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Boundary Layer Flow in Porous Medium Past a Moving Vertical Plate with Variable Thermal Conductivity and Permeability

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***Abstract*—*The present paper is an effort to deal with the problem of a steady two dimensional boundary layer flow of an incompressible viscous fluid past a moving vertical plate with variable permeability, thermal conductivity and suction. In our analysis, we have taken into account the effects of viscous dissipation also. The equations of continuity, motion and energy are transformed into a system of coupled ordinary differential equations in the non-dimensional form which are solved numerically. The effects of various parameters such as, Eckert number, Grashof number and permeability parameter on the velocity and temperature fields are discussed with the help of graphs.*

Keywords—*permeability, thermal conductivity, viscous dissipation, suction velocity, heat transfer,*

I. INTRODUCTION

The problems of convection boundary layer flows and heat transfer of a viscous and incompressible fluid in porous media are encountered quite frequently in geophysics, astrophysics and various engineering and industrial processes. The thermal boundary layer flow induced by a moving surface in a fluid saturated porous medium finds important applications in manufacturing of fiber (optical) materials, chemical engineering and electronics, cooling of nuclear reactors, meteorology and metallurgy etc. Sakiadis [1] and Erickson et al.[2] were the ones who initiated the study of boundary layer flow on a continuous moving surface. In these studies, viscosity and other fluid parameters have been assumed to have constant values hroughout the course of flow. The effect of buoyancy induced pressure gradient on the laminar boundary layer flow about a moving surface with uniform velocity and temperature was studied by Chen and Strobel[3]. It has been reported by Schlichting [4] that the physical properties of the fluids, mainly viscosity, may change significantly with temperature. The fluid flows with temperature- dependent properties are further complicated by the fact that different fluids behave differently with temperature. Different relations between the physical properties of fluids and temperature are given by Kays and Crawferd [5]. Taking into account the variable properties of fluid, Choi[6] studied the laminar boundary layer flow of an isothermal moving flat sheet and moving cylinder by finite difference method. Incorporating a step change in the plate temperature, Jeng et al.[7] studied the heat and momentum transfer about two dimensional plate moving with arbitrary velocity. Benanati and Brosilow[8] have shown that porosity of the medium may not be uniform and a variation ion porosity causes a variation in the medium permeability. Chandrasekhar and Numboodiri [9] carried out an analysis for mixed convection about inclined surfaces in a saturated porous medium incorporating the variation of permeability and thermal conductivity due to packing of particles. Elbashbeshy [10] analyzed the flow of a viscous incompressible fluid along a heated vertical plate taking the variation of the viscosity and thermal diffusivity with temperature in the presence of the magnetic field.. A numerical study on a vertical plate with variable viscosity and thermal conductivity has been treated by Palani and Kim[11]. It is observed that the effects of viscous dissipation are generally ignored in the motion through porous media. However, this effect is quite significant in highly viscous fluids. Anjali Devi and Ganga [11],[12] have considered the viscous dissipation effects on MHD flows past stretching porous surfaces in porous media. Aydin and Kaya [13] considered the laminar boundary flow over a flat plat embedded in a fluid saturated porous medium in the presence of viscous dissipation, inertia effects and suction/injection.

From the preceding investigations, it is clear that inclusion of the variation of medium permeability, thermal conductivity and viscous dissipation in mathematical formulation and analysis may give valuable insight regarding convective fluid flows about moving flat surfaces. Hence, we have considered here a steady two dimensional boundary layer dissipative fluid flow past a moving vertical plate with suction taking into account the variation in the medium permeability and thermal conductivity .

II. NOMENCLATURE

$u -$ Velocity in x-direction

$v -$ Velocity in y-direction

g- Acceleration due to gravity

c_p- Specific heat at constant presure

Pr- Prandtl number

Gr- Thermal Grashof number

u_w - Plate velocity

T_∞ - Temperature of the fluid away from the plate

$\beta -$ Coefficient of volume expansion

ρ_∞ - Ambient fluid density

λ - Thermal conductivity

k - Medium permeability

q_w - Rate of heat transfer

μ - Fluid viscosity

υ - Kinematic coefficient of viscosity

III. MATHEMATICAL FORMULATION

Consider a steady laminar boundary layer flow of an incompressible viscous fluid on an infinite plate, moving vertically with uniform velocity. The x- axis is taken along the plate in the upward direction and y- axis is normal to it. The fluid flow is caused by the motion of the plate with uniform velocity $u = u_w$ as well as by the buoyancy force due to the thermal diffusion across the boundary layer.

