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Reduced Order Observer (DGO) based State Variable Design of Two Loop Lateral Missile Autopilot in Pitch Plane

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Abstract—*In this paper, first the transfer function and block diagram model of a flight path rate demand autopilot (two loop) in pitch plane has been shown and then its state model has been developed. Then state feedback controller has been designed. Thereafter both of Luenberger and Das & Ghosal Observers (DGO) are implemented for the above said two loop autopilot. Finally the system, observer and state feedback controller are integrated in one unit and a comparative study between Luenberger and Das & Ghosal observer is done. It will be shown that the observers are able to track the system states i.e. estimate the system states very quickly and with high degree of accuracy even if the initial states of the plant and observer are mismatched. It has also been established that addition of an observer (an auxiliary dynamic system) to the system does not impair the system stability; it only appends its own poles (Eigen values) with the original system poles.*

Keywords—Missile Autopilot (two loop and three loop), Angle of attack, Pitch/Yaw Motion, Flight path rate demand, Guidance and Control, Gyroscope, Accelerometers, Aerodynamic control, Luenberger Observer, Das & Ghosal Observer, Generalized Matrix Inverse.

I. INTRODUCTION

A lot of research has been done till date on missile autopilot and also published in literature. Both of Lateral and Roll autopilot are necessary for modern day's sophisticated missiles. Lateral autopilot can be implemented for yaw plane or pitch plane. It can be of two types depending on feedback characteristics – Two Loop and Three Loop lateral autopilot. In recent papers two and three loop autopilots have been designed by using frequency domain approach. Many other variants are also proposed. The current paper deals with the time domain design approach (in state space) of two loop missile autopilot in pitch plane. Reduced order Das & Ghosal observer has been implemented to estimate the immeasurable states of the autopilot and state feedback controller has been applied to make the autopilot stable and to get desired dynamic response. Pole placement is carried out by following Ackermann's policy. Numerical values are taken for Matlab simulation. Responses have been plotted and from the results it is established that both of frequency domain approach and state space approach of design are compatible with each other. Finally the well known and well used Luenberger observer is also implemented and by comparing the responses, it is revealed that both of Das & Ghosal observer and reduced order Luenberger observer are performing equally well.

II. AUTOPILOT

Autopilot is an automatic control mechanism for keeping the spacecraft in desired flight path. An Autopilot in a missile is a closed loop system and it is a minor loop inside the main guidance loop. If the missile carries accelerometers and/or rate gyros to provide additional feedback into the missile servos to modify the missile's course of motion then the flight control system i.e. the missile control system is usually called an Autopilot. When the autopilot controls the motion in the pitch or yaw plane, they are called Lateral Autopilot. For a symmetrical cruciform missile, pitch and yaw autopilots are identical. The guidance system detects whether the missile's position is too high or too low, or too much right or left. It measures the deviation or errors and sends signals to the control system to minimize the acceleration (latex) according to the demand from the guidance computer. For aerodynamically controlled skid to run missile, the autopilot activates to move the control-surfaces i.e. wings and fins suitably for orienting the missile body with respect to the desired flight path. This control action generates angle of attack and consequently the latex demand for steering the missile following the desired path. In this paper, such a lateral autopilot (Two Loop) has been designed in pitch plane using reduced order observer based state feedback controller.

III. OBSERVER

To implement state feedback control [control law is given by $u = r - \boldsymbol{K}\hat{\boldsymbol{x}} \ldots\ldots (3.1)$] by pole placement, all the state variables are required to be feedback. However, in many practical situations, all the states are not accessible for direct measurement and control purposes; only inputs and outputs can be used to drive a device whose outputs will approximate the state vector. This device (or computer program) is called State Observer. Intuitively the observer should have the similar state equations as the original system (i.e. plant) and design criterion should be to minimize the difference between the system output $y = C\boldsymbol{x}$ and the output $\hat{y} = C\hat{\boldsymbol{x}}$ as constructed by the observed state vector $\hat{\boldsymbol{x}}$. This is equivalent to minimization of $\boldsymbol{x} - \hat{\boldsymbol{x}}$. Since $\boldsymbol{x}$ is inaccessible, $y - \hat{y}$ is tried to be minimized. The difference $(y - \hat{y})$ is multiplied by a gain matrix (denoted by M) of proper dimension and feedback to the input of the observer. There are two well-known observers namely – Luenberger Observer (1964, 1971) and Das & Ghosal Observer (1981). The second one has some genuine advantages over the first one. Das & Ghosal Observer construction procedure is essentially based on the Generalized Matrix Inverse Theory and Linear space mathematics.

