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Empirical Analysis of Invariance of Transform Coefficients under Rotation

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Abstract:- Rotationally invariant transforms, namely, angular radial transform and polar harmonic transforms such as polar cosine transform, polar sine transform and polar complex exponential transforms, are used to characterize image features for a number of applications like logo recognition, face recognition etc. But the computation of features using these transforms is an expensive process due to their sinusoidal basis functions which are quite computationally expensive. In this paper, the transform coefficients are analyzed to observe the effect of rotation on each coefficient. The experimentation is done to find most robust transform coefficients under rotation for each transform. Further, analysis is done to trace the effect of observed robust transform coefficients in face recognition application. The results show that the recognition rate remains same for cases when all transform coefficients are used for extracting image features and when only 50% to 60% observed robust moments are used whereas the execution time decreases resulting in fast execution of application.

Keywords:- Angular radial transform, polar harmonic transforms, polar cosine transform, polar sine transform, polar complex exponential transform, face recognition, feature extraction, Euclidean distance.

I. INTRODUCTION

Transform theory plays a fundamental role in image processing, as working with the transform of an image instead of the image itself gives more insight into the properties of the image. Image transform is an operation to change the default representation space of a digital image (spatial domain to another domain) so that all the information present in the image is preserved in the transformed domain, but represented differently and the transform is reversible, i.e., can be reverted to the spatial domain. Transforms are called shape descriptors. These capture the significant properties of a function. These are the descriptors that correspond to the projection of the function on a specific basis function. A transform is composed of two parts, i.e., radial and angular part. The radial part of the basis function of a transform is sinusoidal function. Rotationally invariant transforms are used for feature extraction in application related to pattern recognition. These applications are based on properties like rotation invariance, scale invariance, orthogonality etc.

Rotation invariant transforms have the feature to be able to recognize a wide variety of objects irrespective of their rotations. It is done by devising a set of features which are invariant to the image orientation. The magnitude of the coefficients of rotation invariant transforms is rotation invariant. Transforms are rotationally invariant when computed from ideal analog images. Mapping from analog images to digital images causes magnitude invariance of transforms severely compromised due to the discretization errors in computation. This affect the invariance property of transforms making some moments more sensitive and some more robust to rotation. Invariance of transform coefficients is crucial in most of these applications, for e.g. the performance of the pattern recognition critically depends on invariance of employed features with respect to scaling and rotation.

The angular radial transform (ART) [1] and polar harmonic transforms (PHTs) [2] are region based shape descriptors. These transforms are invariant under rotation transformation and can be made scale and translation invariant after applying some geometric transformations. Above mentioned transforms possess low computational complexity and high numerical stability; due to which they are used in many applications related to image processing and pattern recognition like shape retrieval [1], logo recognition system [3], face detection [5], image watermarking [6], and video security systems [4] [7], fingerprint classification [8]. Although these transforms have low computational complexity but still they are expensive for real time applications due to their sinusoidal basis function. The wide use of these transforms in various mentioned applications motivated us to perform an analysis of transform coefficients under rotation using parameters given in [9] to find robust transform coefficients, i.e. the moments having coefficients that are invariant or least invariant under the impact of rotation. These robust transform coefficients can be only used for feature extraction instead of using all coefficients up to some predefined order, thus reducing the computational complexity and making these transforms more efficient for real time applications. Further, an analysis is performed using face recognition application to illustrate that if only some percentage of observed robust transform coefficients are used for above mentioned applications instead of all transform coefficients then whether the same recognition rate or performance level can be achieved resulting in reduced computational complexity. The results are presented based on recognition rate and execution time required for all cases.

The rest of the paper is organized as follows. An overview of considered rotationally invariant transforms is given in Section II. Section III presents computational framework used for transforms. Empirical analysis performed is divided into two parts, first part presents analysis performed on transform coefficients to find robust moments and in second part the affect of observed robust moments is traced in face recognition application. This is presented in Section IV. Section V presents conclusion.

