# Mathematical Modeling and Design Analysis of a Dexterous Endeffector

BSSPM Sharma<sup>1</sup>, M Kiran<sup>2</sup>, V Siva Brahmaiah Rama<sup>3</sup>, Shital Joshi<sup>4</sup>

<sup>1</sup>Mewar University, Electrical & Electronics Division, Rajasthan India <sup>2</sup> VIT University, Mechatronics Division, Vellore India <sup>3</sup> Mewar University, Electrical & Electronics Division, Rajasthan India <sup>4</sup>Mewar University, Electronics and Communication Division, Rajasthan India

**Abstract**—The design and development of dexterous robotic end effectors has been an active research area for a long while. This paper reviews the design and construction of a versatile robotic gripper used to grasp objects of arbitrary shape, size and weight. This is achieved through a mechanical design that incorporates multiple fingers and multiple joints per finger. Human motion can provide a rich source of examples for use in robot grasping and manipulation. Adapting human examples to a robot manipulator is a difficult problem, however, in part due to differences between human and robot hands. Even hands that are anthropomorphic in external design may differ dramatically from the human hand in ability to grasp and manipulate objects due to internal design differences. For example, force transmission mechanisms in robot fingers are generally symmetric about flexion and extension axes, but in human fingers they are focused toward flexion. The design and implementation of a Three-fingered human-size robotic hand intended for dexterous and grasping manipulation applications.

Equations of motion for an index finger was solved numerically using Newton Euler formulation. Matlab code was written in an M-file which helped us in determining the position, velocity and acceleration of individual joints with respect to time.

Tendon driven hands are studied from previous research work to determine its capability in driving the End effectors. Motors located at remote location transmit the driving force on the joints using tendons. Grasp simulation of the designed End effector for various grasp objects are studied in ADAMS. ADAMS is an Dynamic simulation tool that is used for analysis.

Keywords: Dexterous End Effector, Grasp Simulation, Tendon Driven.

## I. INTRODUCTION

Robots have the potential to play a large role in our world. They are currently widely used in industrial applications for labor-intensive operations that require a high level of precision and repetition. In addition, robots can be found in the entertainment industry in the form of toys and animatronics. The function of robots in society is constantly evolving and current research endeavors to bring them further into the realm of domestic assistance, medicine, military, search and rescue, and exploration. In many of these applications, the robot must perform only one specific task and thus can be designed to handle a single operation. However, as the potential use for robots grows, so does their need to interact with objects in their environment. The design of end effectors that can pick up a variety of objects and utilize them as tools is a significant challenge in robot development. Human hand is considered to be the most dexterous end effector with a total of 26 DOF. Its versatility provides motivation for the design of an end effector for industrial Manupualtor. The traditional industrial end effectors are not capable of handling of object in different Size and Shape. There by reducing the capability and adaptability of the end effector during different operation among changing environment. Several Dexterous end effectors have been developed and research work is carried out in this field With this respect, as already demonstrated in the industrial environment, a bottleneck is constituted by the end effector that often is a very simple device with poor sensoriality and limited operational capabilities. Besides the numerous prototypes of articulated robotic hands, developed in more than 30 years of research, mainly in academic environment among many others, limited effort has been devoted to seek and evaluate alternative solutions, maybe simpler from the mechanical point of view than a multi-fingered hand, but with sufficient dexterity to perform in any case non trivial operations on a wide range of objects. Multifingered robot hand is a complex mechanism with multiple-degree-of-freedom, many multifingered robot hands have been developed, such as Utah/MIT hand[1], DLR hand[2], Shadow Dexterous Hand[3], Robonaut hand[4], NAIST-Hand[5], Gifu hand[6].

