International Journal of Engineering Research and Development
e-ISSN: 2278-067X, p-ISSN: 2278-800X, www.ijerd.com
Volume 10, Issue 5 (May 2014), PP.48-55

Experimental Validation of Numerical Simulation of Vibrating Systems having Three Degrees of Freedom using Power Input Method

J.O. Kiran [1], D. Shivalingappa [2], D. Ramesh Rao [3]
[1]*Assistant Professor, Department of Mechanical Engineering, S.T.J Institute of Technology, Ranebennur, Karnataka, India,*
[2]*Professor, Department of Mechanical Engineering, Adhiyamaan College of Engineering, Hosur, Tamilnadu, India*
[3]*Professor, Department of Mechanical Engineering, Bapuji Institute of Engineering and Technology, Davangere, Karnataka, India,*

Abstract:- In this work, the energies dissipated by the spring mass damper system with three degrees of freedom are modelled and simulated for persistent load by using MATLAB. The experimentation is carried out for the numerical model developed. The experimental validation of the numerical solution developed for the persistence excitation system for coupling loss factor of 0.075 is compared.The overall trend of the profile of the experimental values seems to be in good agreement with the theoretical prediction. The difference in the values between the theoretical prediction and the experimental values can be attributed to the approximations made in simplification of the vibrating system for numerical solution. Also, there may be some cross vibrations coming from the floor on which the vibrating system is mounted.

Keywords: - Three degrees of freedom**,** Damping, Coupling loss factor, Persistent load,

I. INTRODUCTION

Vibration and its effect are very common in our daily life. The theory of vibration deals with the systematic analysis of vibratory motions of the body and the forces involved in it. Vibration is a repetitive, periodic, or oscillatory response of a mechanical system. Vibration or oscillation may be defined as any structural deformation that repeats itself periodically **[**1-2].The kinetic energy is stored due to the mass of the structure, potential energy is stored due stiffness and the energy is dissipated due to damping. If the damping is insufficient, the structure that is vibrating at a resonant frequency tends to result in high amplitudes which radiates sound and might ultimately lead to structural failure. Hence it is essential to calculate or predict the resonant frequencies and prevent the structure from high amplitude vibrations by providing sufficient structural damping [3] inside the structure.

There are two types of damping that are inherently present in any structure [4], namely, internal damping of the structure and structural damping at the joints in the structure.

Due to the presence of dynamic loads, the visco-elastic material dissipates energy in the form of heat energy by disrupting the bonds of its long-chain molecules [5].In vibration analysis, physical system can be represented in the form of mathematical model and it is important for analysis to translate mathematical equations and formulations into real conclusions [6].Most engineering systems are continuous and have an infinite number of degrees of freedom. The vibration analysis of continuous systems requires the solution of partial differential equations, which is quite difficult [7].In order to solve equations of motion, the program MATLAB directly use the functions for numerical solution of differential equations with the use of Runge – Kutta method [8].The Statistical energy analysis (SEA) has been developed to predict noise and vibration of complex structures at medium and high frequencies [9-12]. It is a technique for the estimation of average vibration levels in a structure which is excited by broad band random forces. In SEA the internal loss factor is the important parameter to be considered for the analysis of structures.
The following methods which are used by the researchers to estimate the loss factors [13 -15]:

a) Time domain decay-rate method
b) Frequency domain modal analysis, curve-fitting method
c) Flow of Energy and power based method [16]

Each method has its own set of advantages and drawbacks. One such method, the power input method (PIM) is a powerful method for obtaining frequency-averaged loss factors of structures under steady state vibration. In this paper experimentation carried out in the power input method and the behavior of the system is estimated by time averaging.