II. ROTATIONALLY INVARIANT TRANSFORMS

Rotationally invariant transforms are defined on square image functions which do not change its value under rotation. For a given image function, $f(x, y)$, of size $N \times N$ pixels in a two-dimensional Cartesian coordinate system, its rotation invariant transform coefficient, F_{nm}, of order *n* and repetition *m* is defined over a unit disk in the continuous polar domain as follows :

$$F_{nm} = \lambda \int_{0}^{2\pi} \int_{0}^{1} f(r, \theta)\, V_{nm}^{*}\,(r, \theta) r\, dr\, d\theta \tag{1}$$

such that $r = \sqrt{x^2 + y^2}$, $\theta = \arctan(y/x)$, λ is the normalization constant and $V_{nm}^{*}(r, \theta)$ is the complex conjugate of the basis function, $V_{nm}(r, \theta)$, which is separable along the radial and angular directions, i.e.,

$$V_{nm}(r, \theta) = R_{nm}(r) A_m(\theta) \tag{2}$$

Each transform is uniquely defined by its radial basis function, $R_{nm}(r)$, and in order to provide rotational invariance, the angular basis function, $A_m(\theta)$, is given by:

$$A_m(\theta) = e^{jm\theta} \;\; ; j = \sqrt{-1} \tag{3}$$

A. Angular Radial Transform (ART)

The ARTs coefficient, A_{nm}, of a continuous function, $f(r, \theta)$, of order n and repetition m in polar domain are extracted over a unit disk using the following formula:

$$A_{nm} = \frac{1}{2\pi} \int_{0}^{2\pi} \int_{0}^{1} f(r, \theta) V_{nm}^{*}(r, \theta) r\, dr\, d\theta \tag{4}$$

where $n \geq 0$ and $|m| \geq 0$.

The radial function of ARTs is defined as:

$$R_{n}^{ART}(r) = \begin{cases} 1, & n = 0 \\ 2\cos(\pi n r), & otherwise \end{cases} \tag{5}$$

ART is used for invariant shape retrieval [1], logo recognition system [3], face detection [5], image watermarking [6], video security system [4] [7] and cephaloetric X-ray registration [10].

B. Polar Harmonic Transforms (PHTs)

Yap et. al [2] introduced a set of transforms called polar harmonic transforms (PHTs), which can be used to generate rotation invariant features. PHTs consist of three different transforms, namely, polar complex exponential transform (PCET), polar cosine transform (PCT) and polar sine transform (PST).

Polar complex exponential transform (PCET)

The PCETs coefficient of order $|n|\geq 0$ and repetition $|m|\geq 0$ is given in polar domain by:

$$F_{nm}^{PCET} = \frac{1}{\pi}\int_{0}^{2\pi}\int_{0}^{1} f(r,\theta)[V_{nm}^{PCET}(r,\theta)]^{*} r\, dr\, d\theta \tag{6}$$

where $V_{nm}^{PCET}(r,\theta)$ is PCET basis function having radial basis function, $R_{n}^{PCET}(r)$:

$$R_{n}^{PCET}(r) = e^{j2\pi r^{2}} \tag{7}$$

The other two sets of harmonic transforms, PCTs and PSTs, are defined as:

Polar cosine transform (PCT)
PCTs coefficients are defined as:

$$F_{nm}^{PCT} = \lambda\int_{0}^{2\pi}\int_{0}^{1} f(r,\theta)[V_{nm}^{PCT}(r,\theta)]^{*} r\, dr\, d\theta \tag{8}$$

where $n \geq 0, |m| \geq 0$ and

$$V_{nm}^{PCT}(r,\theta) = R_{n}^{PCT}(r)e^{jm\theta} = \cos(\pi n r^{2})e^{jm\theta} \tag{9}$$

where

$$\lambda = \begin{cases} \frac{1}{\pi} & n = 0 \\ \frac{2}{\pi} & n \neq 0 \end{cases} \tag{10}$$

