

## Development of Mathematical Models for Various PQ Signals and Its Validation for Power Quality Analysis

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**Abstract**— PQ analysis for detecting power line disturbances in real-time is one of the major challenging techniques that are being explored by many researchers. In order to understand the causes of electronic equipment being damaged due to disturbances in power line, there is a need for a robust and reliable algorithm that can work online for PQ analysis. PQ signal analysis carried out using transformation techniques have its own advantages and limitations, In this work, Mathematical models for various PQ signal disturbances is developed and validated against real time signal. Based on the mathematical models developed, performances of various transformation techniques are compared. Software reference models for transformation techniques are developed using MATLAB. The developed models are verified using various signal disturbances.

**Keywords**— Power quality, Power system transients, discrete wavelet transforms. Lifting based wavelet transform.

### I. INTRODUCTION

In recent years power quality has become a significant issue for both utilities and customers. This power quality issue is primarily due to increase in use of solid state switching devices, non linear and power electronically switched loads, unbalanced power system, lighting controls, computer and data processing equipments as well as industrial plants rectifiers and inverters. PQ events such as sag, swell, transients, harmonics and flicker are the most common types of disturbances that occur in a power line. Sophisticated equipment's connected to the power line are prone to these disturbances and are affected or get damaged due to disturbances that randomly occur for very short durations. Thus, it is required to estimate the presence of disturbance, and accurately classify the disturbances and also characterize the disturbances. In order to perform the tasks such as detection, classification and characterization, it is required to understand the basic properties of PQ events and their properties. In this section, PQ events (like sag, swell, transients, harmonics and flicker) and their properties are summarized.

### II. REPRESENTATION OF PQ SIGNALS AND THEIR PROPERTIES

PQ signals such as sag, swell, transients, harmonics and flicker have some very basic properties in terms of signal amplitude and frequency as governed by standards ie 220 V and 50Hz signal. Variations in voltage levels and variations in frequencies lead to disturbances in PQ signal. Frequency variations in PQ signals introduce harmonics, random change in loads lead to transients, flicker, sag and swell. Figure.1 shows the different types of disturbances that occur in PQ signal.

From Figure 1 we define the disturbances as follows [1-4]:

Voltage variation: increase or reduction in the amplitude of the voltage normally caused by variations in the load. Voltage variation can be termed as swell or sag.

Voltage sag: sudden reduction in the rms voltage to a value between 0.9 pu and 1pu of the declared value (usually the nominal value) and duration between 10 ms and 1 min.

Voltage swells: sudden increase in the voltage to a value between 1.1 pu to 1.8 pu and duration between 10 ms to 1 min

Transient: oscillatory or non-oscillatory over voltage with a maximum duration of several milliseconds.

Flicker: intensity of the nuisance caused by the flicker, measured as defined by UIE-CEI and evaluated as:

-Short term flicker ( $P_{st}$ ) measured in 10 minutes period

-Long term flicker ( $P_{lt}$ ) defined as 
$$P_{lt} = \sqrt[3]{\sum_{i=1}^{12} \frac{P_{st}^3}{12}}$$

Harmonics: sinusoidal voltage with a frequency which is a multiple of the fundamental frequency mainly caused by non-linear loads.