The PIM method allows an identification of the SEA parameters like the coupling loss factor, damping loss factor, without having to decouple the structure. The PIM has been applied to cars, trucks, helicopters and other complex structures [15,17,18-21].Power input method is one of the energy based method for a comparison of the dissipated energy of a system to the total energy of the system under steady state vibration. This method is theoretically unbiased and it is applicable at all frequencies.The vibration analysis is based on quantities such as force and displacement but this approach is based on energy quantities like energy, energy density, power.The loss factors of a structural system can be defined as, the ratio of the dissipated power per radian to the total energy of the structure [21].

$$\eta(\omega) = \frac{\Delta E}{E_{Se}} \tag{1.1}$$

Where,

E_{Se} is the strain energy,

ΔE is the energy dissipated from damping, and

η is the damping loss factor in the frequency band ω considered

II. ANALYTICAL MODEL

The numerical methods are used to solve the governing equations of the representative systems. The physical systems with three degrees of freedom are usually complex in nature and it is difficult to represent their behavior in mathematical forms. Hence the physical systems with three degrees of freedom are simplified and their representative models can be used for the purpose of studying their behavior. In this work, the vibrating systems are represented directly using lumped masses.

The analytical model is developed for the spring mass damper system having three degrees of freedom as shown in Fig.1. Let m_1, m_2 and m_3 represent the three masses which are connected to four springs of stiffness represented by k_1, k_2, k_3 and k_4 and to four dampers of coefficient c_1, c_2, c_3 and c_4. The force F_2 acts on mass m_2 and the energy is transferred to other masses through the springs and a part of the energy is absorbed by dampers. Springs k_1 and k_4; and dampers c_1 and c_4 are attached to rigid surfaces and masses m_1 and m_3 respectively.

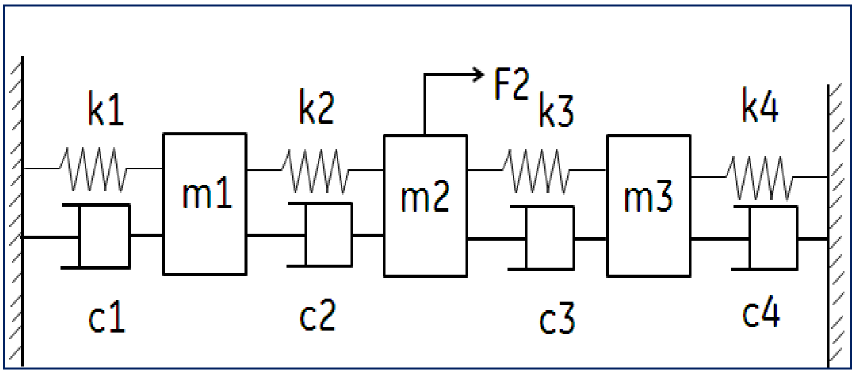

Fig. 1: Mathematical Model for Coupling Loss Factor Estimation

The spring mass damper system is represented by the following equations.

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = 0 \tag{1.2}$$

$$m_2\ddot{x}_2 + (c_2 + c_3)\dot{x}_2 - c_2\dot{x}_1 - c_3\dot{x}_3 + (k_2 + k_3)x_2 - k_2x_1 - k_3x_3 = F_2 \tag{1.3}$$

$$m_3\ddot{x}_3 + (c_3 + c_4)\dot{x}_3 - c_3\dot{x}_2 + (k_3 + k_4)x_3 - k_3x_2 = 0 \tag{1.4}$$

The above three governing equations can be represented in matrix form as

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad (1.5)$$

$$C = \begin{pmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 + c_4 \end{pmatrix} \quad (1.6)$$

$$K = \begin{pmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{pmatrix} \quad (1.7)$$

$$F = \begin{Bmatrix} 0 \\ F_2 \\ 0 \end{Bmatrix} \quad (1.8)$$

$$M\ddot{x} + C\dot{x} + Kx = F \quad (1.9)$$

$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{pmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 + c_4 \end{pmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{pmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_2 \\ 0 \end{Bmatrix} \quad (1.10)$$

III. EXPERIMENTAL WORK

The experimental setup consists of three plates which are connected with bolt and nuts. One bolt each consists of one contact point, the plates are joined by a spacer of 20mm thickness having outside diameter are 40 mm and inside diameter 20 mm. Eight spacers used are made up of aluminum material. The spacers are replaced with the dampers for different cases, during the experimentation for different models. All the four edges of the plates are free. The four connecting locations are away from the edges. In this configuration three plates are connected at discrete points using bolts with a small gap between them.

The three plates are identical and they are numbered as 1,2 & 3. Plate 2 is inner plate and plates 1&3 are outer plates. It is to be noted that plate 2 is not connected to plate1 or 3.The plate 1 and plate 3 are fixed to the frame through the bolts, but they are coupled through the spacers.