Polar sine transform (PST)
PSTs coefficients are defined as:

$$F_{nm}^{PST} = \lambda\int_{0}^{2\pi}\int_{0}^{1} f(r,\theta)[V_{nm}^{PST}(r,\theta)]^{*} r\, dr\, d\theta \tag{11}$$

where $n \geq 1, |m| \geq 0$ and

$$V_{nm}^{PST}(r,\theta) = R_{n}^{PST}(r)e^{jm\theta} = \sin(\pi n r^{2})e^{jm\theta} \tag{12}$$

where

$$\lambda = {}^{2}/_{\pi} \tag{13}$$

PHTs are used in applications like fingerprint classification [8].

III. COMPUTATIONAL FRAMEWORK FOR TRANSFORMS

In digital image processing, the image function, $f(x,y)$, is discrete, whereas transform invariants are defined over the unit disk in continuous polar domain. However, the discrete image function can be converted into polar domain using a suitable interpolation process [11] but this mapping introduces interpolation error which affects the accuracy in computation [12]. To avoid interpolation error, the traditional method is used to compute transform coefficients in the Cartesian domain directly using zeroth-order approximation (ZOA) of the double integration involved in Eq. (1) as follows:

$$F_{nm} = \lambda\sum_{i=0}^{N-1}\sum_{k=0}^{N-1} f(i,k)\, R_{nm}\,(r_{ik})\, e^{-jm\,\theta_{ik}}\,\Delta x\,\Delta y \tag{14}$$

such that λ is the normalization constant depending on the transform used and

$$\Delta x = \Delta y = \tfrac{2}{D}, \quad r_{ik}^{2} = x_{i}^{2} + y_{k}^{2} \leq 1 \text{ and } \quad \theta_{ik} = \arctan({}^{y_k}/_{x_i}) \tag{15}$$

and the coordinates (x_i, y_k) are given by

$$x_i = \frac{2i-N+1}{D}, y_k = \frac{2k-N+1}{D}, i, k = 0,1, \dots N-1 \tag{16}$$

where

$$D = \begin{cases} N & for\ inner\ disk \\ N\sqrt{2} & for\ outer\ disk \end{cases} \tag{17}$$

The coordinate (x_i, y_k) is the centre of the square pixel grid (i, k). This mapping converts the square domain into an approximated unit disk. Many transform-based applications use inner disk for computing the coefficients therefore we have taken $D = N$ and $\Delta x = \Delta y = \frac{2}{N}$.

IV. EMPIRICAL ANALYSIS RESULTS

A. Empirical analysis done to find most robust moments

The algorithms for the computation of transforms are implemented in Microsoft Visual Studio 2010 under Microsoft Window 7 environment on Intel 2.26 GHz processor with 3 GB RAM. The experimentation is performed with twelve standard 256-level gray scale images of size 128×128 pixels given in Table 1.

The rotation invariance property of the transforms is analysed for maximum order, $n_{max} = 10$ and maximum repetition $m_{max} = 10$. Let F_{nm}^t represent transform coefficient at order n and repetition m for a particular transform t. The parameter $C_{nm}(\theta)$ at $\theta = 0^\circ, 5^\circ, \dots, 45^\circ$ is computed for each transform using the formula [9]:

$$C_{nm}(\theta) = |F_{nm}(\theta)| / |F_{nm}(0^\circ)| \tag{18}$$

Further, the deviation is measured by computing the parameter:

$$\sigma^2 = \frac{1}{10} \sum_{i=1}^{10} (C_{nm}(\theta_i) - 1)^2 \tag{19}$$

where $\theta_i = 0^\circ, 5^\circ, \dots, 45^\circ$.