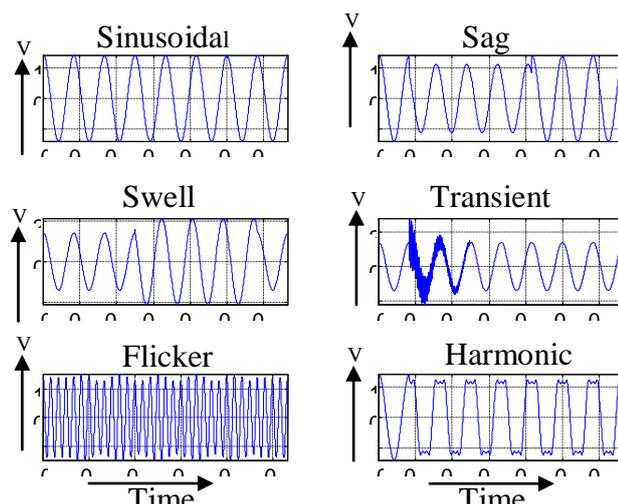


Figure.1 graphical representations of PQ signal disturbances

### III. MATHEMATICAL MODELING OF PQ SIGNALS

Verification of software reference models for PQ analysis requires PQ signals; which are generated using software programs. In order to represent the real time signals accurately, mathematical equations are used to generate PQ Synthetic signals. Synthetic signals generated need to have all the properties of a real-time signal and should match the real-time signal in all aspects. Performance of PQ analysis techniques is dependent of PQ signals, hence it is required that the synthetic signals which are generated using software programs are accurately modeled. Development of software programs to generate PQ signal disturbances are governed by standard definitions [5-7] for events as given in Table 1 is generated.

Table1. Mathematical Models of PQ Signals

PQ disturbances	Model	Parameters
Normal	$x(t)=\sin(\omega t)$	
swell	$x(t)=A(1+\alpha(u(t-t_1)-u(t-t_2)))\sin(\omega t)$ $t_1 < t_2, u(t) = 1, t \geq 0$ $0, t < 0$	$0.1 \leq \alpha \leq 0.8$ $T \leq t_2 - t_1 \leq 9T$
Sag	$x(t)=A(1-\alpha(u(t-t_1)-u(t-t_2)))\sin(\omega t)$	$0.1 \leq \alpha \leq 0.9; T \leq t_2 - t_1 \leq 9T$
Harmonic	$x(t)=A(\alpha_1 \sin(\omega t) + \alpha_3 \sin(3\omega t) + \alpha_5 \sin(5\omega t) + \alpha_7 \sin(7\omega t))$	$0.05 \leq \alpha_3 \leq 0.15,$ $0.05 \leq \alpha_5 \leq 0.15$ $0.05 \leq \alpha_7 \leq 0.15, \sum \alpha_i^2 = 1$
Outage	$x(t)=A(1-\alpha(u(t-t_1)-u(t-t_2)))\sin(\omega t)$	$0.9 \leq \alpha \leq 1; T \leq t_2 - t_1 \leq 9T$
Sag with harmonic	$x(t)=A(1-\alpha(u(t-t_1)-u(t-t_2)))$ $(\alpha_1 \sin(\omega t) + \alpha_3 \sin(3\omega t) + \alpha_5 \sin(5\omega t))$	$0.1 \leq \alpha \leq 0.9, T \leq t_2 - t_1 \leq 9T$ $0.05 \leq \alpha_3 \leq 0.15,$ $0.05 \leq \alpha_5 \leq 0.15;$ $\sum \alpha_i^2 = 1$
Swell with harmonic	$x(t)=A(1+\alpha(u(t-t_1)-u(t-t_2)))$ $(\alpha_1 \sin(\omega t) + \alpha_3 \sin(3\omega t) + \alpha_5 \sin(5\omega t))$	$0.1 \leq \alpha \leq 0.9, T \leq t_2 - t_1 \leq 9T$ $0.05 \leq \alpha_3 \leq 0.15,$ $0.05 \leq \alpha_5 \leq 0.15;$ $\sum \alpha_i^2 = 1$
Interruption	$x(t)=A(1-(u(t_2)-u(t_1)))\cos(\omega t)$	
Transient	$x(t)=A[ \cos(\omega t) + k \exp(-(t-t_1)/\tau) \cos(\omega_n(t-t_1))(u(t_2)-u(t_1))]$	$K=0.7$ $\tau=0.0015$ $\omega_n=2\pi f_n$ $900 \leq f_n \leq 1300$

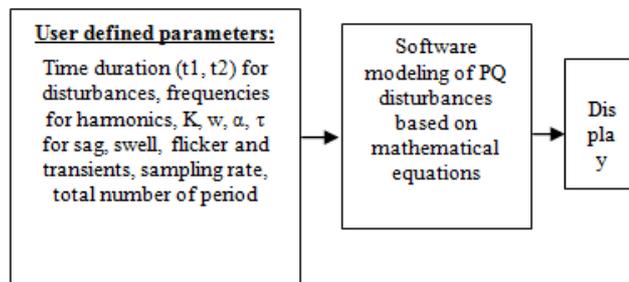
$x(t)$ - PQ signal, A-Amplitude(constant),  $\omega_n$ -angular frequency, t-time,  $\alpha_n$ -time duration of event occurrence (constant), T-time duration,  $f_n$ -frequency, K-constant,  $\tau$ -constant