Each plate has dimensions of 1100 mm x 900 mm and the thickness of 2 mm. The plates used are made up of aluminum material. The young’s modulus of the material is 70 GPa and the Poisson’s ratio is 0.3. The density of material is 2800 kg/m^3

Fig.2a: Experimental setup three degrees of freedom system for persistent load

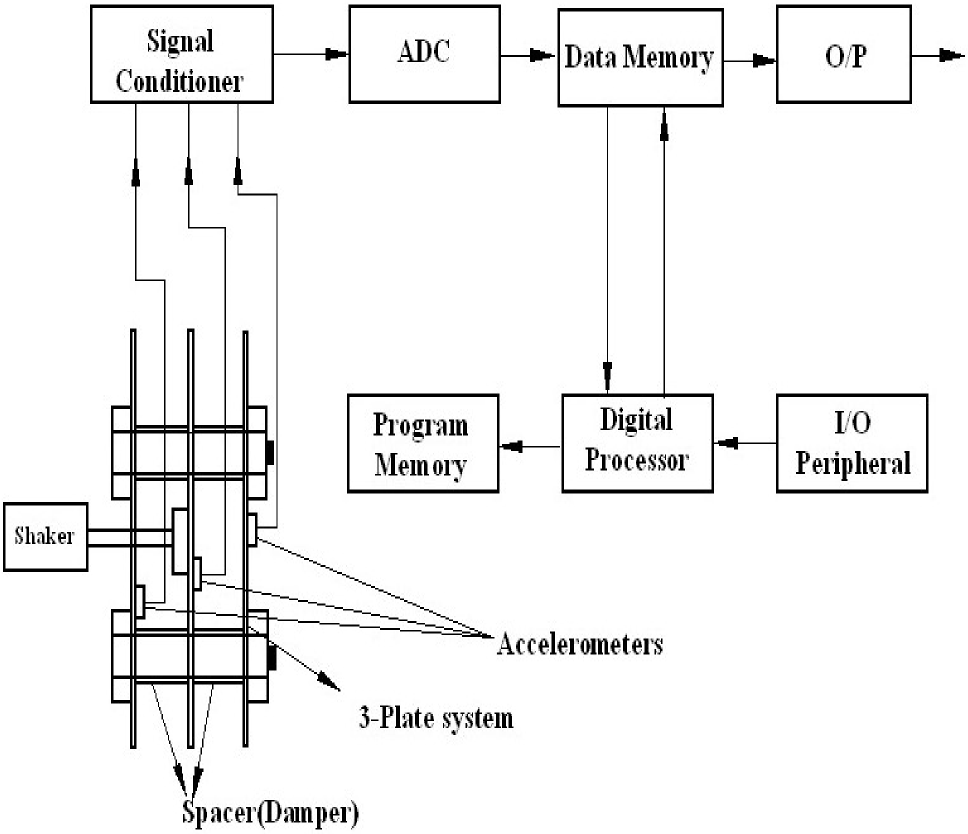

Fig.2b: Experimental setup three degrees of freedom system for persistent load

IV. SIMULATION RESULTS

A sample problem with given input loss factors is numerically solved with the power input method to estimate the loss factors and check the behavior of the system under steady load condition. The experimentation is carried out for Known damper having a loss factor of 0.075 to evaluate the theoretical results with experimental results. The experimentation three plate systems in the form of our theoretical model of persistence load in Fig.1. The simulation and experimental parameters are shown in table 1.The power input method has no theoretical limitations, but it is practically limited to the range of 0.5 $<\eta<$0.001 [22].

Plate	Mass (kg)	Stiffness (N/m)	Natural frequency (rad/s)	Force (N)	Time (S)	Modulus of rigidity (Gpa)
Plate 1	5.39	1514	16.75	NA	NA	26.9
Plate2	5.42	1514	16.74	5	1	
Plate 3	5.38	1600	17.24	NA	NA	

Table.1: Simulation and Experimental Parameters for Persistent Load

A. Verification of Displacements:

Fig. 3 shows the comparison of displacement of plates 1, 2 &3 for the theoretical prediction and the experimentally observed values. Since the load is applied on plate 2, there is a very small displacements are noticed in plate 1. The same is found to be true when verified experimentally. The experimental data is collected for every 0.1 s. In plate 2 the experimentally measured values are higher than the theoretical prediction when there is a peak in the profile and are below for profile at other than the peaks. The maximum displacement error occurs in between the time span of 0.1 to 0.5s. Since the force is excited at the second plate, the plate has come to the steady state as time increases.