The transform coefficients for which the value of parameter σ^2 is least are considered more robust compared to moments that have high value. The results were obtained by considering the average deviation of all the 12 images given in Table 1. It was observed that some transform coefficients at particular order and repetition show high deviation for a particular image which were excluded during calculation of average deviation. These high deviating order and repetition indicated that the obtained most robust transform coefficients are image dependent and the results also show that PST has least deviating nature among all other transforms considered followed by ART. Figure 1 show the graph plotted for minimum and maximum moments and their corresponding average $C_{nm}(\theta)$ values for each transform under investigation.

As the results obtained from this analysis were observed to be image dependent so to further trace the effect of robust transform coefficients using face recognition application the same analysis is conducted on the training image set used in face recognition application. This is done for only least deviating transforms (i.e. polar sine transform and angular radial transform) among transforms under investigation that is concluded after the analysis done on the 12 images given in Table 1.

Table 1. Twelve standard gray scale images used for experimentation

			
Barbara	**Bridge**	**Cameraman**	**Clock**

Fishing Boat

Gold Hill

House

Jet Plane

Mandrill

Peppers

Pirate

Tree

A. Analysis to trace applicability of results of sub-section A

The results obtained after analysis of transform coefficients using training image set were applied to face recognition application. The face recognition application employed is used to recognize the most relevant face from the database when query face image is presented to the application. The Euclidean distance (ED) is used as a similarity measure between the query image features and the features extracted from the images in the database. Euclidean distance is defined as:

$$ED(F^D, F^Q) = \sqrt{\sum_{i=1}^{n}(F_i^Q - F_i^D)^2} \tag{20}$$

where, F^D is the images in database and F^Q is the query image presented to the application, $F^D = F_1^D, F_2^D, \dots, F_n^D$ are features extracted from the images in database, $F^Q = F_1^Q, F_2^Q, \dots, F_n^Q$ are features extracted from the query image and n is the number of these features. The database image having the least Euclidean distance with the presented query image is considered the best match for the query image. To trace the applicability of above analysis results to face recognition application the AT&T "The Database of Faces" (formerly "The Olivetti Research Lab (ORL) Database of Faces") is used. The AT&T contains 400 face images of 40 different persons with 10 images of each person in a different folder.

