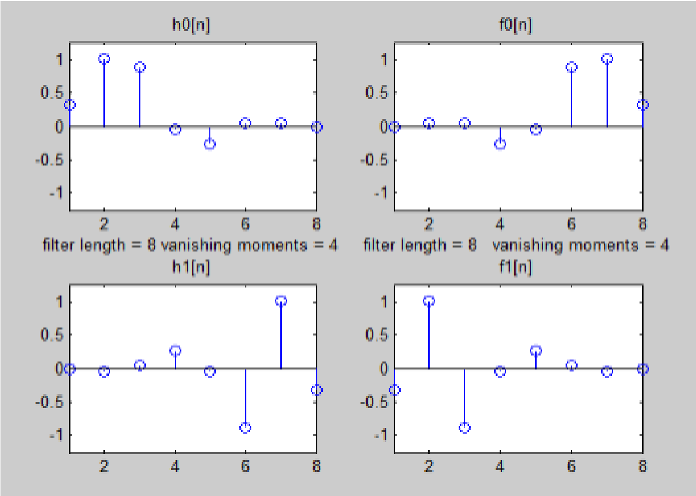

Fig.6: Impulse response of decomposition and reconstruction filters.

IV. Conclusion

We have constructed the wavelets via the two channel perfect reconstruction filter bank. This filter bank is orthogonal and the product filters have a degree of 4p-2. The low-pass filters will have 'p' zeros at z = -1 and have a total of '2p' coefficients which is the length of all filters. The results for tap 8 orthogonal wavelet filter are examined and coefficients of the filters associated with it are derived. The location of zeros of the filters in this construction were found. These constructed wavelets coefficients can be used in any image processing applications.

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