Sag and swell events are modeled using step functions defined for periods between  $t_1$  and  $t_2$ , amplitude of step function is used to control the intensity of voltages thus defining sag and swell. Time intervals  $t_1$  and  $t_2$  define the duration of sag and swell. Harmonic signal consists of frequency components that are multiples of 1 KHz. In a PQ signal the disturbances in terms of harmonics will have more than three frequency components and are odd multiples of fundamental component. The odd harmonics also have voltage Variations and thus affect the fundamental signal. Parameters that define amplitudes of harmonic components given by  $a_1, a_2, a_3$  and  $a_4$  have variations from 0.05 to 0.15 and the square sum of all components should not exceed unit magnitude. These parameters are used to control the harmonic variation in a pure sine wave signal.

In signal disturbances that create exponential variations within a short duration of time interval are transients. Parameters such as  $\alpha, w$  and  $K$  control the variations in transients in a given pure sine wave as shown in table 1. Transients occur for a short time interval, controlled by  $t_1$  and  $t_2$ . During this time interval, the signal behaviors consists an exponential variation controlled by  $\tau$ . Flicker is due to randomness in sag and swell affecting pure sine wave. Mathematical models for flicker are similar to sag and swell, but the duration of sag and swell is randomly added to the pure sine wave signal.

#### IV. ALGORITHM FOR PQ SIGNAL GENERATION USING MATHEMATICAL MODELS

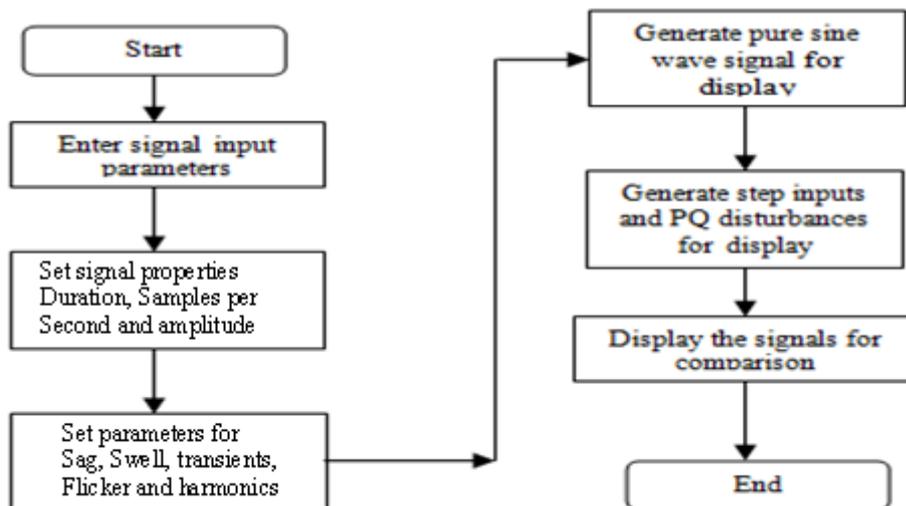
Based on the discussions and numerical equations presented in the previous section software programs are developed for modeling PQ signals with disturbances as shown in Figure 2. Control parameters for variations in distortions or events are defined as user inputs. The known disturbances are introduced in the signal; graphical display helps in verifying the PQ events.