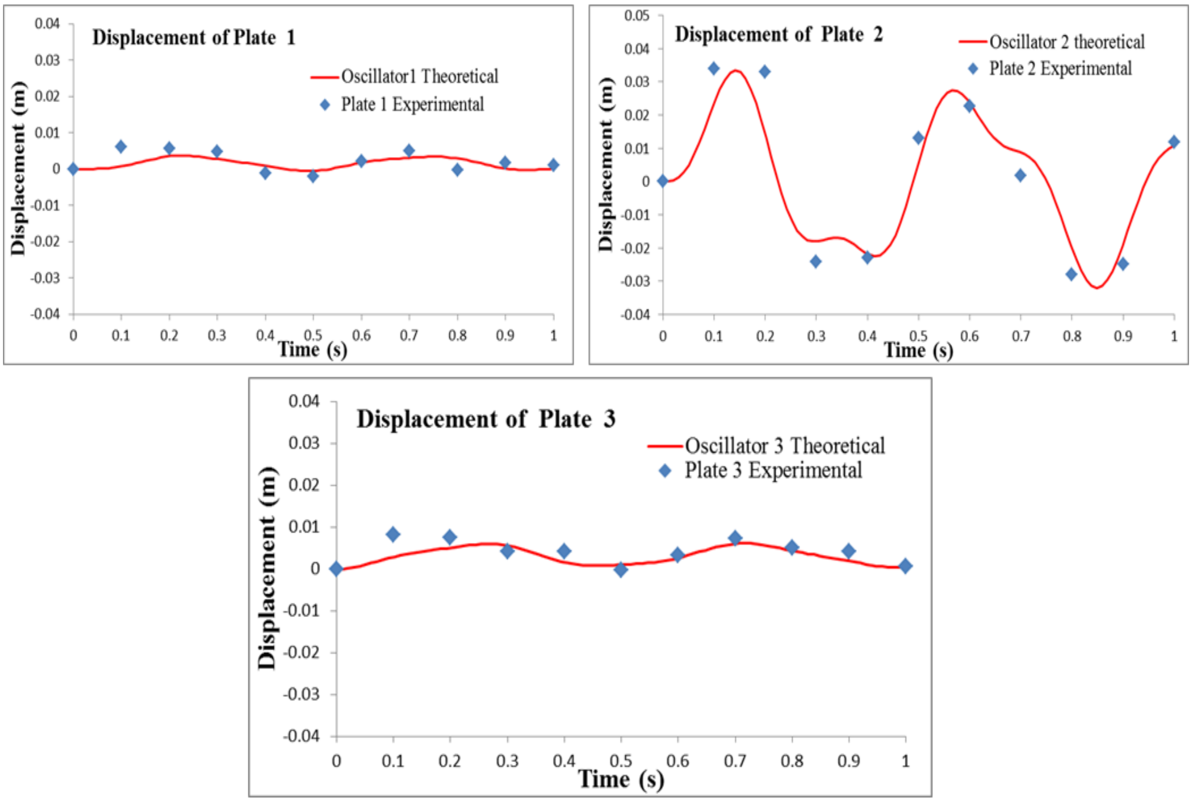

Fig.3Numerical &Experimental Displacement of Oscillator1, 2 and 3

B. Verification of Velocities:

Fig.4 shows the theoretical and experimental velocity simulation of the plates. It can be observed that the higher velocity occurs for plates 1 and 3 as the force directly acting on plate 2 and there is no controlling load on plate 1 and 3. The velocity of plate 2 is very low compared to the plate 1 and 3. In the experiment the system behavior is same as the theoretical behavior. In between 0.1 to 0.5s a small variation in the velocity of plates 1 and 3 as the time increases the velocity becomes stable.