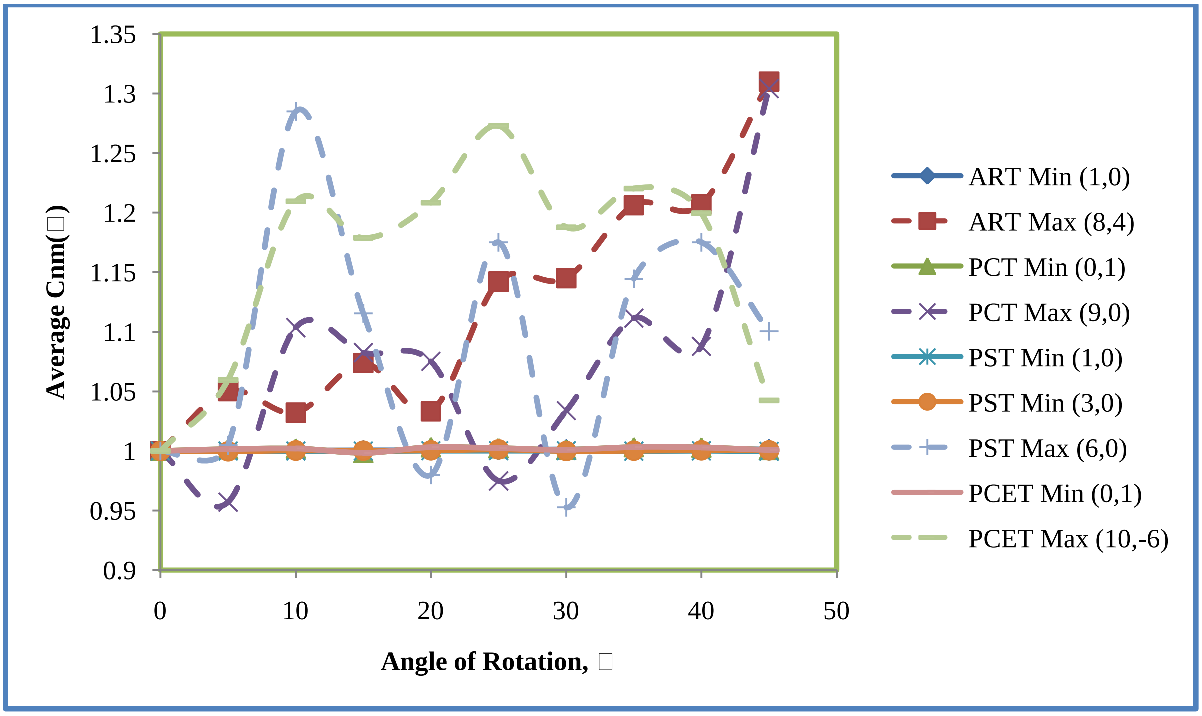

Fig. 1. Avg.$C_{nm}(\theta)$ values vs. angle of rotation for least and most deviating transform coefficients

For this analysis we have taken 100 face images (first 5 of first 20 subjects) in the training set which acts as database and all train and test images are resized to canvas size 128×128 . The test database consists of 1000 images which were developed using other 5 images of same 20 subjects by rotating (inner disk rotation) them at multiple angles from $\theta = 0^{\circ}, 5^{\circ}, \ldots, 90^{\circ}$. Some used train and test images are given in Table 2.

Table 2. Examples of train and test images used for face recognition analysis

Train Images	Test Images
	   
	   
	   

In this analysis the recognition rate of faces is considered against execution time needed. The comparison is done by considering all and reduced number of transform coefficients for feature extraction. The observed robust transform coefficients obtained from analysis of train images are considered in ascending order of their deviation. Then from these ascending transform coefficients the face recognition analysis is performed by taking 10% transform coefficients to 100% transform coefficients for feature extraction. The results are presented based on recognition rate and execution time needed. The performance of the face recognition application is based on how well the query faces can be matched with correct faces in database in presence of rotation at various angles. In this experimentation PST and ART are used for feature extraction at maximum order, $n_{max} = 10$ and maximum repetition $m_{max} = 10$, making a total of 110 transform coefficients for PST and 121 for ART (taking $n \geq 0$ and $m \geq 0$).

After experimentation, it was observed that the rate of recognition becomes stable when 50% - 60% transform coefficients are used for feature extraction. This means instead of using all transform coefficients for feature extraction only 50% - 60% robust transform coefficients can be utilized, thus decreasing the time complexity.

Table 3. Percentage of robust transform coefficients used for feature extraction against recognition rate obtained and execution time needed for PST

Percentage of moments used	No. of Moments	Recognition Rate	Time taken (in sec)
10%	11	84.8%	25.396
20%	22	88.3%	51.152
30%	33	90.6%	76.518
40%	44	92%	129.167
50%	55	93%	130.511
60%	66	92.6%	166.664
70%	77	93%	179.416
80%	88	93%	207.153
90%	99	93%	231.77
100%	110	93%	255.498

Table 3 and Table 4 represent the percentage of robust transform coefficients used for feature extraction against recognition rate obtained and execution time needed for PST and ART respectively. Figure 2 and Figure 3 shows the graph plotted against percentage of transform coefficients used and recognition rate for PST and ART respectively. The results show that the execution time reduces by about half when only 50% transform coefficients are used.