*Figure 2:* Block diagram for signal generation using mathematical models

#### V. FLOW CHART FOR PQ SIGNAL DISTURBANCE GENERATION USING MATLAB

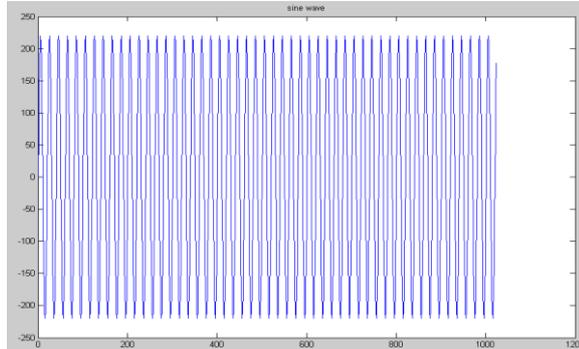
Figure 3 presents the flow chart for the PQ signal disturbances generation using MATLAB. PQ signal of 50 Hz for duration of 10 seconds with sampling rate of 1 kpsps consisting of 400,000 samples are generated. Generated signals are divided into frames of 1024 samples. The MATLAB program is simulated with the required parameters depending upon user requirements. The disturbances are introduced and generated signals are displayed for visualizations.



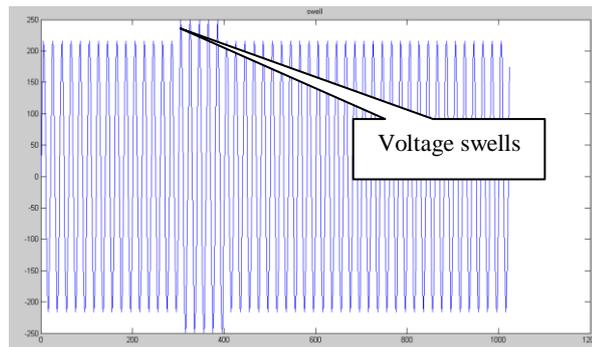
*Figure 3* Flow chart for PQ signal disturbance generation using Matlab

## VI. SIMULATION RESULTS OF PQ SIGNALS USING SOFTWARE PROGRAMS

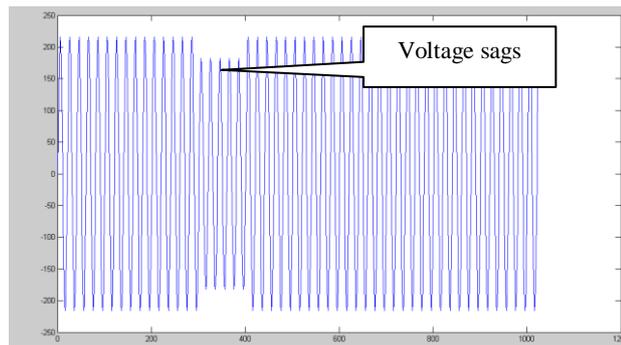
Software programs discussed in previous section is simulated using MATLAB, the results of the same is presented in this section. Pure sine wave, sag, swell, transients, harmonics and flickers signals are generated and displayed as shown in Figure 4(a-f). PQ signal with 50 Hz, 220 V peak is sampled at 1 Ksps, is introduced with disturbances and is plotted.