It provides a promising base for supplanting human hand in execution of tedious, complicated and dangerous tasks, more precisely than a human hand, in situations such as manufacturing, space, the seabed, grasp planning is one of key issues for robotic dexterous hands to accomplish the desired tasks. In the field of robot hand programming, grasp planning involves determining the hand placement relative to the object of interest as well as the posture of the hand assumed in grasping object. In the study of robotics, we are constantly concerned with the location of objects in three dimensional spaces. The objects are the links of the manipulator, the parts and tools with which it deals and other objects in the manipulator's environment. At a crude but important level, these objects are described by just two attributes: position and

orientation.one topic of immediate interest is the manner in which we represent these quantities and manipulate them mathematically.

# II. DESIGN OF ANTHROPOMORPHIC END EFFECTOR

Based on the discussion given in the previous sections, it was decided to design a new anthropomorphic robot hand that incorporated two fingers and an opposable thumb. Each of the fingers in the new design has three joints allowing flexion motion, equivalent to the MCP, PIP and DIP joints of the human finger, here in after referred to as finger joints 1, 2, and 3 respectively. Each finger has three independent degrees-of-freedom, The thumb has two joints and here In after referred to as thumb joints 1 and 2 respectively. Hence making the design of total 8 DOF.Each joints where to be controlled by different actuators, the actuators where to be located at a remote location and to drive the joints using tendons connected to pulley's at respective joints.

Table 1. Dimensions of the designed End Effector.							
Parameters (mm)	Proximal Phalange	Middle Phalange	Distal Phalange				
Length	50	38	24				
Breadth	20	15	15				
Thickness	1	1	1				
Diameter of Pulley	18	8	4				

AND FEFTURE

Fig.1. End Effector Designed in Solid works.

# III. ACTUATION MECHANISM

We employed a tendon-driven mechanism to drive the fingers in order to design a small hand part with a sufficient fingertip force. The driving forces from the actuators are transmitted to the fingers through a gear mechanism at the wrist. The gear mechanism at the wrist makes the hand part and the actuator part split able. By splitting the hand part and the actuator part, we can separately maintain either parts, and moreover, we can replace the actuator part to meet conditions. The gears for driving the fingers are arranged in line In order to develop a human-sized finger, we have integrated a special thin and small pulley. The pulley consists of a cover and a base with a wire guiding groove. The wire is set along the guiding groove and is held by screwing the cover part. The developed pulley is used in every joint. The dimensions of the developed robot hand is 200[mm] (length)×70[mm] (width)×20[mm] (thickness). and it has at least 10[N] at the fingertip in the current configuration. The simulation test in ADAMS showed the effectiveness of the developed mechanism in grasping objects of various shape and size.





Fig.2. End Effector Simulation in ADAMS.



Fig.3. End Effector Simulation in ADAMS Grasping a ball.

## IV. MODEL OF A PROPOSED MULTI FINGER ROBOTIC HAND

The finger has 3 active joints. DIP joint has connection with PIP joint. The thumb is designed by having 2 active joints. The joint of each link of MFRH model is a frame to determine the kinematic derivation. The frames are named by number according to which they are attached. The convention that was used to locate the frame on the links is known as the D-H convention which is given below:

The *z* - axis of frame  $\{i\}$ , called  $\{zi\}$ , is coincident with the joint *i*. The origin of frame  $\{i\}$  is located where the  $\alpha$ i perpendicular intersects the joint *i* axis. *xi* points along *a* i in the direction from joint *i* to joint *i* +1. Assuming that the frames have been attached to the links according to the D-H convention, the following definitions of the link parameters are

valid .Rotate the frame xi-1 yi-1z i-1 about the z i-1 axis through an angle  $\theta$  i. Translate the current frame xi-1 yi-1 zi-1 along the current z i-1 axis by di units. Translate the current frame xi-1 yi-1 zi-1 along the current xi axis by ai units. Rotate the current frame xi-1 yi-1 zi-1 about the xi axis through an angle ai. Fig. 4 shows that the finger has four frames with three joints. The first frame also known as the base frame is x0, y0, z0 and the subsequent frames are assigned as per the figure starting with x1, y1, z1 and ending with x4, y4, z4. The forward kinematic solution of a finger will be assigned using homogenous matrix.

