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Skolem Mean Labeling Of Nine Star Graphs

[1]D. S. T. Ramesh, [2]I. Gnanaselvi, [3]S. Alice Pappa, [4]P. Alayamani
[1]*Department of Mathematics Margoschis College, Nazareth – 628 617 Tamil Nadu, INDIA*
[2]*Department of Mathematics Jayaraj Annapackiam C.S.I. College of Engineering Nazareth – 628 617, Tamil Nadu, INDIA*
[3]*Department of Mathematics, Margoschis College, Nazareth-628 617, Tamil Nadu, India.*
[4]*Department of Mathematics, Pope's College, Sawyerpuram-628251, Tamil Nadu, India.*

Abstract:-A graph G = (V, E) with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $\{1, 2, ...p\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, ...p\}$ defined by $f^*(e = uv) = \frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u) + f(v) + 1}{2}$ if f(u) + f(v) is odd then the resulting edges get distinct labels from the set $\{2, 3, ..., p\}$. In this paper, we prove that nine star graph $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + 7\ell$ for ℓ = 2, 3, . . .

Keywords: Skolem mean graph, skolem mean labeling, star graphs

I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [3]. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. A graph with p vertices and q edges is called a (p, q) graph. In this paper, we prove that six star graph $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + 4\ell$ for ℓ = 2,3, . . .

II. SKOLEM MEAN LABELING

2.1.0 Definition: A graph G is a non empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively. $|V(G)| = q$ is called the size of G. A graph of order p and size q is called a (p,q)-graph. If e = uv is an edge of G, we say that u and v are adjacent and that u and v are incident with e.

2.1.1 Definition: A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). Similarly, an edge labeling of a graph G is an assignment of labels to the edges of G that induces for each vertex v a label depending on the edge labels incident on it. Total labeling involves a function from the vertices and edges to some set of labels.

2.1.2 Definition: A graph G with p vertices and q edges is called a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0, 1, …, q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then the resulting edge labels are distinct. The labeling f is called a mean labeling of G.

2.1.3 Definition: A graph G = (V,E) with p vertices and q edges is said to be skolem mean if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 1, 2, ..., p in such a way that when the edge e = uv is labelled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from 2, 3, ..., p. f is called a skolem mean labeling of G.

A graph G = (V, E) with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $\{1,2,...,p\}$ such that the induced map f^* from the edge set of G to $\{2,3,..,p\}$ defined by

$$f^*(e=uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even}\\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd}\end{cases}$$

, then the resulting edges get distinct labels from the set $\{2,3,...,p\}$

2.1.4 Definition: The r-star is the disjoint union of $K_{1a_1}, K_{1a_2}, K_{1a_3},...,K_{1a_r}$ where $a_1, a_2,...,a_r$ are positive integers and K_{1a_i} is a star of length a_i for $1 \le i \le r.$ We denote it by $K_{1a_1} \cup K_{1a_2} \cup K_{1a_3} \cup ... \cup K_{1a_r}$. G has $a_1 + a_2 + ... + a_r + r$ vertices and $a_1 + a_2 + ... + a_r$ edges.

2.1.5 Theorem: The nine star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where $\ell \le m$ is a skolem mean graph if $|m-n| = 4+7\ell$ for ℓ = 2, 3, 4, . . .

Proof: Consider the graph G = $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where V(G) = $\{v_{i,j} : 1 \le i \le 7; 0 \le j \le \ell\} \cup \{v_{8,j} : 0 \le j \le m\} \cup \{v_{9,j} : 0 \le j \le n\}$ and E(G) = $\{v_{i,0}v_{i,j} : 1 \le i \le 7; 1 \le j \le \ell\} \cup \{v_{8,0}v_{8,j} : 1 \le j \le m\} \cup \{v_{9,0}v_{9,j} : 1 \le j \le n\}.$ Then G has $7\ell+m+n+9$ vertices and $7\ell+m+n$ edges.

Without loss of generality, we assume that $m < n$ where $n = 7\ell+m+4$ for ℓ = 2, 3, . . . and m = 2, 3, . . . Let us consider the case that $|m-n| = 4+7\ell$ for ℓ = 2, 3, . . . Let us prove that G is a skolem mean graph. The vertex labeling $f : V(G) \to \{1,2,3,...,7\ell+m+n+9\}$ is defined as follows:

$$f(v_{i,0}) = \begin{cases} i & 1 \le i \le 7\\ 9 & i = 8\\ 7\ell+m+n+8 & i = 9\end{cases}$$

$$f(v_{i,j}) = \begin{cases} 2\ell(i-1)+2j+9 & 1 \le i \le 7; 1 \le j \le \ell\\ 2\ell(i-1)+2j+9 & i = 8; 1 \le j \le m\end{cases}$$

$$f(v_{9,j}) = \begin{cases} 6+2j & 1 \le j \le n-2\\ 7\ell+m+n+7 & j = n-1\\ 7\ell+m+n+9 & j = n\end{cases}$$

The corresponding edge labeling F is defined as

$$F(uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even}\\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd}\end{cases}$$

Then, $$F(v_{i,0}v_{i,j}) = \begin{cases} (i-1)\ell + j + 5 & i = 1; \quad 1 \le j \le \ell \\ (i-1)\ell + j + 6 & 2 \le i \le 3; \; 1 \le j \le \ell \\ (i-1)\ell + j + 7 & 4 \le i \le 5; \; 1 \le j \le \ell \\ (i-1)\ell + j + 8 & 6 \le i \le 7; \; 1 \le j \le \ell \end{cases}$$

$$F(v_{8,0}v_{8,j}) \quad = 7\ell + j + 9 \qquad 1 \le j \le m$$

$$F(v_{9,0}v_{9,j}) \quad = 7\ell + m + j + 9 \qquad 1 \le j \le n \text{ and } m = 2,3,4,\ldots$$