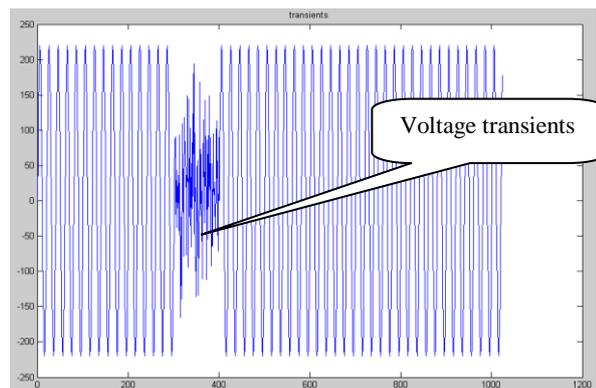
**Figure 4 (a):** sine wave



**Figure 4 (b):** Voltage swell



**Figure 4 (c):** Voltage sag



**Figure 4 (d):** Voltage transients

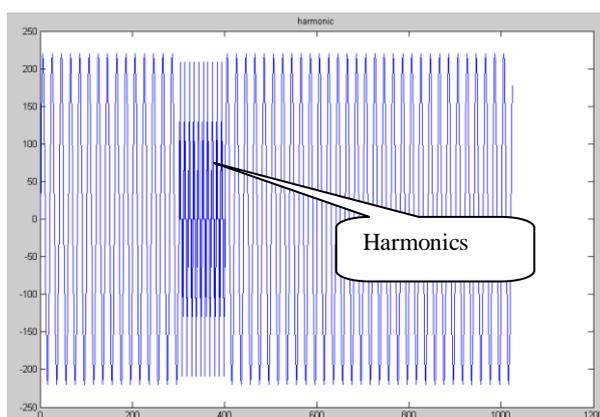


Figure 4 (e): Harmonics

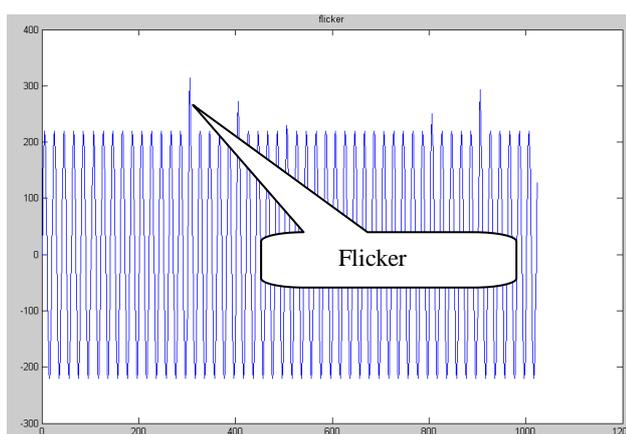


Figure 4(f): Flicker

Disturbances in PQ signal with variations in various parameters like amplitude and duration are generated and recorded for the analysis. Database of more than 120 signals are generated and are used for classification and characterization. Table 2 presents various test cases that have been generated using the software models and the generated signal is compared with real time signals.

## VII. REAL TIME GENERATION OF PQ EVENTS:

In order to validate the mathematical models that are used to generate PQ disturbances, it is required to verify the correctness of synthetically generated signal against real time signals. Hence it is required to generate and capture real time disturbances. Figure 5 shows the block diagram for the experimental set-up to acquire real time PQ disturbances for the analysis. Experimental set up consists of a power line connected to data acquisition system, loads such as resistive (lighting load or lamp), inductive load (induction motors) and both. Voltage variation across the power line is captured using the data acquisition system consisting of differential module, ADC to digitize the analog signals to 12-bit resolution and software for data acquisition and data processing. The DAQ (Data Acquisition) card (PCL818HG) [62] and driver software SNAP MASTER is used for data capture and AUTO SIGNAL ANALYSIS software for analysis.. The line voltage is captured from the distribution node, and conditioned to step down the voltage to  $\pm 10V$ . The stepped down voltage is converted to its digital equivalent using the DAQ card, the digital data is further processed using software modules. The processed data is displayed for visualization and information is extracted for analysis.

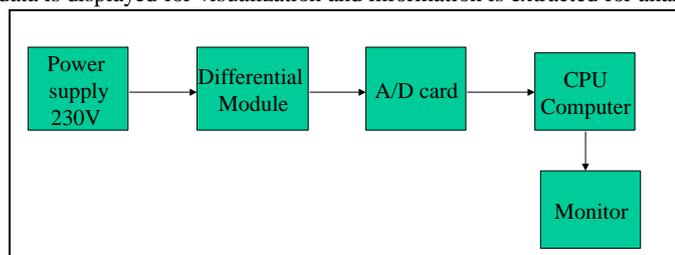


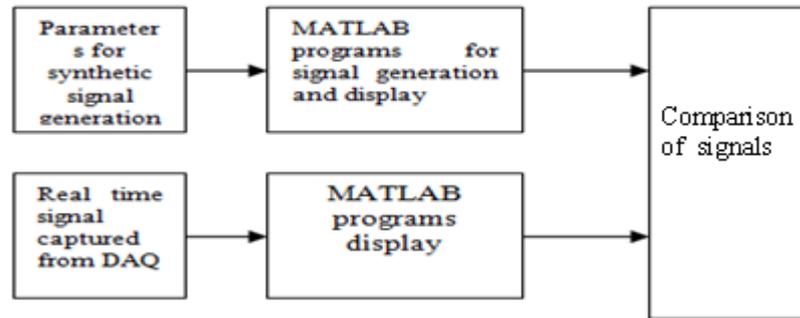
Figure 5: Block diagram for the experimental set-up