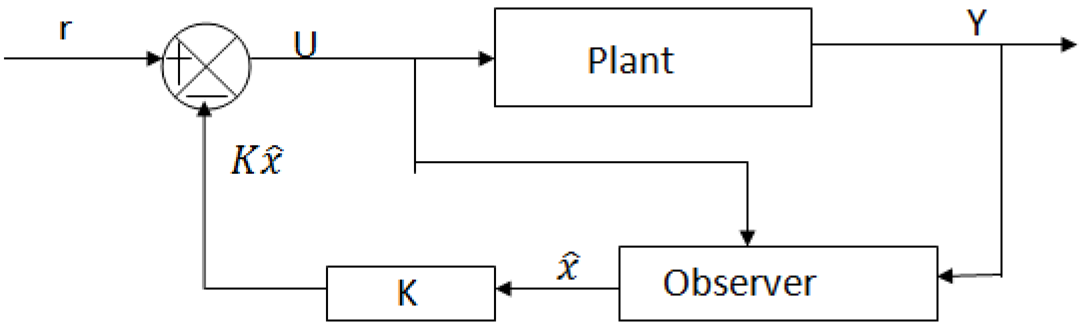

Fig 3.1: Observer General Block Diagram

IV. DEVELOPMENT OF TWO LOOP AUTOPILOT FROM THE CONVENTIONAL ONE

The following block diagram represents the transfer function model of flight path rate demand two loop autopilot in pitch plane **[2][3].**

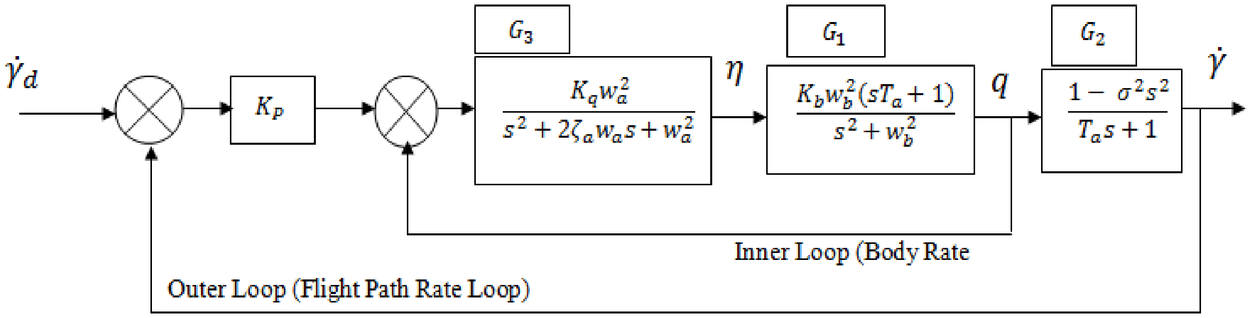

Fig 4.1: Conventional Two Loop Lateral Autopilot Transfer Function Model

The above transfer function model has been converted into its state variable equivalent model as presented in fig. (4.2).

G_1, G_2 are Aerodynamic Transfer Function while $G_3$is the Actuator Transfer Function.