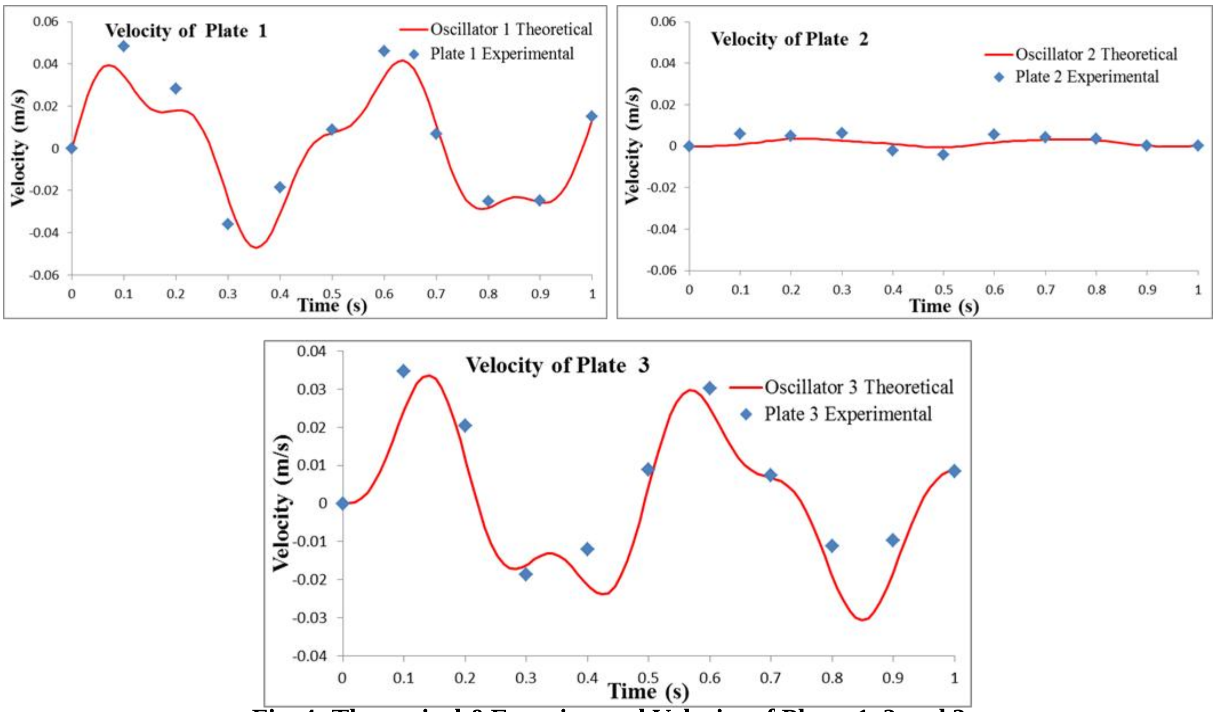

Fig. 4: Theoretical &Experimental Velocity of Plates 1, 2 and 3

C. Verification of Energies for Plates:

The experimental values are good matches in between 0.5s onwards because the system has to be taken time for equilibrium state. There are some errors in the experimental values that is because of small phase errors in the two measurement channels and an insufficient number of measurement points were taken.

. The overall trend of the profile of the energies of the experimental values seems to be in good agreement with the theoretical prediction.