A GUI is developed to automate the experimental setup. This helps in selecting the instruments required for measurement. Types of ADCs, properties of ADCs, signal interfaces and display mode settings can be configured using

the GUI [63]. GUI also helps in defining sampling rate, duration of frame length, and number of frames that needs to be captured for data processing. The start duration and stop duration that can be set in the GUI helps in capturing the PQ signal at the required time for processing. Data captured using the data acquisition unit stores the captured data in ASCII format and \*.plt format is used for analysis. Loads are varied to introduce PQ disturbances and the captured data is taken for analysis. Variations in load such as resistive and inductive are randomly introduced and multiple experiments are conducted.

### VIII. VALIDATION OF MATHEMATICAL MODELS

In order to check the correctness of the developed mathematical models and it is required to validate the generated synthetic signal against real-time signals. In this work, an experimental setup is developed to generate real-time signal and disturbances are introduced by varying different loads. The data acquisition unit developed captures the signal along with the disturbances. The recorded signal is stored in a text file. The MATLAB model developed reads the text file and the contents are plotted. The software program developed based on mathematical model is used to generate similar kind (real-time) of signal by varying the input parameters to the software model. The output of both the software model and the real-time signal are compared and modifying the mathematical models minimizes the error between them. Various test cases are captured using the real-time experimental setup, which are used to verify the synthetic signal generated using software programs. The synthetic signal is validated against real-time signal and is used for PQ analysis. Figure 6 shows the experimental set up for PQ signal validation.



**Figure 6:** Experimental setup for PQ signal validation

Comparison or equivalence check is carried out to compare the similarities between real time PQ signal and synthetic PQ signal. Synthetically generated PQ signal is compared with the results of real time signals. In order to compare the signals Mean Square Error (MSE) [64] is computed between the real time signal and the synthetic signal as in equation

$$MSE = \frac{1}{Q} \sum_{k=1}^Q e(k)^2 = \frac{1}{Q} \sum_{k=1}^Q (t(k) - a(k))^2$$

$t(k)$  is the real time signal samples that are compared with  $a(k)$  which is the generated synthetic signal,  $Q$  is number of samples. The difference in samples values are squared and accumulated to compute MSE. Real time signals generated for various disturbances is compared with synthetic signal, the MSE is verified, MSE is well above threshold value of 2%, mathematical model presented in Table 1 is modified to minimize the MSE well below the threshold.

### IX. SUMMARY OF VALIDATIONS

Real time signals generated consisting PQ disturbances are compared with the results of synthetic signals. For validation purpose, with the experimental setup shown in Figure 6 known disturbance is introduced into the real time signal by changing the load signals (resistive and inductive). Variations observed due to load variations or PQ events or disturbances that are captured are noted and the same set of parameters is applied to generate synthetic signals. MSE is computed between the synthetic and real time signal. Voltage sags and swells intensities and duration of occurrence is noted from real time results, the same parameters are set in the synthetic model. Similarly all other events are generated based on the know set of input parameters obtained from real time signal. The results of the same are presented in Table 2

Iteration	PQ disturbance		MSE (real time, synthetic signal) In %	Remarks
	Event	Real time parameters		
1	Sag	0.19 ms 211 V	12%	Constant value of a needs to be matched appropriately
2	Swell	0.2 ms 232 V	8%	Constant value of a needs to be matched appropriately
3	Transients	0.5 ms	7%	Time duration and noise dB levels to be accurately selected
4	Harmonics	0.17 ms 0.15, 0.12, 0.2, 0.1	14%	Frequencies are not only odd multiples but also even multiples
5	Flicker	0.01 ms, 0.03 ms, 0.10 ms, 0.12ms	16%	Time duration needs to accurately chosen

Table 2 Real time signal validation results

From the results obtained it is observed that MSE is greater than the required threshold level which is 2 %, this variations is due to the fact that the mathematical models that have been tabulated in Table 3 does not accurately model the real time signals. In order to minimize the error, it is required to appropriately update the mathematical model for PQ signal generation. The major limitation is that the real time signal parameters obtained have to be appropriately introduced in to the mathematical model i.e need to choose appropriate time interval, intensity values for events and select appropriate harmonics levels. The modified mathematical models that match with the real time signals are presented in Table 3