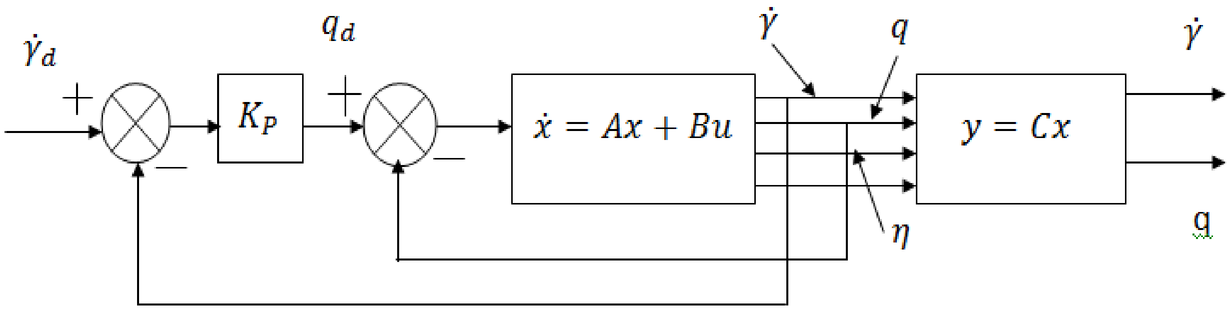

Fig 4.2: Two Loop Autopilot State space Model

Further the configuration shown in fig. 4.2 has been modified to take the form given in fig. (4.3).

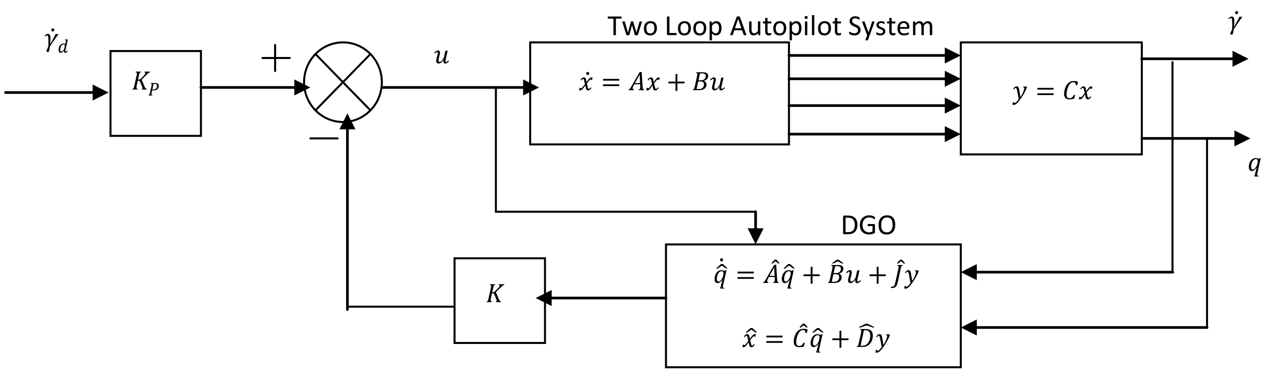

Fig 4.3: State Feedback Configuration of Classical Two Loop Autopilot with DGO

V. THE STATE SPACE MODEL OF TWO LOOP AUTOPILOT

The transfer function model of the two loop autopilot (fig. 4.2) can be easily converted to the corresponding state space model (fig. 4.3) which gives the following state equations based on the 4 states variables:

$\boldsymbol{x_1 = \dot{\gamma}}$ (***Flight path rate demand***);
$\boldsymbol{x_2 = q}$ (***pitch rate***);
$\boldsymbol{x_3 = \eta}$ (***elevator deflection***);
$\boldsymbol{x_4 = \dot{\eta}}$ (***rate of chance of elevator deflection***)

- Out of them x_1 *and* x_2 have been considered to be as outputs.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}-\frac{1}{T_a} & \frac{1+\sigma^2 w_b^2}{T_a} & -\frac{K_b\sigma^2 w_b^2}{T_a} & -K_b\sigma^2 w_b^2\\ -\frac{1+w_b^2T_a^2}{T_a(1+\sigma^2 w_b^2)} & \frac{1}{T_a} & K_b w_b^2 T_a-\frac{(1+w_b^2T_a^2)K_b\sigma^2 w_b^2}{T_a(1+\sigma^2 w_b^2)} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -w_a^2 & -2\zeta_a w_a\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ K_q w_a^2\end{bmatrix}u \ldots\ldots (5.1)$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}\ldots\ldots (5.2)$$

VI. REDUCED ORDER DAS & GHOSAL OBSERVER (DGO) APPLIED TO TWO LOOP AUTOPILOT

Reduced order Das and Ghosal observer **[1]** is governed by the following equations and conditions.