Table 4. Percentage of robust transform coefficients used for feature extraction against recognition rate obtained and execution time needed for **ART**

Percentage of moments used	No. of Moments	Recognition Rate	Time taken (in sec)
10%	12	79.3%	30.607
20%	24	91%	60.107
30%	36	91.9%	87.766
40%	48	90%	115.846
50%	60	92%	128.263
60%	72	93%	160.587
70%	84	92.4%	186.67
80%	96	93%	218.534
90%	108	92.2%	244.667
100	121	93%	298.444

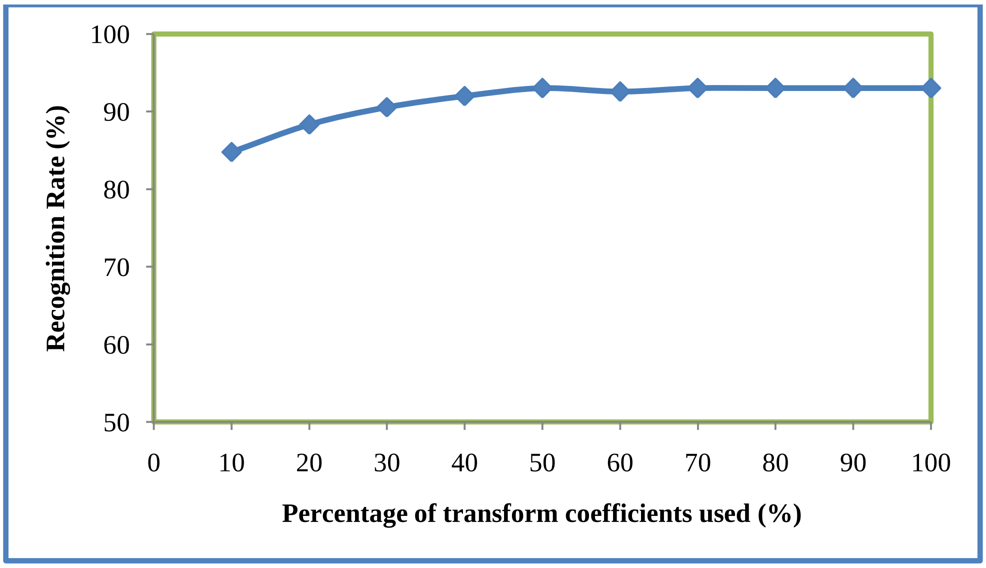

Fig. 2. Recognition rate vs. percentage of transform coefficients used for feature extraction using PST

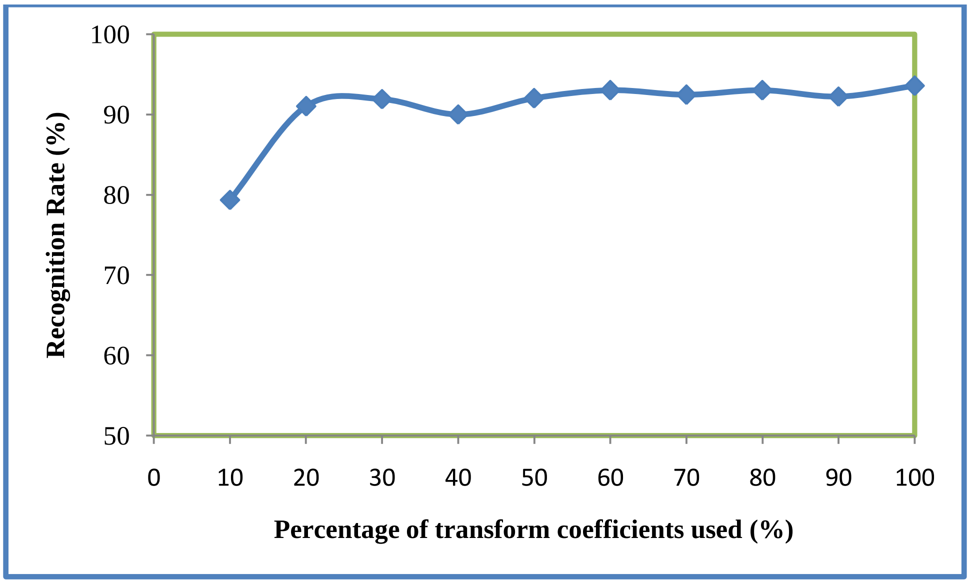

Fig. 3. Recognition rate vs. percentage of transform coefficients used for feature extraction using ART