# 1) Forward Kinematic



Fig.4. Model of Index finger

Forward Kinematic is used to determine the position and orientation of MFRH to determine the position and orientation of the robot hand relative to the robot base coordinate system. The derivation of forward kinematic equation based on TableI

Table I.	DH	Table
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i	$\theta_{i}$	i	<i>a</i> <sub><i>i</i>—1</sub>	α <sub>i-1</sub>
1	$\theta_{_1}$	0	0	0
2	$\theta_{_2}$	0	L1(MCP)	0
3	$\theta_{_3}$	0	L2(PIP)	0
4	90	0	L3(DIP)	0

#### V. DYNAMIC MODELING

Manipulator dynamics is concerned with the equations of motion, the way in which the manipulator moves in response to torques applied by the actuators, or external forces. The history and mathematics of the dynamics of serial-link manipulators is well covered by Paul and Hollerbach. There are two problems related to manipulator dynamics that are important to solve:

- *inverse dynamics* in which the manipulator's equations of motion are solved for given motion to determine the generalized forces.
- *direct dynamics* in which the equations of motion are integrated to determine the generalized coordinate response to applied generalized forces.

The equations of motion for an *n*-axis manipulator are given by

$$\boldsymbol{Q} = \mathbf{M}(\boldsymbol{\theta}) \boldsymbol{\ddot{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \boldsymbol{\dot{\theta}}) \boldsymbol{\dot{\theta}} + \mathbf{F}(\boldsymbol{\dot{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) \quad (1)$$

where

•

heta is the vector of generalized joint coordinates describing the pose of the manipulator,

 $\dot{\theta}$  is the vector of joint velocities,

 $\hat{\theta}_{\text{ is the vector of joint accelerations,}}$ 

M is the symmetric joint-space inertia matrix, or manipulator inertia tensor,

C describes Coriolis and centripetal effects-Centripetal torques are proportional to  $\dot{\theta}^{2}i$ ,

while the Coriolis torques are proportional to  $\dot{\theta}_i \dot{\theta}_j$ ,

F describes viscous and Coulomb friction and is not generally considered part of the rigidbodydynamics,

G is the gravity loading,

Q is the vector of generalized forces associated with the generalized coordinates heta .

#### A.. Recursive Newton-Euler formulation

The recursive Newton-Euler (RNE) formulation computes the inverse manipulator dynamics, that is, the joint torques required for a given set of joint angles, velocities and accelerations. The forward recursion propagates kinematic information — such as angular velocities, angular accelerations, linear accelerations — from the base reference frame (inertial frame) to the end-effector. The backward recursion propagates the forces and moments exerted on each link from the end-effector of the manipulator to the base reference frame3.



Fig. 5 The Frames of the link with reference to the previous link

## 1) Outward recursion, $1 \le i \le n$ .

If axis *i*+1 is rotational <sup>*i*+1</sup>  $W_{i+1} = {}^{i+1}\mathbf{R}_{i}^{i} W_{i} + \dot{\theta}_{i+1} {}^{i+1} \hat{z}_{i+1}$ <sup>*i*+1</sup>  $\dot{W}_{i+1} = {}^{i+1}\mathbf{R}_{i}^{i} \dot{W}_{i} + {}^{i+1}\mathbf{R}_{i}^{i} W_{i} \times \dot{\theta}_{i+1} {}^{i+1} \hat{z}_{i+1} + \ddot{\theta}_{i+1} {}^{i+1} \hat{z}_{i+1}$ <sup>*i*+1</sup>  $\dot{V}_{i+1} = {}^{i+1}\mathbf{R}_{i} ({}^{i} \dot{V}_{i} + {}^{i} W_{i} \times {}^{i} p_{i+1})$ <sup>*i*+1</sup>  $\dot{V}_{i+1} = {}^{i+1}\mathbf{R}_{i} ({}^{i} \dot{W}_{i} \times {}^{i} p_{i+1} + {}^{i} W_{i} \times ({}^{i} W_{i} \times {}^{i} p_{i+1}) + {}^{i} \dot{V}_{i})$ <sup>*i*+1</sup>  $\dot{V}_{i+1} = {}^{i+1} \dot{W}_{i+1} \times {}^{i+1} p_{i+1} + {}^{i+1} W_{i+1} \times {}^{i+1} W_{i+1} \times {}^{i+1} p_{i+1} + {}^{i+1} \dot{V}_{i+1}$ <sup>*i*+1</sup>  $F_{i+1} = m_{i+1} {}^{i+1} \dot{V}_{i+1}$ <sup>*i*+1</sup>  $\dot{V}_{i+1} = {}^{Ci+1} \mathbf{I}_{i+1} {}^{i+1} \dot{W}_{i+1} + {}^{i+1} W_{i+1} \times {}^{Ci+1} \mathbf{I}_{i+1} {}^{i+1} W_{i+1}$ 