$x = C^g y + L\,h \ldots\ldots\ldots (6.1)$ (eqn. 13 of [1])

$\dot{h(t)} = L^g AL\,h(t) + L^g AC^g\,y(t) + L^g\,B\,u(t) \ldots\ldots\ldots (6.2)$ (eqn. 15 of [1])

$\dot{y} = CALh + CAC_g\,y + CB\,u \ldots\ldots (6.3)$ (eqn. 18 of [1])

$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g B - MCB)u + M\dot{y} \ldots\ldots (6.4)$ (eqn. 19 of [1])

$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)M\}y + (L^g B - MCB)u \ldots\ldots (6.5)$ (eqn. 20 of [1])

$where\ \hat{q} = \hat{h} - My \ldots\ldots (6.6)$ (Page-374 of [1])

$And\ \hat{x} = L\hat{q} + (C^g + LM)y \ldots\ldots (6.7)$ (eqn. 21 of [1])

VII. MATLAB SIMULATION AND RESULTS

The following numerical data for a class of missile have been taken for Matlab simulation:

$T_a = 0.36\,sec;\ \sigma^2 = 0.00029\ \mathrm{sec}^2;$
$w_b = 11.77\frac{rad}{sec};\ \zeta_a = 0.6;\ K_b = -10.6272 per\ sec;$
$v = 470\frac{m}{sec};\ K_p = 5.51;$
$K_q = -0.07;\ w_a = 180\frac{rad}{sec}; K_i = 22.02;$

Using these values, the state space model given by eqns. (5.1) & (5.2), becomes eqn. (7.1a & 7.1b)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}-2.77 & 2.77 & 0.0003207 & -0.0001154\\ -52.6488 & 2.77 & -494.2238 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -32400 & -216.00\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ -1944\end{bmatrix}u \ldots\ldots(7.1a) and$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}\ldots\ldots(7.1b)$$

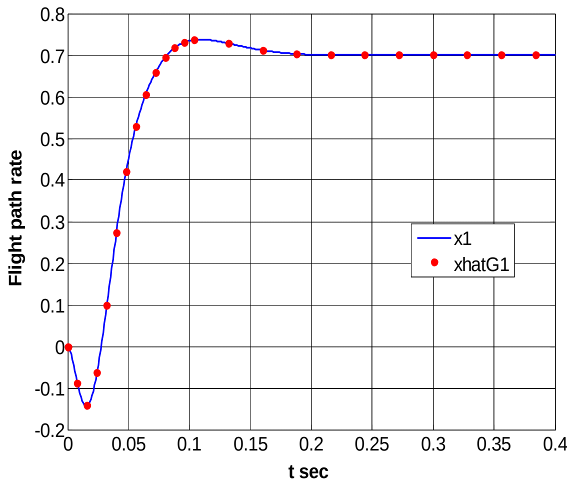

Figure 7.1: x1 is plant state and xhatG1 is estimated state

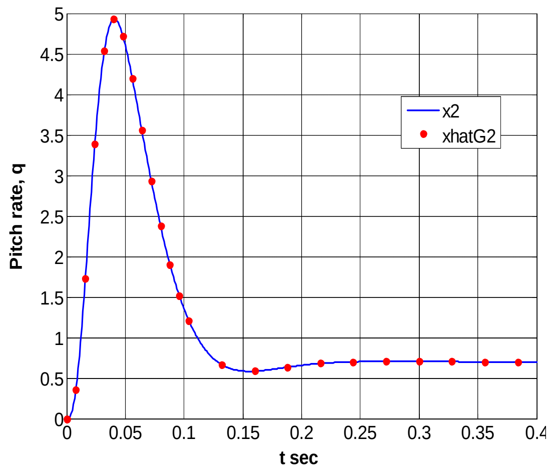

Figure 7.2: x2 is plant state and xhatG2 is estimated state

N.B. Blue continuous line in all the figures (responses) indicates the plant response obtained from state space model and the red dotted chain line in the above figures indicates the estimated states by Das & Ghosal Observer (DGO).