V. CONCLUSION

Rotationally invariant transforms, namely, angular radial transform and polar harmonic transforms such as polar cosine transform, polar sine transform and polar complex exponential transforms, are used for feature extraction in various image processing applications like logo recognition, face recognition etc. However, mapping from analog images to digital images causes magnitude invariance of transforms severely compromised due to the discretization errors in computation. This affect the invariance property of transforms making some moments more sensitive and some more robust to rotation. Invariance of transform coefficients is crucial in most of these applications. These features are analyzed in this paper and the effect of obtained results is traced using face recognition application and concluded as:

- Among all transforms under investigation PSTs have the least deviating nature in case of rotation invariance followed by ART.
- It was also observed that the robust moments are image dependent showing high deviating nature at some particular orders and repetitions for particular images.
- The effect of robust transform coefficients traced in face recognition application illustrates that reduced number of coefficients (robust) between 50% to 60% can be used instead of using all coefficients to achieve same recognition rate but at low computational complexity.

REFERENCES

[1]. Bober M., MPEG-7 Visual shape descriptors. IEEE Trans. on Circuits and Systems for Video ., 11 (6) (2001) 716-719.
[2]. Yap P.T., Jiang X., Kot A.C., Two-dimensional polar harmonic transforms for invariant image representation. IEEE Trans. on Pattern Analysis and Machine Intelligence. 32 (7) (2010) 1259-1270.
[3]. Wahdan O.M., Omar K. and Nasrudin M.F., Logo recognition system using angular radial transform descriptors. Journal of Computer Science. 7 (9) (2011) 1416-1422.
[4]. He D., Sun Q. and Tian Q., A semi-fragile object based video authentication system. ISCAS (2003).
[5]. Fang J. and Qiu G., Human face detection using angular radial transform and support vector machines.
[6]. Singh C. and Ranade S.K., Geometrically invariant and high capacity image watermarking scheme using accurate radial transform. Optics and Laser Technology. 54 (2013) 176-184.
[7]. Lee S.H., Sharma S., Sang L., Park J.-I. and Park Y.G., An intelligent video security system using object tracking and shape recognition. ACVIS LNCS Springer-Verlag Berlin Heidelberg. 6915 (2011) 471-482.
[8]. Liu M., Jiangz X., Chichung A. K. and Yap P.-T., Application of polar harmonic transforms to fingerprint classification. World Scientific Publishing. (2011).
[9]. [Singh C., Sharma P. and Upneja R., On image reconstruction, numerical stability, and invariance of orthogonal radial moments and radial harmonic transforms. Pattern Recognition and Image Analysis. 21 (4) (2011) 663-676.
[10]. Singh C. and Kaur A., Cephalometric X-Ray Registration using Angular Radial Transform. International Conference on Recent Advances and Future Trends in Information Technology. (2012) 18-22.
[11]. Xin Y., Pawlak M. and Liao S., Accurate calculation of moments in polar co-ordinates. IEEE Trans. on image processing. 16 (2007) 581-587.
[12]. Singh C. and Walia E., Computation of Zernike moments in improved polar configuration. IET J. Image Processing. 3 (2009) 217-227.
[13]. Rodtook S. and Makhanov S. S., Numerical experiments on the accuracy of rotation moments invariants. Image and Vision Computing. 23 (2005) 577-586.
[14]. Omaia D., Poel J. D. V. and Batista L. V., 2D-DCT distance based face recognition using a reduced number of coefficients.
[15]. Singh C. and Aggarwal A., A noise resistant image matching method using angular radial transform. Digital Signal Processing. 33 (2014) 116-124.