2) Inward iterations: i: 4 to 1

$$\begin{aligned} & \stackrel{i}{f_{i}}_{i} = \stackrel{i}{R_{i+1}} \stackrel{i+i}{f_{i+1}} \stackrel{i+i}{f_{i+1}} \stackrel{i+i}{F_{i}}_{i} \\ & \stackrel{i}{n_{i}}_{i} = \stackrel{i}{N_{i}} \stackrel{i}{R_{i+1}} \stackrel{i+i}{n_{i+1}} \stackrel{i+i}{n_{i+1}} \stackrel{i}{+} \stackrel{i}{p_{ci}}_{i} \times \stackrel{i}{F_{i}} \stackrel{i}{+} \stackrel{i}{p_{i+1}} \times \stackrel{i}{R_{i+1}} \stackrel{i+i}{f_{i+1}}_{i} \\ & \mathcal{T} = \stackrel{i}{n_{i}} \stackrel{i}{n_{i}} \stackrel{i}{\mathcal{I}} \stackrel{i}{\mathcal{I}} \quad . \end{aligned}$$

# VI. TRANSFORMATION MATRICES OF DIFFERENT FRAMES USING DENAVIT HARTENBERG

While it is possible to carry out all of the analysis using an arbitrary frame attached to each link, it is helpful to be systematic in the choice of these frames. A commonly used convention for selecting frames of reference in robotic applications is the Denavit- Hartenberg, or D-H convention. In this convention, each homogeneous transformation  $\mathrm{Hi}^{11}$  is represented as a product of four basic transformations

$$H^{i-1}_{i} = \begin{pmatrix} c \theta_{i} & -s \theta_{i} & 0 & a_{i-l} \\ s \theta_{c} \alpha_{i-1} & c \theta_{c} \alpha_{i-1} & -s \theta_{i-1} & -s \alpha_{i-1} d_{i} \\ c \theta_{s} \alpha_{i-1} & c \theta_{s} \alpha_{i-1} & c \alpha_{i-1} & c \alpha_{i-1} d_{i} \\ 0 & 0 & 1 \end{pmatrix}$$

$$H^{0}_{4} = H^{0}_{1} H^{1}_{2} H^{2}_{3} H^{3}_{4}$$

$$H^{0}_{4} = \begin{bmatrix} c_{123} & -s_{123} & 0 & l_{l} c_{1} + l_{2} c_{12} + l_{3} c_{123} \\ s_{123} & c_{123} & 0 & l_{l} s_{1} + l_{2} s_{12} + l_{3} s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where

$$Cos \ \theta_{1}cos \ \theta_{2} - sin \ \theta_{1}sin \ \theta_{2} = cos(\theta_{1} + \theta_{2}) = c_{12}$$
  

$$Sin \ \theta_{1}cos \ \theta_{2} + cos \ \theta_{1}sin \ \theta_{2} = sin(\theta_{1} + \theta_{2}) = s_{12}$$
  

$$Cos \ \theta_{12}cos \ \theta_{3} - sin \ \theta_{12}sin \ \theta_{3} = cos(\theta_{1} + \theta_{2} + \theta_{3}) = c_{123}$$
  

$$Sin \ \theta_{12}cos \ \theta_{3} + cos \ \theta_{12}sin \ \theta_{3} = sin(\theta_{1} + \theta_{2} + \theta_{3}) = s_{123}$$
  