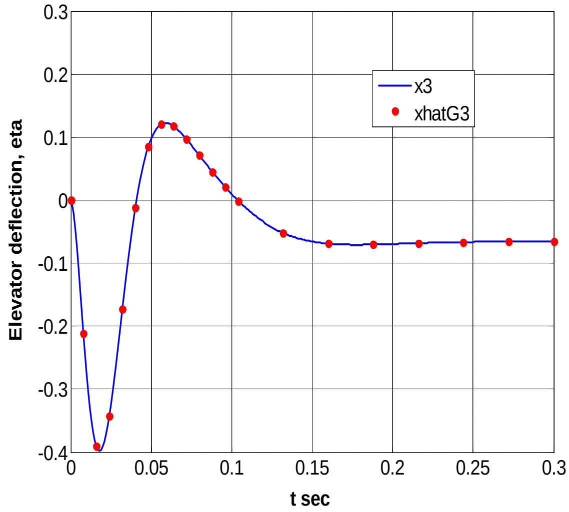

Figure 7.3: x3 is plant state and xhatG3 is estimated state

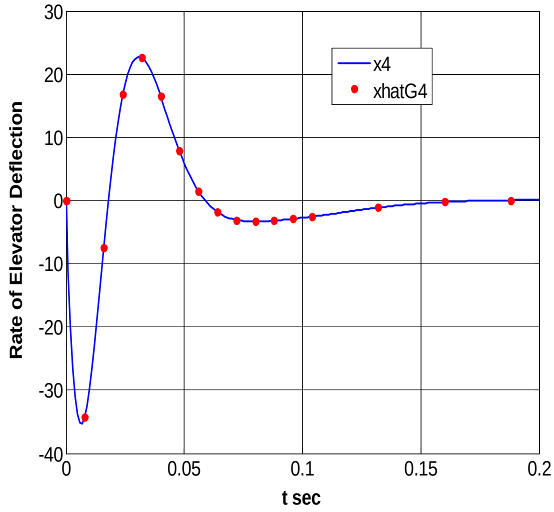

Figure 7.4: x4 is plant state and xhatG4 is estimated state

VIII. OBSERVATION AND DISCUSSION

In this paper, flight path rate demand two loop autopilot has been designed in state space model corresponding to the transfer function model given in literature **[2] & [3].** Flight path rate $\dot{\gamma}$ and the pitch rate q have been used as outputs. In practical missiles these are generally measured by gyros and accelerometers. Reduced order Das & Ghosal observer is applied to measure the other two states i.e. elevator deflection η and rate of change of elevator deflection $\dot{\eta}$. Finally four states have been fedback to input to implement state feedback control. It is seen from the simulation graphs that the original states (blue continuous line) obtained from transfer function model and state space model overlap with each other indicating that both the modeling schemes are compatible. It has also been established through the simulation that Das & Ghosal observer has successfully caught the system states within less than 0.02 seconds and without any steady state error or oscillations. Further the observation has also been carried out by using the very well known and well used Luenberger method **[4], [5], [6] & [7]** and it is seen that both of Luenberger and Das & Ghosal observer are giving exactly same dynamic performance (red dotted chain line indicates both of the observed states). So it can be inferred that Das & Ghosal observer is at par with reduced order Luenberger observer.

FUTURE SCOPE OF WORK

This design methodology of lateral autopilot can be extended to three loop lateral autopilot and roll autopilot also. Robustness study and parameter variations of the missile can be explored through this method.

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