The resultant edge labels of G are 6, 7, . . . , $\ell+5$, $\ell+7$, . . . , $2\ell+6$, $2\ell+7$, . . . , $3\ell+6$, $3\ell+8$, . . . , $4\ell+7$, $4\ell+8$, . . . , $5\ell+7$, $5\ell+9$, . . . , $6\ell+8$, $6\ell+9$, . . . , $7\ell+8$, $7\ell+10$, . . . , $7\ell+m+9$, $7\ell+m+10$, . . . , $7\ell+m+n+9$ and has $7\ell+m+n$ distinct labels. Hence the induced edge labels of G are distinct. Hence the graph G is a skolem mean graph. ■

EXAMPLE: Figure 1 gives an example of skolem mean labeling of

$K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,3} \cup K_{1,21}$

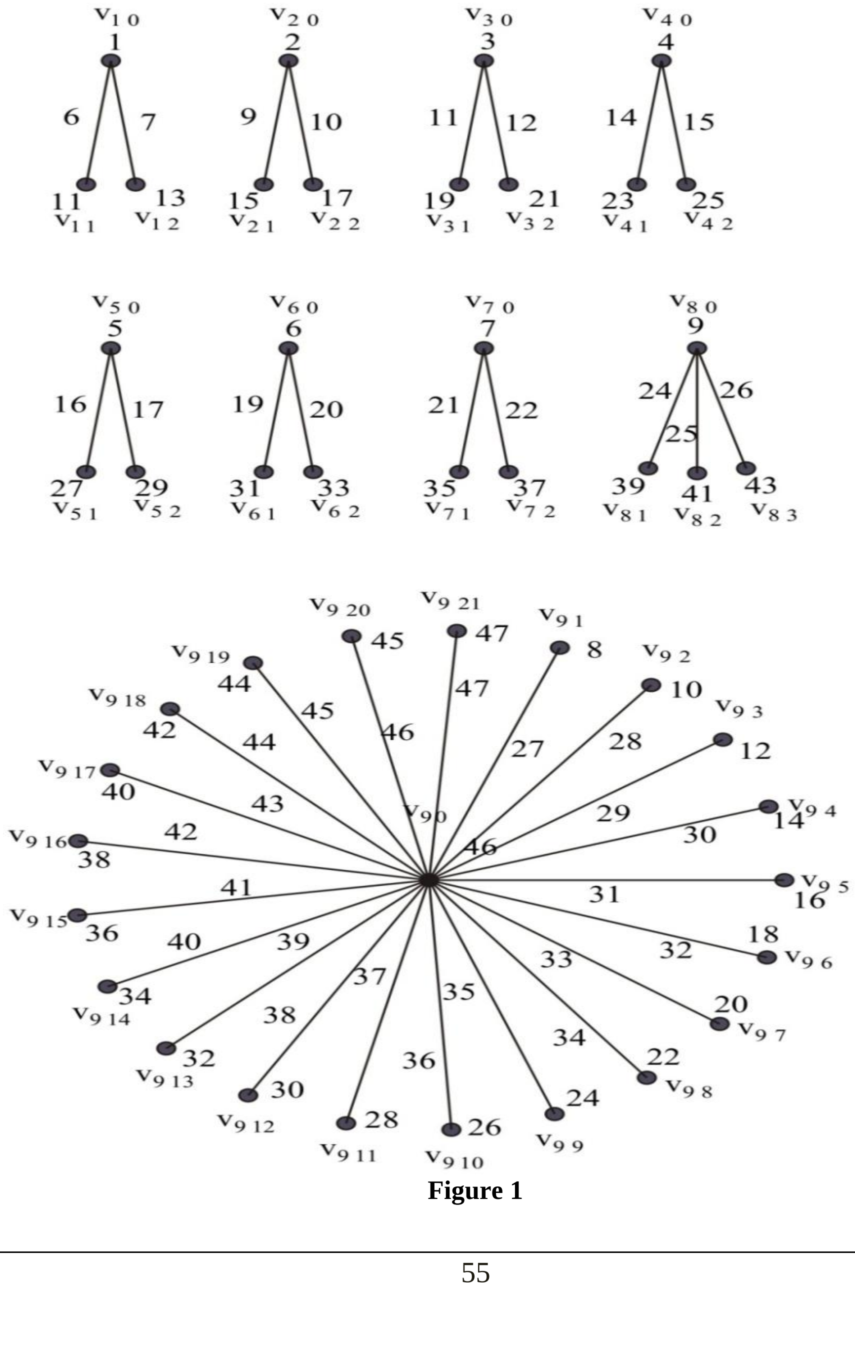

Figure 1

REFERENCES

[1] V. Balaji, D. S. T. Ramesh and A. Subramanian, *Skolem Mean Labeling***,** Bulletin of Pure and Applied Sciences, Vol. 26E No. 2 (2007), 245 –248.

[2] V. Balaji, D. S. T. Ramesh and A. Subramanian, *Some Results On Skolem Mean Graphs***,** Bulletin of Pure and Applied Sciences, Vol. 27E No.1(2008), 67–74.

[3] J. A. Gallian, *A Dynamic Survey of Graph Labeling***,** The Electronic Journal of Combinatorics, 14(2007), #DS6.

[4] F. Harary, *Graph Theory***,** Addison – Wesley, Reading Mars, (1972).