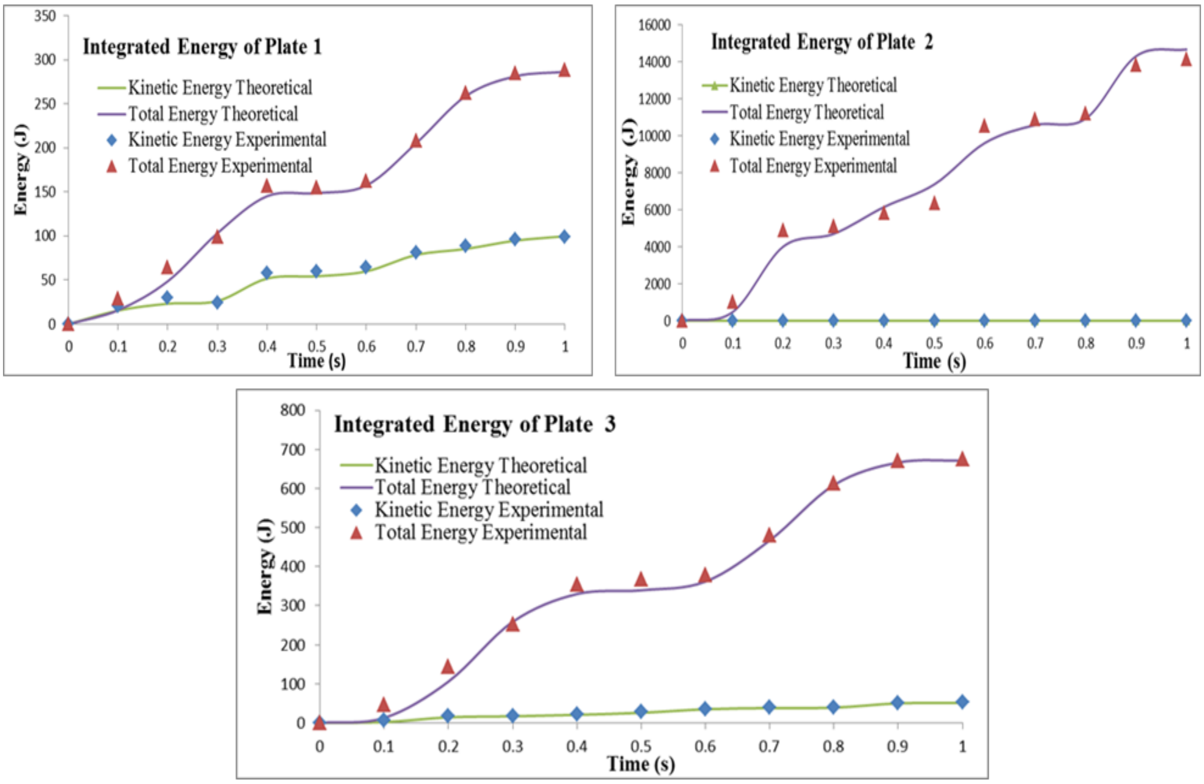

Fig. 5: Numerical & Experimental Integrated energy of Plates 1, 2&3

D. Verification of Total transferred Energies of Plates:

The total energy transferred from the plates is found out by theoretical and experimental as shown in Fig. 6. The plate 2 is excited with a persistent load of 5 Newton.