All the intrinsic fluid properties are assumed to be uniform except the density in the body force term. Assuming that Darcy law and Boussinesq approximations are valid, the equations governing the present two dimensional, steady and laminar boundary layer flow can be written as:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) - \frac{\mu u}{k\rho_\infty} \tag{2}$$

$$v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right) + \frac{\mu}{\rho_\infty C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mu}{k\rho_\infty C_p}u^2 \tag{3}$$

Where, $\alpha(y) = \dfrac{\lambda}{\rho C_p}$ is the variable thermal conductivity and on the right hand side of the equation (3) the second term is due to viscous dissipation and third term is the modification in the viscous dissipation modeling as suggested by Nield and Bejan(1992)(hereafter written as N-B modification).

From equation (1), we find

$$v = v_0\text{, a constant.}$$

Also, $v_0 < 0$, as a constant suction is applied at the plate.

The relevant boundary conditions with prescribed heat and mass flux then are:

$$u = u_w \, , \, v = -v_0 \, , \quad -\lambda \frac{\partial T}{\partial y} = q_w \, , \qquad \text{at} \quad y = 0 \qquad (5)$$

$$u \to 0, \quad T \to T_\infty , \qquad \text{as} \quad y \to \infty \qquad (6)$$

The first condition on the velocity at the plate follows from the no slip condition and the condition for temperature at the plate is that of uniform heat flux.
Introducing following non-dimensional variables-

$$\eta = \frac{\rho_\infty v_0}{\mu_0} y \, , \quad f(\eta) = \frac{u}{u_w} \, , \; \theta(\eta) = \left(T - T_\infty\right) \frac{v_0 \lambda}{q_w \upsilon} \, , \qquad (7)$$

where, η is the similarity variable, $f(\eta)$ is dimensionless form of velocity and θ is non-dimensional temperature.
Following Chandrasekhara et al. as stated above, the variations in the permeability and thermal conductivity have been taken in the following forms-

$$\begin{aligned} k(\eta) &= k_0 (1 + d^* e^{-\eta}) \\ \alpha(\eta) &= \alpha_0 [\varepsilon_0 (1 + d e^{-\eta}) + b\{1 - \varepsilon_0 (1 + d e^{-\eta})\}] \end{aligned} \qquad (8)$$

where d^* and d are constants, α_0, k_0 and ε_0 are the values of the diffusivity, permeability and porosity respectively at the edge of the boundary layer, b being the ratio of the thermal conductivity of the solid to that of the fluid.
Thus, with these assumptions on the physical parameters, the equations (2), (3) and (4) with the help of equation (7), reduce to the following ordinary differential equations:

$$f'' + f' + Gr\theta - \delta f = 0 \qquad (9)$$

$$\frac{1}{P_r}\left([\varepsilon_0(1+de^{-\eta}) + b\{1-\varepsilon_0(1+de^{-\eta})\}]\theta'' + \varepsilon_0 d(b-1)e^{-\eta}\theta'\right) + \theta' + E_c\left(f'^2 + \left(\frac{\delta f^2}{(1+d^* e^{-\eta})}\right)\right) = 0 \qquad (10)$$

where,

$$Gr = \frac{q_w \upsilon^2 g \beta}{\lambda v_0^3 u_w} \, , \qquad \delta = \frac{\upsilon^2}{k {v_0}^2} \, , \qquad (11)$$

$$\Pr = \frac{\upsilon}{\alpha} \, , \qquad Ec = \frac{{u_w}^2 \lambda v_0}{C_p \upsilon q_w} \, , \quad .$$

The transformed boundary conditions are:

$$f = 1 \; , \; \theta' = -1 \; , \qquad at \;\; \eta = 0 \qquad (12)$$

$$f = 0 \; , \; \theta = 0 \, , \qquad as \;\; \eta \to \infty \qquad 13)$$

IV. RESULTS AND DISCUSSION

The equations (9) and (10) form a system of coupled non linear ordinary differential equations which are to be solved subject to the boundary conditions (12) and (13). They have been solved numerically using boundary value problem solver code. The results have been presented graphically. There are six graphs in all and they have been numbered as figures 1-3. Each figure contains two graphs , one each for velocity f and the temperature θ showing the effects of various parameters on them.

In figure 1, the effects of permeability parameter δ on the flow and temperature fields are shown. Due to exponential variation of permeability, we find quite new features in the velocity and temperature profiles.Velocity and temperature both steadily attain the ambient fluid conditions . Initially both show a decreasing trend with a decrease in permeability parameter and then velocity and temperature both increase.

Figure 2 shows the effects of Grashof number on the velocity and temperature profiles. Velocity, in the vicinity of the plate, first increases and attains a maximum and then starts decreasing and uniformly mixes with the ambient fluid. Due to exponential variation of permeability and thermal conductivity, we find here new patterns of variation in the velocity field. For Gr=4 and 6, we observe that there is a kind of oscillatory character in the velocity profile.

Temperature profile also shows the similar behaviour.

Figure 3 shows the effect of Eckert number Ec on the flow field. We find that an increase in the Eckert number has the decreasing effect on the velocity field. We have also considered the effect on the velocity and temperature when there is no N-B modification incorporated and the result is shown by dotted lines in the graphs.. It has the effects of decreasing the velocity and temperature profiles. Also, we have considered the case when there is no variation in the thermal conductivity and this result is shown by dashed lines .