PQ disturbances	Model Parameters and mathematical models	Modified mathematical models and parameters
Normal	$x(t)=\sin(\omega t)$	$x(t)=\sin(\omega t)$
Swell	$x(t)=A(1+\alpha(u(t-t_1)-u(t-t_2)))\sin(\omega t)$ $0.1 \leq \alpha \leq 0.8$ $T \leq t_2 - t_1 \leq 9T$	$x(t)=A(1+\alpha(u(t-n*t_1)-u(t-m*t_2)))\sin(\omega t)$ $0.1 \leq \alpha \leq 0.8$ $T \leq t_2 - t_1 \leq 9T$
Sag	$x(t)=A(1-\alpha(u(t-t_1)-u(t-t_2)))\sin(\omega t)$ $0.1 \leq \alpha \leq 0.9; T \leq t_2 - t_1 \leq 9T$	$x(t)=A(1-\alpha(0.2*u(t-n*t_1)-u(t-m*t_2)))\sin(\omega t)$ $0.1 \leq \alpha \leq 0.9; T \leq t_2 - t_1 \leq 9T$
Harmonic	$x(t)=A(\alpha_1 \sin(\omega t) + \alpha_3 \sin(3\omega t) + \alpha_5 \sin(5\omega t) + \alpha_7 \sin(7\omega t))$ $0.05 \leq \alpha_3 \leq 0.15, 0.05 \leq \alpha_5 \leq 0.15$ $0.05 \leq \alpha_7 \leq 0.15, \sum \alpha_i^2 = 1$	$x(t)=A(\alpha_1 \sin(\omega t) + \alpha_3 \sin(3\omega t) + \alpha_5 \sin(5\omega t) + \alpha_7 \sin(7\omega t) + \alpha_4 \sin(4\omega t))$ $0.05 \leq \alpha_3 \leq 0.17, 0.05 \leq \alpha_5 \leq 0.25, 0.05 \leq \alpha_4 \leq 0.12$ $0.05 \leq \alpha_7 \leq 0.35, \sum \alpha_i^2 = 1$
Transient	$x(t)=A[\cos(\omega t) + k \exp(-(t-t_1)/\tau) \cos(\omega_n(t-t_1))(u(t_2)-u(t_1))]$ $K=0.7 \tau=0.0015$ $\omega_n=2 \pi f_n$ $900 \leq f_n \leq 1300$	$x(t)=A[\cos(\omega t) + k \exp(-(t-t_1)/\tau) \cos(\omega_n(t-n*t_1))(u(m*t_2)-u(n*t_1))]$ $K=0.7 \tau=0.0015$ $\omega_n=2 \pi f_n$ $900 \leq f_n \leq 1300$
x(t)- PQ signal, A-Amplitude(constant), $\omega_n$ -angular frequency, t-time, $\alpha_n$ -time duration of event occurrence (constant), T-time duration, $f_n$ -frequency, K-constant, $\tau$ -constant		

Table 3 Modified mathematical models

The modifications are that the time periods of event disturbances are accurately controlled by  $n$  and  $m$  constants. The intensities of events are also modified by appropriately choosing the constants. The modified equations are modeled using MATLAB software constructs and the results generated are compared with real time signals after several iterations, the MSE was brought below 2% threshold. Thus the mathematical models that have been modified are matching the real time PQ event disturbances and thus are used for analysis.

## X. CONCLUSION

In this paper mathematical model for PQ signal generation is developed and is validated against real time PQ signal. PQ events such as sag, swell, transient, and harmonics are generated using the mathematical model developed. Real time signals generated consisting PQ disturbances are compared with the results of synthetic signals. MSE is computed between the synthetic and real time signal. Voltage sags and swells intensities and duration of occurrence is noted from real time results, the same parameters are set in the synthetic model. From the results obtained it is observed that MSE is greater than the required threshold level which is 2%. In order to minimize the error, it is required to appropriately update the mathematical model for PQ signal generation.

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