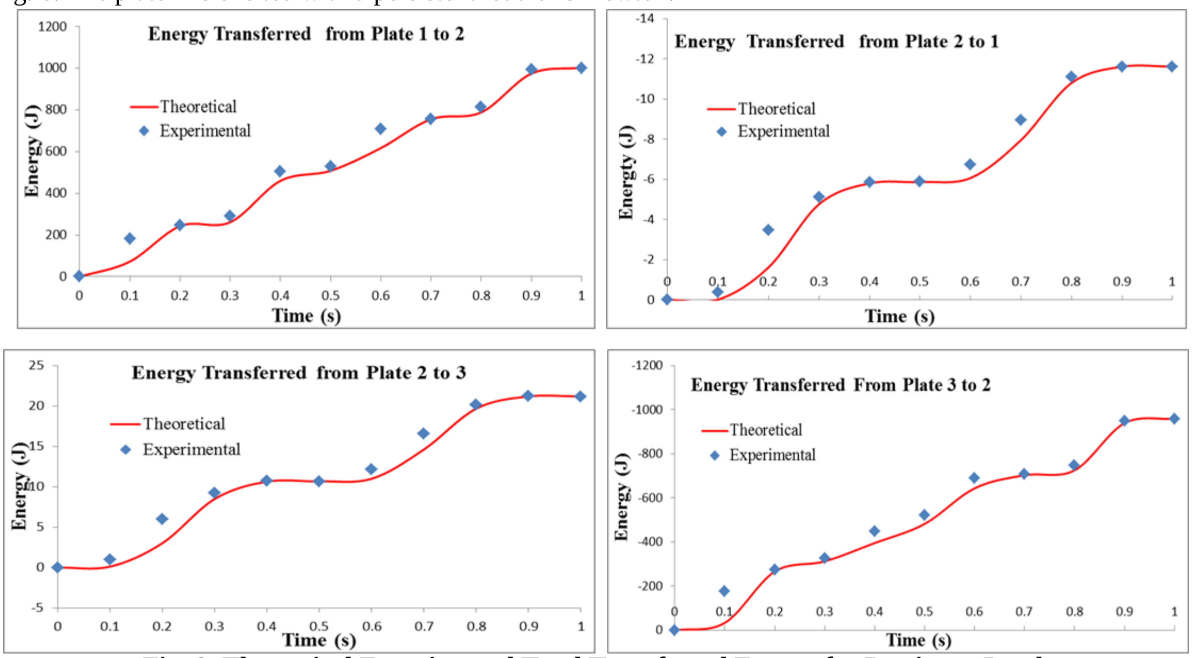

Fig. 6: Theoretical Experimental Total Transferred Energy for Persistent Load

It is observed from Fig.6, that the total energy transferred from plate 1 to plate 2 is 1000 J. But from plate 2 to 1 these values are negative since very less amount of energy is transferred from plate 2 to 1 due to equilibrium condition. The energy transferred from plate 2 to 3 at 1 second is 20 J. The energy transferred from plate 3 to 2 is -900 J. The experimental values are matching very close with the numerical values.Positive energies are from plate 1 to 2 and from 2 to 3. Negative energies are from 2 to 1 and from 3 to 2 in the reverse direction.

V. CONCLUSIONS

Validations of Numerical Simulation results are done by conducting experiment on the three degrees of freedom Vibrating System by power input method. Thedisplacement of oscillators for the theoretical prediction values are observed experimentally. The overall trend of the profile of the displacement of the experimental values seems to be in good agreement with the theoretical prediction. The difference in the values between the theoretical prediction and the experimental values can be attributed to the approximations made in simplification of the vibrating system for numerical solution.

From the velocity simulation of the plates of the model, it can be observed that the higher velocity occurs for plates 1 and 3 as the force directly acting on plate 2 and there is no controlling load on plate 1 and 3 the velocity. The velocity of plate 2 is very low compared to the oscillator 1 and 3, In the experiment the system behavior is same as the theoretical behavior.

From the energy plots we observed that the positive energies are from oscillator 1 to 2 and from 2 to 3. Negative energies are from 2 to 1 and from 3 to 2 in the reverse direction, which is evident experimentally also.
The overall trend of the profile of the experimental values seems to be in good agreement with the theoretical prediction. The difference in the values between the theoretical prediction and the experimental values can be attributed to the approximations made in simplification of the vibrating system for numerical solution. Also, there may be some cross vibrations coming from the floor on which the vibrating system is mounted.

REFERENCES

[1] R.H. Lyon and R.G. DeJong,"*Theory and Application of Statistical Energy Analysis"* Elsevier science & technology, Books, 1995.

[2] S.S. Rao,"*Mechanical Vibrations"* Prentice Hall Inc, Book, 2004.

[3] A.D. Nashif, "*Vibration Damping"* Wiley John & Sons, Book, 1985.

[4] C.W. DeSilva, "*Vibration Fundamentals and Practice"* CRC Press,Book, 1999.

[5] W. Liu,"*Experimental and Analytical Estimation of damping in beams and plates with damping treatments"* PhD Thesis, Lawrence, Aerospace Engineering,Univercity of Kansas, 2008.

[6] S.S. Sastry, "*Intoductory Methods of Numerical Analysis"* Fourh Edition, Prentice-Hall, 2005.

[7] Willam.T.Thomsons, Maric Dillon Dahleh,"*Theory of Vibrations and with applications"* 5th Edition,Persons, 1998.

[8] D.W. Zingg, T.T. Chisholm, "Runge–Kutta methods for linear ordinary differential equations" *Applied Numerical Mathematics,* Vol. 31, pp 227–238, 1999.

[9] R.H. Lyon and R.G. DeJong, "Theory and Application of Statistical Energy Analysis" Butterworth, London, 1995.

[10] C.B. Burroughs, R.W. Fischer, F.R. Kern, "An introduction to statistical energy analysis" The Journal of the Acoustical Society of America, Vol.101, pp 1779–1789,1997.