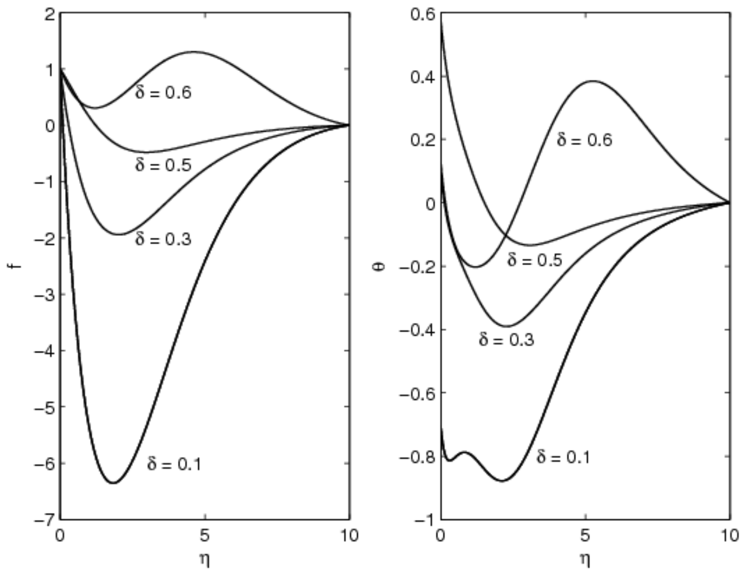

Figure 1

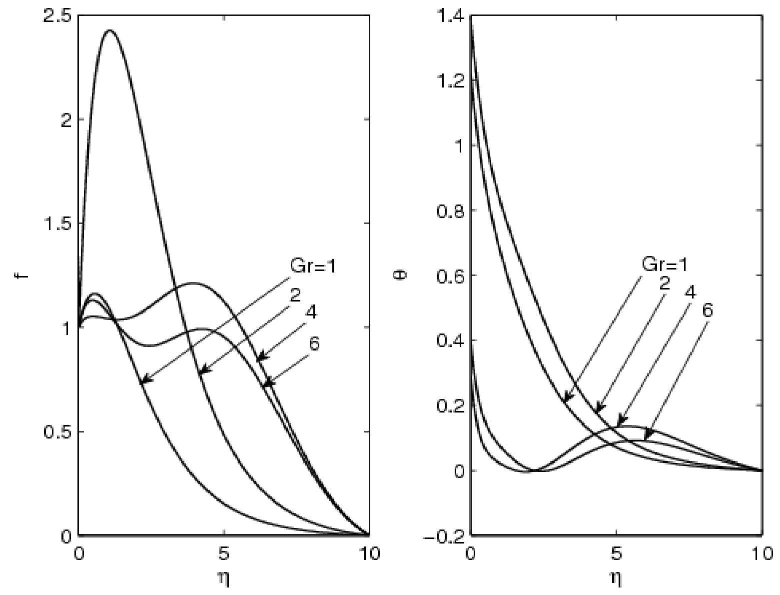

Figure-2

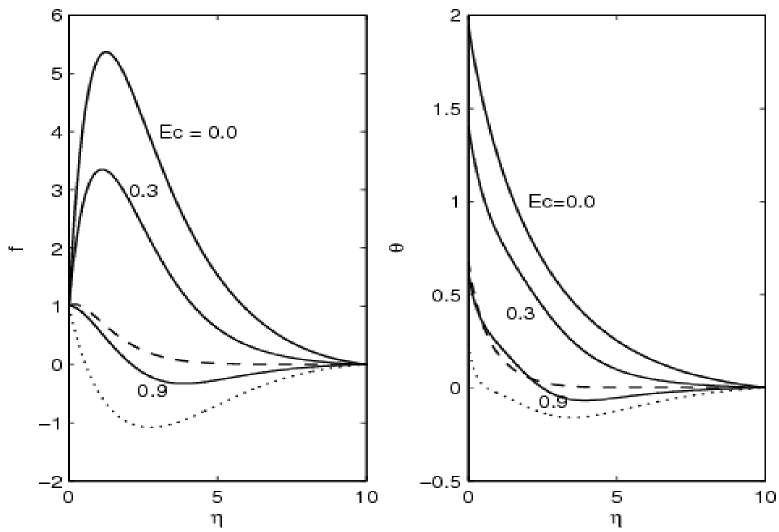

Figure-3

V. CONCLUSION

We observe that the permeability parameter and Grashof number both have quite significant effects on the velocity and temperature profiles. The effects of exponential variation of thermal conductivity and permeability are prominently visible in the graphs. Also, an increase in the Eckert number has the effect of decreasing the velocity and temperature both.

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