The positon of the endeffector frame to the reference frame  
can be determined.

$$\begin{split} & \underset{\mathbf{X}_{n}=1_{1}c_{1}+1_{2}c_{12}+1_{3}c_{123};}{\underset{\mathbf{X}_{n}=1_{1}s_{1}+1_{2}s_{12}+1_{3}s_{123};}{\boldsymbol{\theta}=\boldsymbol{\theta}_{1}+\boldsymbol{\theta}_{2}+\boldsymbol{\theta}_{3}+\boldsymbol{\theta}_{4,i};}\\ & \boldsymbol{\theta}=\boldsymbol{\theta}_{1}+\boldsymbol{\theta}_{2}+\boldsymbol{\theta}_{3}+\boldsymbol{\theta}_{4,i};\\ & \boldsymbol{\theta}_{V}=\mathbf{\theta}_{R_{4}}\mathbf{x}^{4}v_{4} \\ & \boldsymbol{\theta}_{V}=\mathbf{\theta}_{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \\ & \dot{\mathbf{x}}=\begin{pmatrix} l_{1}s_{1}-l_{2}s_{12}-l_{3}s_{123}&-l_{2}s_{12}-l_{3}s_{123}&-l_{3}s_{123}&\dot{\boldsymbol{\theta}}_{1}\\ l_{1}c_{1}+l_{2}c_{12}+l_{3}c_{123}&l_{2}c_{12}+l_{3}c_{123}&l_{3}c_{123}&\dot{\boldsymbol{\theta}}_{2}\\ & \dot{\boldsymbol{\theta}}=\begin{pmatrix} l_{1}s_{1}-l_{2}s_{12}-l_{3}s_{123}&-l_{2}s_{12}-l_{3}s_{123}&-l_{3}s_{123}\\ l_{1}c_{1}+l_{2}c_{12}+l_{3}c_{123}&-l_{2}s_{12}-l_{3}s_{123}&-l_{3}s_{123}\\ & 1& & \dot{\boldsymbol{\theta}}_{3} \\ \end{split}$$
Hence  $^{0}j(\boldsymbol{\theta})=\begin{pmatrix} l_{1}s_{1,2}l_{2}s_{12}-l_{3}s_{123}&-l_{2}s_{12}-l_{3}s_{123}&-l_{3}s_{123}\\ l_{1}c_{1}+l_{2}c_{12}+l_{3}c_{123}&l_{2}c_{12}+l_{3}c_{123}&-l_{3}s_{123}\\ l_{1}c_{1}+l_{2}c_{12}+l_{3}c_{123}&l_{2}c_{12}+l_{3}c_{123}&l_{3}c_{123}\\ l_{1}& 1& 1 \end{pmatrix}$ 

Static Forces in Manipulator is obtained by taking the Transpose of the jacobian .

$$\tau = J^{T}F$$

# VII. RESULTS AND DISCUSSIONS

The Dexterous End effector design has a total 8 DOF, with 3 fingers which are to be actuated independently through motors placed at remote location. The objective of the design was to achieve the aesthetic and dexterity of human hand such that it can be used in work floor of an industry to perform various operation without the need of swapping of end effectors during operation cycles.

The kinematic and dynamic analysis where derived manually to determine the position, velocity, acceleration and torque with respect to time.

Dynamic simulation was carried out using ADAMS which gave us the simulation regarding various grasp positions regarding different size and objects. The maximum finger tip force that can be obtained using this design was about 8–15 N which is similar to that several designs developed in the past GIFU hand was able to produce only 4-9 N.



Fig 6. Reaction force developed in End Effector during grasp simulation in ADAMS.



*Fig7*. The variable q (or)  $\theta$  varying with respect to time, plot obtained in Matlab

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