[11] E.E. Ungar, "Fundamentals of statistical energy analysis of vibrating systems" US Air Force, AFFDL-TR 66–52, April, 1966.

[12] L. Maxit, J.L. Guyader, "Estimation of SEA coupling loss factors using a dual formulation and FEM modal information Part 1: theory Part 2: numerical applications, Journal of Sound and Vibration, Vol. 239, pp. 907–948.2001.

[13] F.H. Chu and B.P. Wang, "Experimental determination of damping in materials and structures" Edited by Torvik P.J.,*Damping application for vibration control ASME winter anual meeting.* Chicago, pp113-122, 1980.

[14] M. Carfagni and M. Pieriai, "Determining the loss factor by the power input method(PIM) Part I Numerical Investigation" *Journal of sound and acoustics,* Vol 121, pp 417-421, 1997.

[15] D.A. Bies and S. Hamid, "In situ determination of loss and coupling loss factors by power injection method" *Journal of Sound and Vibration,* Vol.70, pp 187-204,1980.

[16] W. Liu and M.S. Ewing, "Experimental and Analytical Estimation of Loss factors by Power Input Method" *AIAA Journal,*Vol.45(2), pp 477-484, 2007.

[17] K.De Langhe, "High Frequency vibrations: Contributions to experimental and Computational SEA parameters Identification Technique" Ph.D. Thesis, Belgium, 1996.

[18] R.H. Lyon, "Statistical Analysis of Power Injection and Response in Strucures and Rooms" *Journal of Acoustical Society of America,* Vol. 45, 1969.

[19] M.F. Ranky and B. L. Clarkson, "Frequency average loss factors of plates and shells" *Journal of Sound and Vibration*, Vol. 89(3), pp 309-323, 1983.

[20] Jocobsen and Finn, "*Measurement of structural loss factors by the power input method"* Technical University of Denmark, 1986.

[21] J. Plunt, "Power injection method for vibration damping determination of body panels with applied damping treatments and trim" *SAE Transactions*, Vol. 100, pp 1563-1571, 1991.

[22] Bolduc, Maxime, "Acquiring Statistical Energy Analysis Damping Loss Factor for Complex Structures with Low to High Damping Characteristics" Ph.D dissertation, University of Sherbrooke, Quebec Canada, August 2007.

[23] H. Dande, "Panel Damping Loss Factor Estimation using the random Decrement Technique" Master of Science, Aerospace Engineering, University of Kansas, Lawrence, 2009.

[24] K.Vatti, "Damping Estimation of Plates for Statistical Energy Analysis" Master of Science, Aerospace Engineering, University of Kanasa, Lawrence, 2010.

[25] B.C. Bloss and M.D. Rao, "*Estimation of Frequency–Averaged Loss Factors by the Power Injection and the Impulse Response Decay Methods*" Journal of the Acoustical Society of America, Vol.117(1), pp 240–249, 2005.

[26] M.S. Ewing, "Validation of Panel Damping Loss Factor Estimation Algorithms using a Computational Model" *The 50th AIAA/ASME/ASCE/AHS/ASC, Structures Structural Dynamics and Materials conference*, 2009.

[27] J. He and Z.F. Fu, "*Modal Analysis"* Butterworth-Heinemann, 2001.

[28] F.H. Chu and B.P. Wang, "Experimental determination of damping in materials and structures" Edited by Torvik P.J.,*Damping application for vibration control ASME winter annual meeting,* Chicago, pp 113-122,1980.

[29] Adhikari. Sondipon, "Damping Models for Structural Vibration" A Ph.D dissertation submitted to the University of Cambridge, September 2000.

[30] S.S. Rao, "*The Finite Element Method in Engineering" 2nd Edition,* Oxford: Pergamon Press, 1989.

[31] E.K. Blum, "A modification of the Runge–Kutta fourth-order method," *Math. Comp* Vol.16, pp 176–187,1962.

[32] S. Finnveden, "A Symmetric Formulation for Experimental Statistical Energy Analysis" *Journal of Sound and vibration*, Vol. 223(1), pp 161-169,1999.

[33] J.Wood House, "An approach to the theoretical background of statistical energy analysis applied to structural vibration" *Journal of the Acoustical Society of America*,Vol. 69 ,pp1695-1709,1981.